EconS 305 - Intermediate Microeconomics without Calculus
Homework Set 3 Solutions

Assignment 3-1

1. Consider the following market with two goods, x and z. The consumer's utility function is

$$\bar{U} = x^{0.2}z^{0.8}$$

   (a) Derive the demand curves for x and z (Remember to include p_x, p_z and Y in there as unknowns. Look at the last example if you need help.)

   First, we need to calculate the marginal rate of substitution, which we do by applying our formula,

$$MRS = -\frac{MU_x}{MU_z} = -\frac{0.2x^{-0.8}z^{0.8}}{0.8x^{0.2}z^{-0.2}} = -\frac{z}{4x}$$

   Also, we need the marginal rate of transformation, which is just the ratio of prices

$$MRT = -\frac{p_x}{p_z}$$

   Our tangency point will occur when the marginal rate of substitution equals the marginal rate of transformation

$$\begin{aligned} MRS &= MRT \\ -\frac{z}{4x} &= -\frac{p_x}{p_z} \end{aligned}$$

   Rearranging,

$$4p_x x - p_z z = 0$$

   Now, we use this equation and the budget constraint,

$$p_x x + p_z z = Y$$

   to find a solution. Adding the two equations together

$$\begin{aligned} 4p_x x - p_z z + (p_x x + p_z z) &= 0 + Y \\ 5p_x x &= Y \\ x^* &= \frac{Y}{5p_x} \end{aligned}$$

   which is the demand curve for x. To find the demand curve for z, we substitute x^* back into our tangency point

$$4p_x\left(\frac{Y}{5p_x}\right) - p_z z = 0$$

   Solving for z, we have our demand curve

$$z^* = \frac{4Y}{5p_z}$$

(b) Let $p_x = 2$, $p_z = 4$ and $Y = 50$. Find the equilibrium quantities demanded of x and z.

All we need to do is plug these values into our demand curves to obtain a solution

$$x^* = \frac{50}{5(2)} = 5 \qquad z^* = \frac{4(50)}{5(4)} = 10$$

Intuitively, the utility function tells us that the consumer values good z much more than good x. If the prices were equal, the consumer would consume 4 times as much of good z than good x, as reflected in the marginal rate of substitution. However, since the price of good z is double that of good x, the consumer only consumes twice as much good z.

Assignment 3-2

1. Return to the same utility function we analyzed in assignment 3-1.

$$\bar{U} = x^{0.2}z^{0.8}$$

(a) Holding prices constant at $p_x = 2$ and $p_z = 4$, pick three values for income and plot the optimal quantities of x and z on a figure. Use these points to derive the wealth expansion path graphically.

Using our demand functions from assignment 3-1,

$$x^* = \frac{Y}{5p_x} \qquad z^* = \frac{4Y}{5p_z}$$

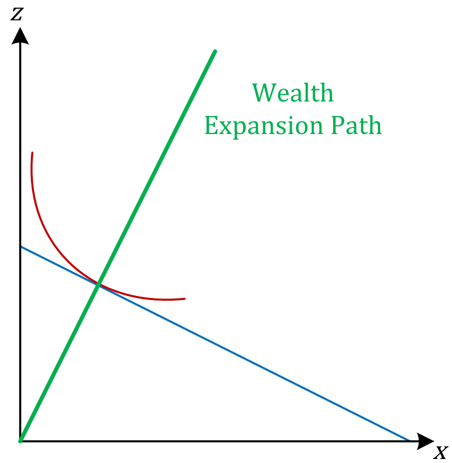
and our given prices, they reduce to

$$x^* = \frac{Y}{5(2)} = \frac{Y}{10} \qquad z^* = \frac{4Y}{5(4)} = \frac{Y}{5}$$

Now, we can pick any 3 values of Y (Any value is valid). I have included several,

Y	x^*	z^*
5	0.5	1
10	1	2
15	1.5	3
20	2	4
25	2.5	5

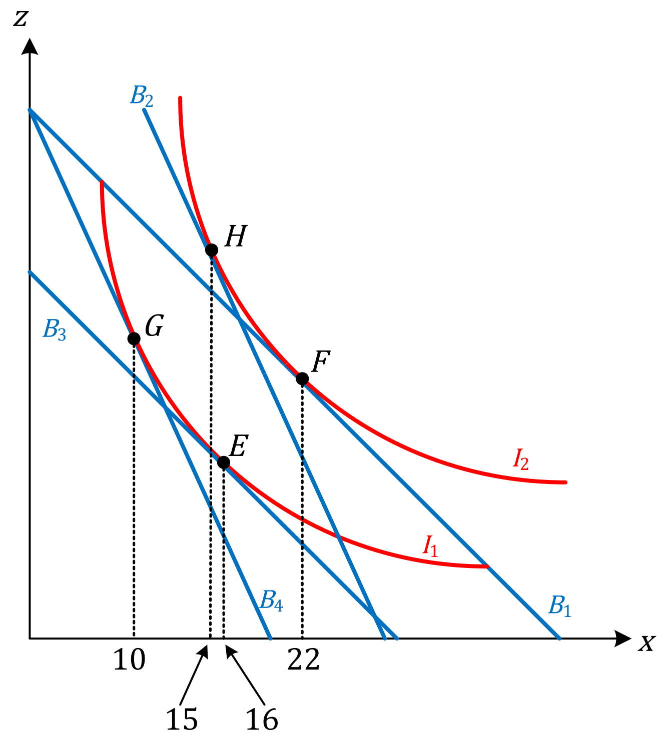
From here, a simple plot will give the wealth expansion path



Assignment 3-3

I have put a collection of budget lines, pseudo budget lines and indifference curves in the figure below. In each of the below scenarios, I want you to identify the following items.

- Whether it is a price increase or decrease. Explain what on the figure tells you this.
- Which curve is the pseudo budget line. Explain what on the figure tells you this.
- The total effect. (the answer for this and the next two should all be numbers)
- The substitution effect.
- The income effect.



a. The initial equilibrium is at point F and the new equilibrium is at point G

- It is a price increase. The initial budget line is B_1 and it rotates in to B_4.
- The pseudo budget line is B_2 It is parallel to the new budget line, but tangent to the old utility.
- The total effect is $10 - 22 = -12$
- The substitution effect is $15 - 22 = -7$
- The income effect is $10 - 15 = -5$

b. The initial equilibrium is at point G and the new equilibrium is at point F

- It is a price decrease. The initial budget line is B_4 and it rotates in to B_1.
- The pseudo budget line is B_3 It is parallel to the new budget line, but tangent to the old utility.
- The total effect is $22 - 10 = 12$
- The substitution effect is $16 - 10 = 6$
- The income effect is $22 - 16 = 6$

Assignment 3-4 and 3-5

1. Return to the cost of living example with Jesse relocating to Mexico.

   (a) Calculate Jesse's intermediate bundle.
   Recall Jesse's demand functions

   $$x^* = \frac{Y}{2p_x} \qquad z^* = \frac{Y}{2p_z}$$

   We calculated the income that jesse would need in Mexico to be at his original utility level $Y_C = 141.42$. Plugging this income level in with the new prices, we obtain the intermediate bundle.

   $$x_C^* = \frac{141.42}{2(10)} = 7.07 \qquad z_C^* = \frac{141.42}{2(10)} = 7.07$$

   (b) Calculate the total effect.
   Recall the initial and final bundles for Jesse

   $$\begin{aligned} x_A^* &= 10 \qquad z_A^* = 5 \\ x_B^* &= 5 \qquad z_B^* = 5 \end{aligned}$$

   The total effects are

   $$\begin{aligned} x_B^* - x_A^* &= 5 - 10 = -5 \\ z_B^* - z_A^* &= 5 - 5 = 0 \end{aligned}$$

   (c) Calculate the substitution effect.
   The substitution effects are

   $$\begin{aligned} x_C^* - x_A^* &= 7.07 - 10 = -2.93 \\ z_C^* - z_A^* &= 7.07 - 5 = 2.07 \end{aligned}$$

   (d) Calculate the income effect.
   The income effects are

   $$\begin{aligned} x_B^* - x_C^* &= 5 - 7.07 = -2.07 \\ z_B^* - z_C^* &= 5 - 7.07 = -2.07 \end{aligned}$$