EconS 425 - Third-Degree Price Discrimination

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Industrial Organization

Introduction

- Today, we'll finish our discussion of first-degree price discrimination with a look at block pricing.
- Then we'll move on to third-degree price discrimination.

Block Pricing

- Last time, we looked at two-part pricing, or as I like to call it, the "Costco model."
  - Firms set an access fee, or fixed price for the right to purchase from the monopolist, and then just charge a price of marginal cost per unit sold.
  - By setting the fixed price equal to the consumer surplus under perfect competition, the monopolist can capture all of the surplus for themself.
- Do you see any problem with this?

Block Pricing

- Consider two college best friends, Nathaniel and Wade. They both just graduated and are sharing an apartment while looking for jobs in their degree field (neither of them majored in economics).
  - They both love to eat chimichangas, with Wade liking them a bit more than Nate.
  - Their local chimichanga stand knows this, and has implemented a two-part pricing scheme that charges Wade a much higher access fee than Nate.
- What can Nate and Wade do?

Block Pricing

- Nate and Wade could just have Nate go to the chimichanga stand, pay his access fee, then buy enough chimichangas for both himself and Wade.
    - In fact, Wade could even give Nate some money to cover part of Nate's access fee.
- The monopolist loses more than half of its profits, while Nate and Wade get quite a bit of consumer surplus (in total among both of them).
    - And chimichangas.

Block Pricing

- Two-part pricing, by design, cannot prevent arbitrage.
  - A consumer with a lower valuation can simply buy enough for everyone else.
- This is why we need another method: block pricing.
  - In block pricing, the firm restricts the quantity sold to the consumer. Rather than letting a consumer pay an access fee and a per-unit cost, the firm charges a total fee $T(x, q)$ to consumer x for a specific quantity q.

Block Pricing

- The firm can do this because they know each consumer's willingness to pay $v(x, q)$.
  - From this information, they can infer how much each consumer would buy in a perfectly competitive market, q^C.
- Then they just set the total fee equal to the consumer's valuation for the perfectly competitive quantity,

$$T(x, q^C) = v(x, q^C)$$

Block Pricing

- Now the consumers pay exactly the same amount as they would have under two-part pricing while purchasing the same amount.
  - They simply can't choose how much they want to purchase.
- As before, this is an economically efficient arrangement, as everyone who should be in the market is in the market.
  - Just a few more calculations.

Third-Degree Price Discrimination

- Third-degree price discrimination, also known as group pricing, can be utilized by a firm when they know that their market can be split into two or more groups, and they can also identify which consumers belong to which groups.
    - These groups are usually easily identifiable characteristics, like age, gender, etc.
- As long as the groups have different demand functions, the firm can charge different prices to each group to extract more profits.

Third-Degree Price Discrimination

- Fortunately, third-degree price discrimination is fairly straightforward.
  - If a firm can segregate their market into subgroups, then they can treat each subgroup as if they were a separate market.
- Consider a case where there are n different identifiable subgroups within a market. The firm's profit maximization problem is

$$\max_{q_1,q_2,\ldots q_n} \underbrace{p_1 q_1 - c(q_1)}_{\text{Profit from subgroup 1}} + \underbrace{p_2 q_2 - c(q_2)}_{\text{Profit from subgroup 2}} + \ldots + \underbrace{p_n q_n - c(q_n)}_{\text{Profit from subgroup } n}$$

$$= \max_{q_1,q_2,\ldots q_n} \sum_{i=1}^{n} p_i q_i - c(q_i)$$

Third-Degree Price Discrimination

$$= \max_{q_1, q_2, \ldots q_n} \sum_{i=1}^{n} p_i q_i - c(q_i)$$

- From here, we can take first-order conditions, and we'll end up with n equations and n unknowns, with i from 1 to n,

$$\frac{\partial \pi}{\partial q_i} = p_i'(q_i) q_i + p_i(q_i) - c'(q_i) = 0$$

  where $p_i(q_i)$ is the inverse demand function for subgroup i.
- The nice thing is that none of the first-order conditions overlap. What we end up is an equation that is virtually identical to a monopolist's problem.

Third-Degree Price Discrimination

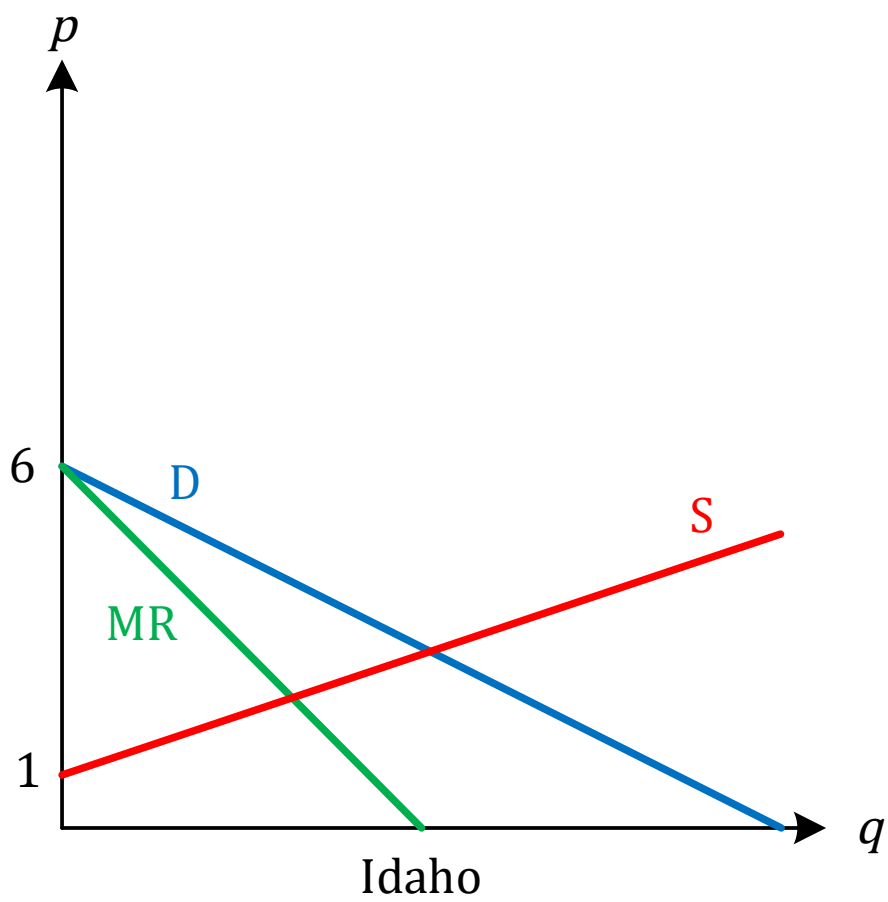
- You are a manager for a ski resort located in northern Idaho. Your boss has come to you with new market research data that indicates that your market demand function can be broken into two individual demand functions: the demand for people from Idaho, and the demand for people from the rest of the world.
- The inverse demand functions are as follows:

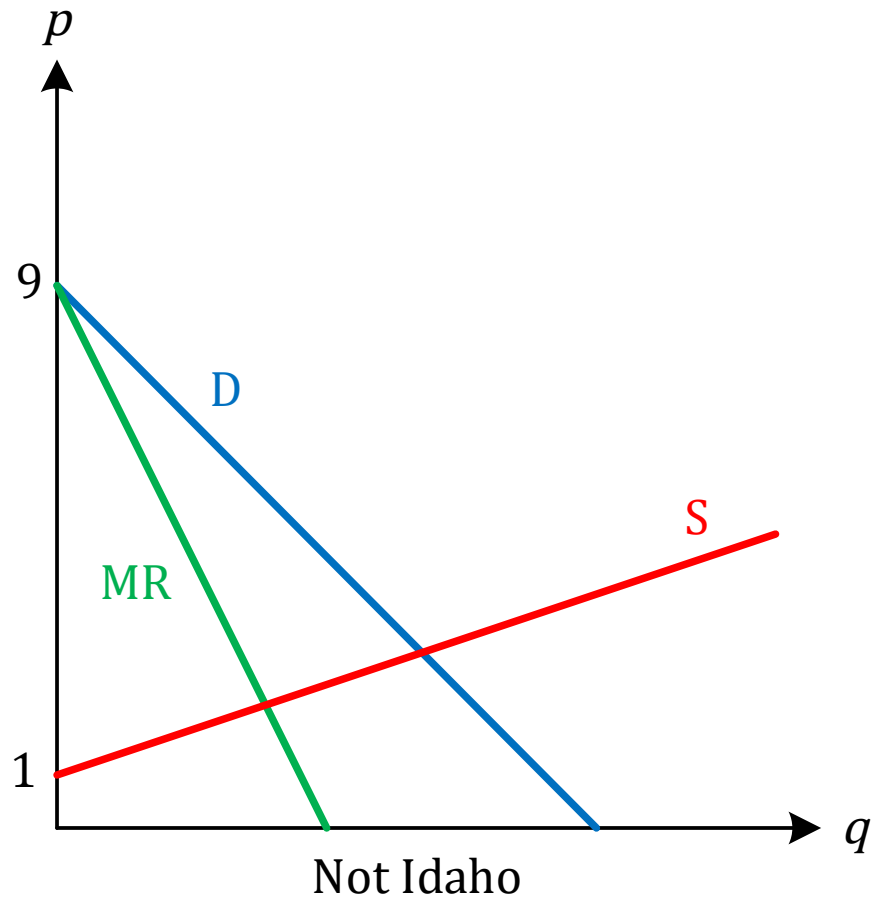
$$\begin{aligned} \text{Idaho:} \qquad & p = 6 - \frac{1}{2}q \\ \text{Not Idaho:} \qquad & p = 9 - q \end{aligned}$$

and the total cost function is:

$$TC = q + \frac{1}{6}q^2$$

Third-Degree Price Discrimination





Third-Degree Price Discrimination

$$\begin{aligned} \text{Idaho:} \qquad & p = 6 - \frac{1}{2}q \\ \text{Not Idaho:} \qquad & p = 9 - q \end{aligned}$$

- If we could only charge one price to this market, we would need to know the aggregate inverse demand function. First, we need to convert these two inverse demands back to demand functions,

$$\begin{aligned} \text{Idaho:} \qquad & q = 12 - 2p \\ \text{Not Idaho:} \qquad & q = 9 - p \end{aligned}$$

then add them to obtain

$$Q = 12 - 2p + (9 - p) = 21 - 3p$$

Third-Degree Price Discrimination

$$Q = 21 - 3p$$

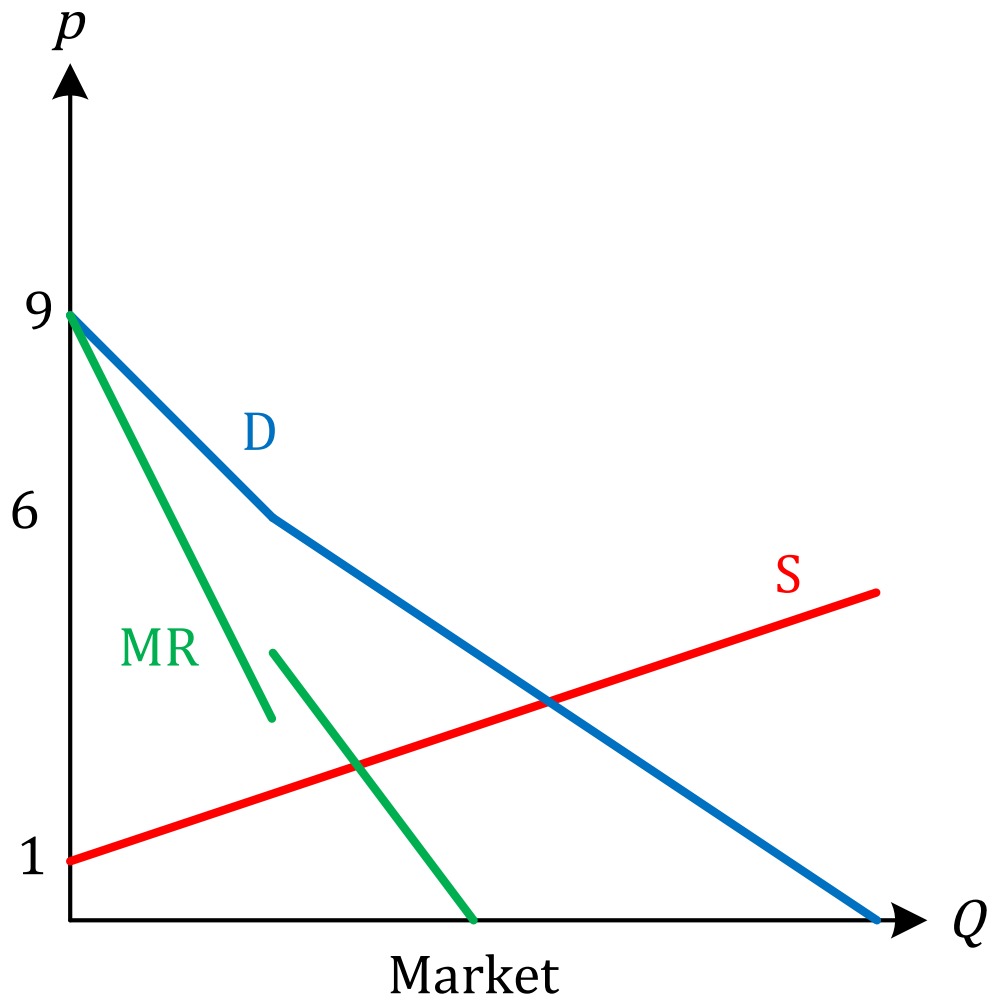
- From here, we convert back to an aggregate inverse demand function to obtain,

$$p = 7 - \frac{1}{3}Q$$

- Note: when $p \geq 6$, the Idaho group will not purchase any goods. Thus, our piecewise inverse aggregate demand function is

$$p = \begin{cases} 9 - Q & \text{if} \quad p \geq 6 \\ 7 - \frac{1}{3}Q & \text{if} \quad p < 6 \end{cases}$$

Third-Degree Price Discrimination



Third-Degree Price Discrimination

- Visually, it appears that our solution exists where both subgroups of consumers are served ($p < 6$).
  - Note: this may not always be the case. When a firm can only charge one price, it might be more profitable to cut out a segment of the market.
- Thus, we'll use that segment of the aggregate inverse demand function in our profit maximization problem,

$$\max_Q \quad \left(7 - \frac{1}{3}Q\right)Q - \left(q + \frac{1}{6}q^2\right)$$

with first-order condition,

$$\frac{\partial \pi}{\partial Q} = 7 - \frac{2}{3}Q - 1 - \frac{1}{3}Q = 0$$

Third-Degree Price Discrimination

$$\frac{\partial \pi}{\partial Q} = 7 - \frac{2}{3}Q - 1 - \frac{1}{3}Q = 0$$

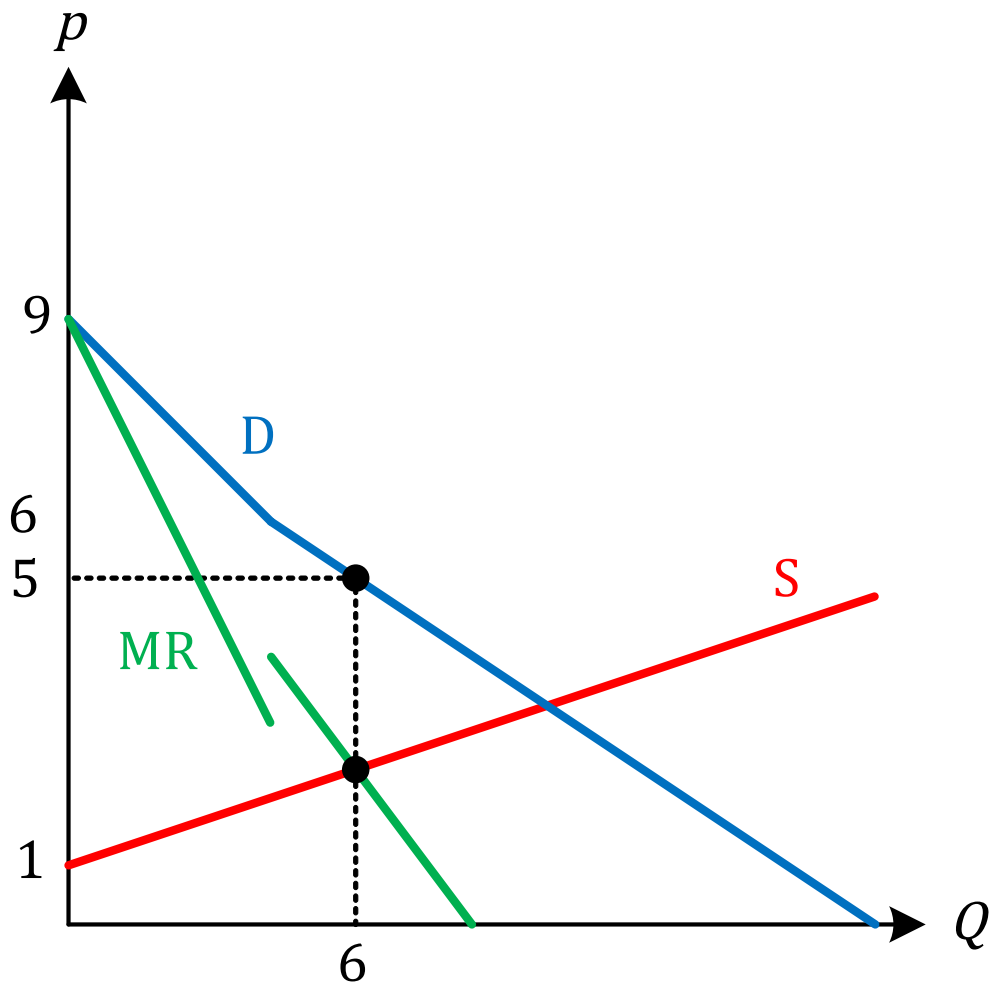
- Rearranging terms,

$$Q^* = 6$$

and plugging this back into the aggregate inverse demand function gives us our market price,

$$p^* = 7 - \frac{1}{3}Q^* = 5$$

Third-Degree Price Discrimination



Third-Degree Price Discrimination

- Now, let's look at what happens when we can price discriminate. Let Idaho residents be subgroup 1 while non-Idaho residents are subgroup 2. The firm's maximization problem is now

$$\max_{q_1,q_2} \quad \left(6-\frac{1}{2}q_1\right)q_1-\left(q+\frac{1}{6}q_1^2\right)+(9-q_2)q_2-\left(q+\frac{1}{6}q_2^2\right)$$

with first-order conditions,

$$\begin{aligned}\frac{\partial\pi}{\partial q_1} &= 6-q_1-1-\frac{1}{3}q_1=0\\ \frac{\partial\pi}{\partial q_2} &= 9-2q_2-1-\frac{1}{3}q_2=0\end{aligned}$$

Third-Degree Price Discrimination

$$\begin{aligned}\frac{\partial \pi}{\partial q_1} &= 6 - q_1 - 1 - \frac{1}{3}q_1 = 0 \\ \frac{\partial \pi}{\partial q_2} &= 9 - 2q_2 - 1 - \frac{1}{3}q_2 = 0\end{aligned}$$

- We can solve both of these first-order conditions individually. Starting with the first, rearranging terms,

$$\begin{aligned}\frac{4}{3}q_1 &= 5 \\ q_1^* &= \frac{15}{4}\end{aligned}$$

- And the second, rearranging terms,

$$\begin{aligned}\frac{7}{3}q_2 &= 8 \\ q_2^* &= \frac{24}{7}\end{aligned}$$

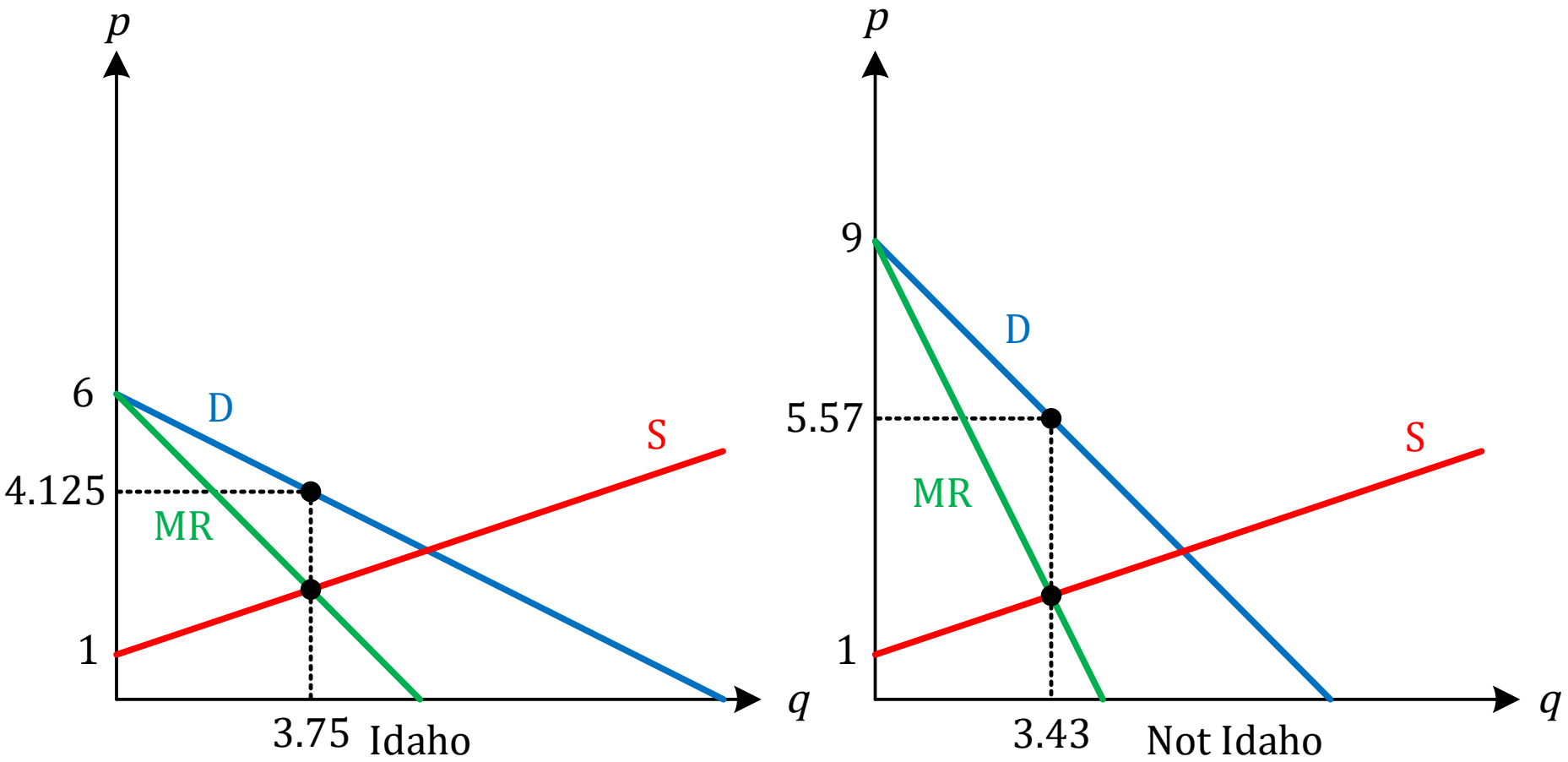
Third-Degree Price Discrimination

$$q_1^* = \frac{15}{4} = 3.75 \qquad q_2^* = \frac{24}{7} \approx 3.43$$

- Plugging these back into their respective inverse demand functions yield the group prices,

$$\begin{aligned} p_1^* &= 6 - \frac{1}{2}q_1^* = 6 - \frac{1}{2}\left(\frac{15}{4}\right) = \frac{33}{8} = 4.125 \\ p_2^* &= 9 - q_2^* = 9 - \frac{24}{7} = \frac{39}{7} \approx 5.57 \end{aligned}$$

Third-Degree Price Discrimination



Third-Degree Price Discrimination

- Comparing our results before and after implementing price discrimination,

	Idaho - U	Not Idaho - U	Idaho - NU	Not Idaho - NU
p	5	5	4.125	5.57
q	2	4	3.75	3.43
π	6	12	8.41	10.39

- The main thing to notice is that the Idaho residents are charged a lower price while the Not Idaho residents are charged a higher price relative to uniform pricing.
  - This matches up with their relative elasticities. Lowering the price for Idaho residents brings in several new consumers to the market, while raising the price for Not Idaho residents does not drive that many out.

Third-Degree Price Discrimination

- Furthermore, total profit for the monopolist also increases from 18 under uniform pricing to 18.8 under price discrimination (a 4% increase).
    - We should expect this. Price discrimination should always increase profits for the monopolist.

Third-Degree Price Discrimination

- The result where price decreased for the more elastic group but increased for the less elastic group holds generally. We can even prove it by looking at a general profit maximization function,

$$\max_p \quad pq(p) - c(q(p))$$

and taking a first-order condition with respect to p,

$$q + p\frac{dq}{dp} - c'\frac{dq}{dp} = 0$$

Rearranging terms,

$$q + p\frac{dq}{dp} = c'\frac{dq}{dp}$$

Third-Degree Price Discrimination

$$q + p\frac{dq}{dp} = c'\frac{dq}{dp}$$

- Dividing both sides by $\frac{dq}{dp}$ and moving the q in the first term to the denominator,

$$\frac{1}{\frac{dq}{dp}\frac{1}{q}} + p = c'$$

then I factor p out of the left-hand side of the equation,

$$\begin{aligned} p\left(\frac{1}{\frac{dq}{dp}\frac{p}{q}} + 1\right) &= c' \\ p\left(\frac{1}{\varepsilon} + 1\right) &= c' \end{aligned}$$

Third-Degree Price Discrimination

$$p\left(\frac{1}{\varepsilon}+1\right)=c'$$

- Rearranging this a bit more, I obtain price as a function of marginal cost and the price elasticity of demand,

$$\begin{aligned} p\left(\frac{1+\varepsilon}{\varepsilon}\right) &= c' \\ p &= c'\left(\frac{\varepsilon}{\varepsilon+1}\right) \end{aligned}$$

- This is known as the inverse elasticity pricing rule, and it's a common shortcut used to find a monopoly price when you have data on marginal cost and the price elasticity of demand.

Third-Degree Price Discrimination

$$p = c'\left(\frac{\varepsilon}{\varepsilon + 1}\right)$$

- Now, let there be two different subgroups, and we'll assume that the price for subgroup 1 is higher than the price for subgroup 2,

$$p_1 > p_2$$

- Substituting the inverse elasticity pricing rule,

$$\begin{aligned} c'\left(\frac{\varepsilon_1}{\varepsilon_1 + 1}\right) &> c'\left(\frac{\varepsilon_2}{\varepsilon_2 + 1}\right) \\ \frac{\varepsilon_1}{\varepsilon_1 + 1} &> \frac{\varepsilon_2}{\varepsilon_2 + 1} \end{aligned}$$

Third-Degree Price Discrimination

$$\frac{\varepsilon_1}{\varepsilon_1 + 1} > \frac{\varepsilon_2}{\varepsilon_2 + 1}$$

- Cross multiplying,

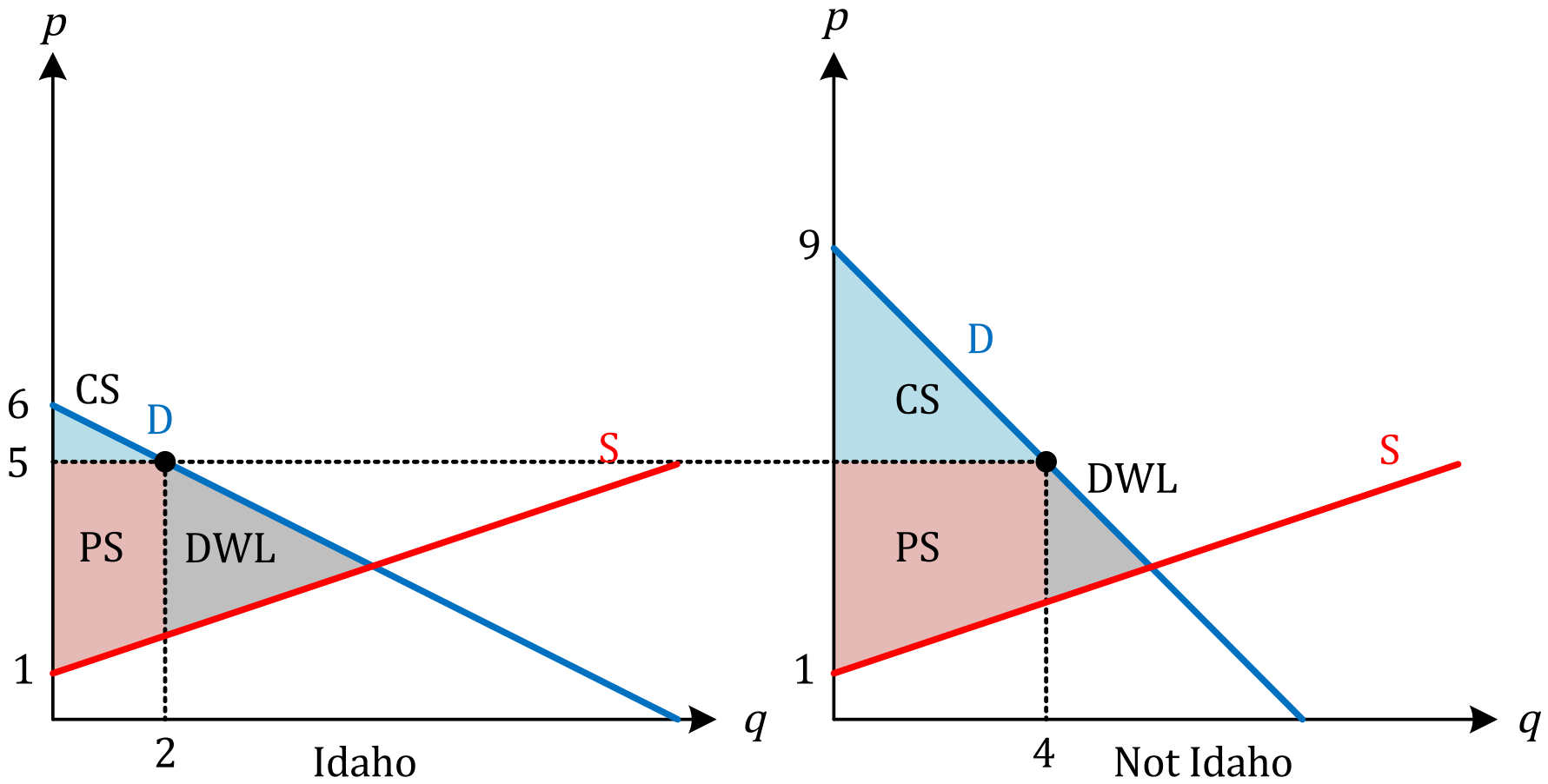
$$\begin{aligned} \varepsilon_1(\varepsilon_2 + 1) &> \varepsilon_2(\varepsilon_1 + 1) \\ \varepsilon_1\varepsilon_2 + \varepsilon_1 &> \varepsilon_1\varepsilon_2 + \varepsilon_2 \\ \varepsilon_1 &> \varepsilon_2 \end{aligned}$$

- Now we have to remember that the *price elasticity of demand is negative* while well behaved. Thus, when $\varepsilon_1 > \varepsilon_2$, this implies that subgroup 1 is less elastic than subgroup 2.
  - Thus, the subgroup with the more inelastic demand pays the higher price.

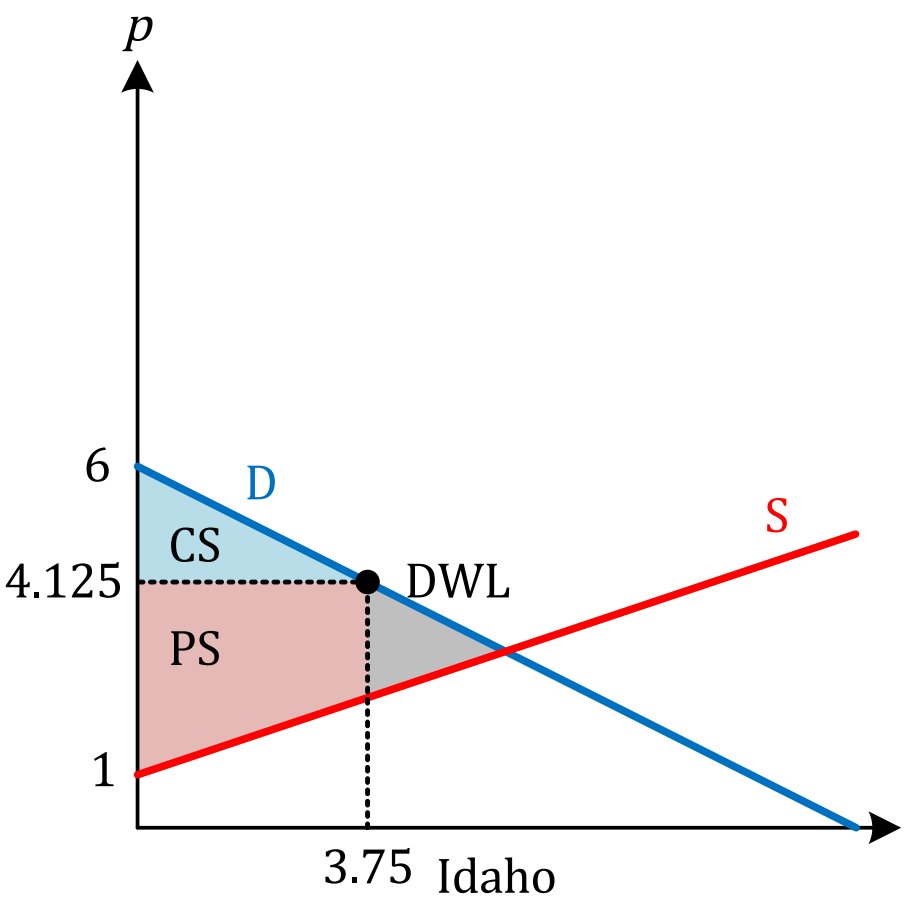
Third-Degree Price Discrimination

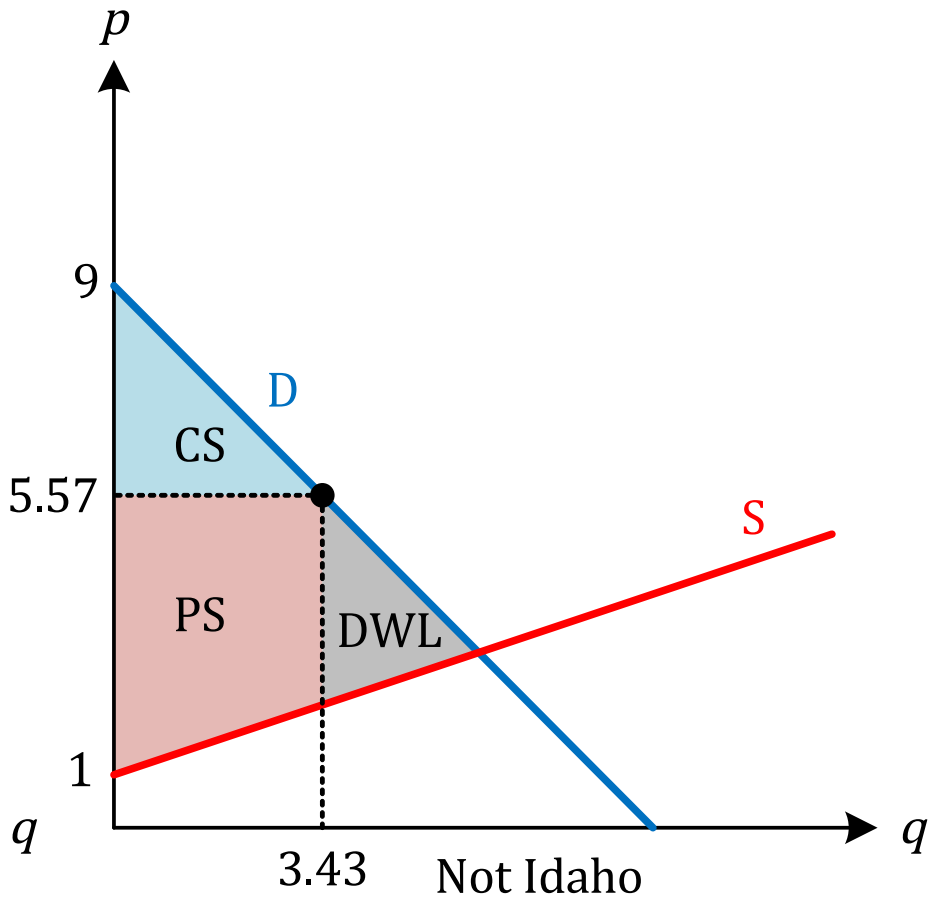
- To calculate welfare effects under third-degree price discrimination, we must look at each subgroup individually, regardless of whether we used uniform or price discrimination.
- Let's look at it graphically.

Third-Degree Price Discrimination



Third-Degree Price Discrimination





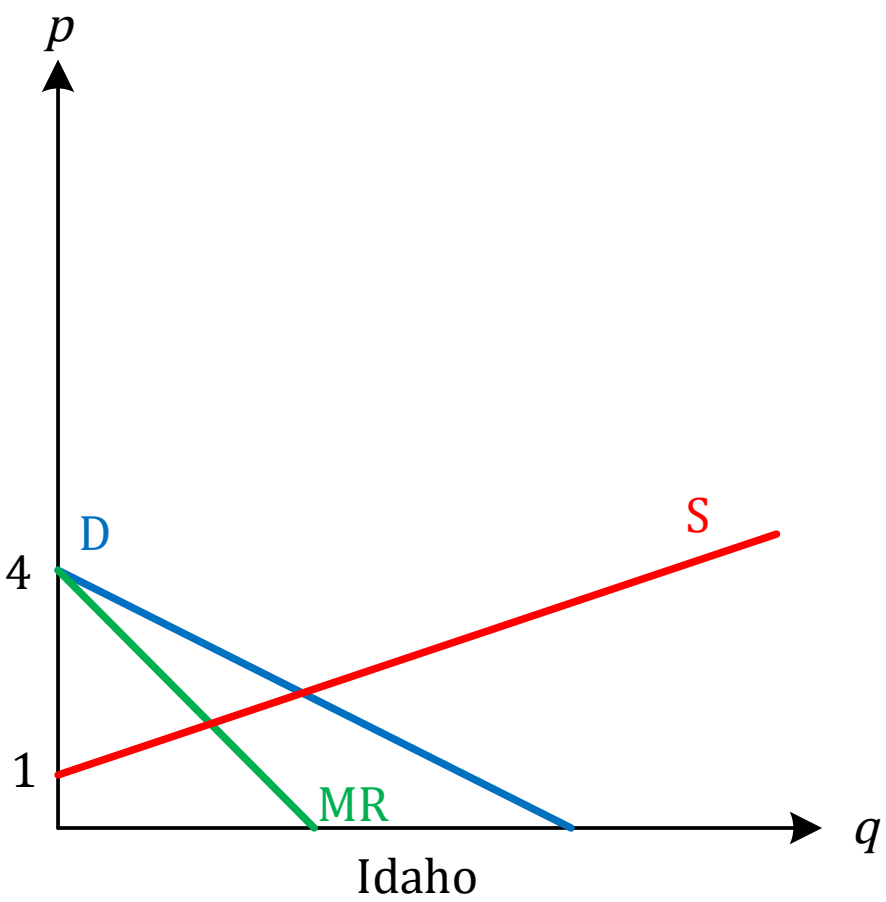
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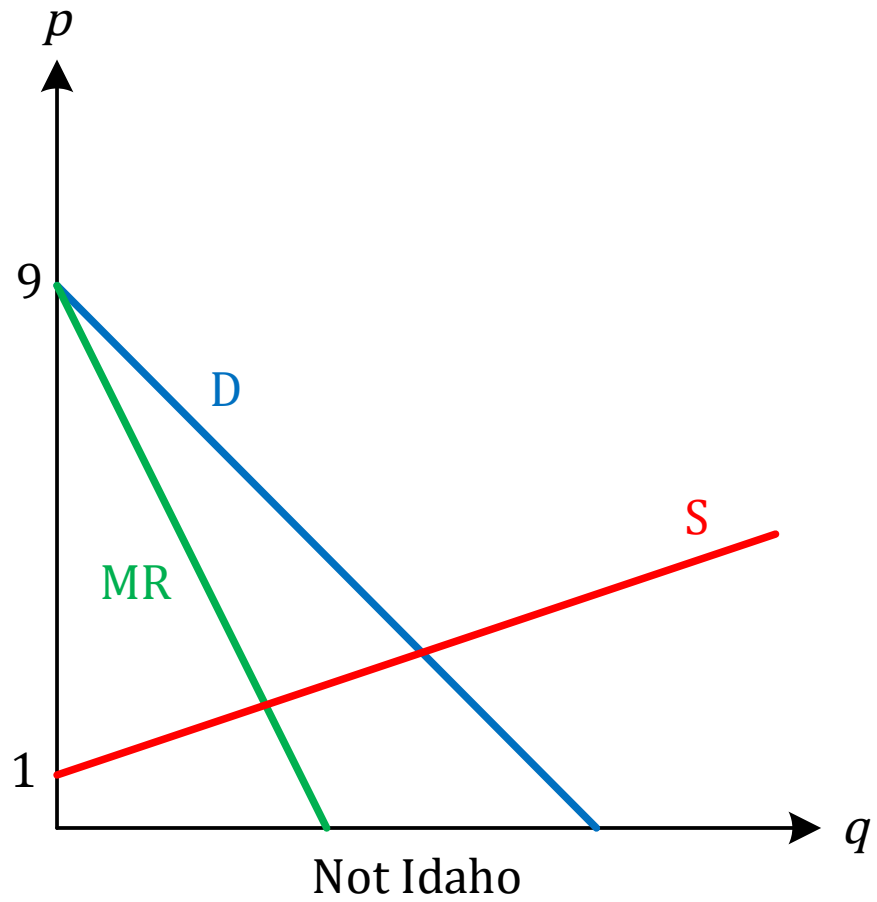
- For the majority of cases, welfare decreases under third-degree price discrimination.
  - Producer surplus rises, consumer surplus and deadweight loss can either rise or fall.
- Under specific demand functions, welfare can increase. But those are rarely seen in the real world.
  - There is one noteworthy special case, though!

Third-Degree Price Discrimination

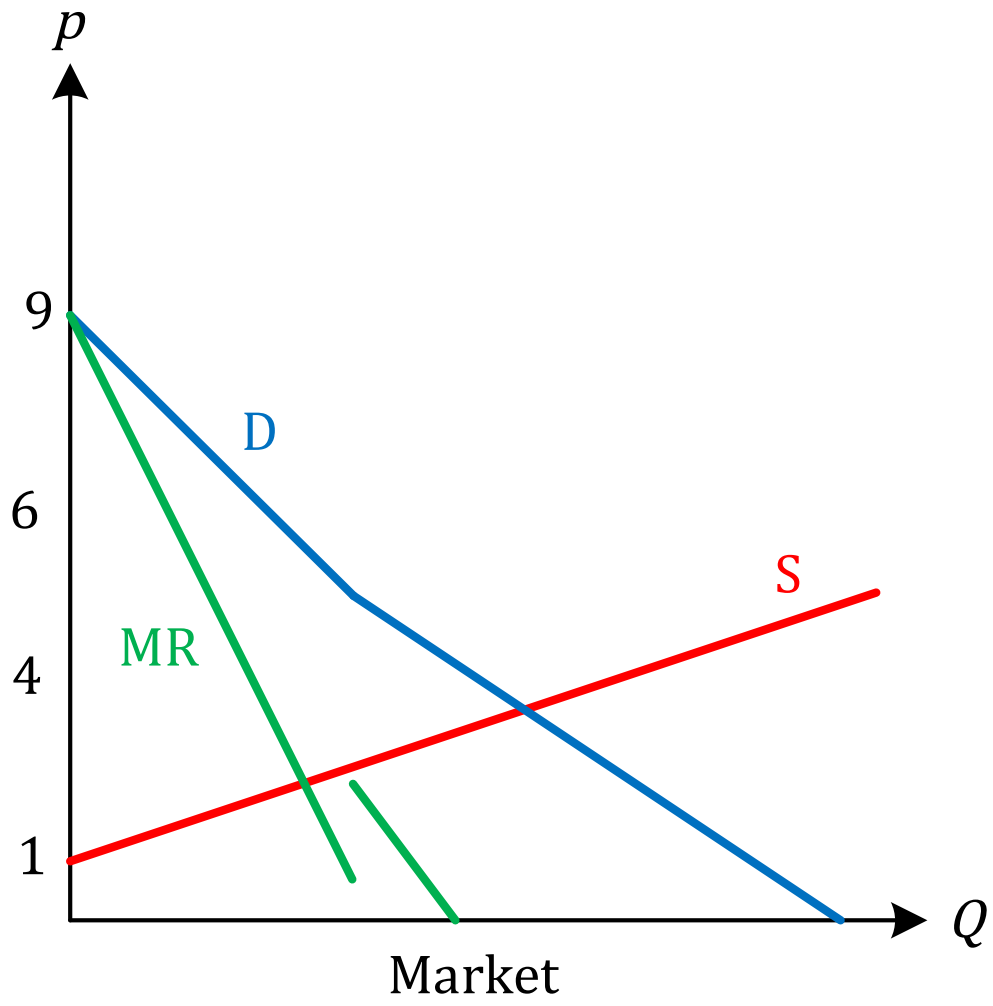
- What if, when under uniform pricing, all but one subgroup was left out of the market?
  - Basically, the monopolist could earn higher profits by selling only to the consumers who will accept the high price.
- All of the surplus in the subgroup that was left out is deadweight loss.
  - Let's adjust our resort example to see that.

Third-Degree Price Discrimination





Third-Degree Price Discrimination



Third-Degree Price Discrimination

- In this case, under uniform pricing, the firm simply sets the monopoly price for the Not Idaho subgroup and ignores the Idaho subgroup.
  - The firm receives the same profits from the Not Idaho subgroup as they would under third-degree price discrimination, but nothing for the Idaho group. It's all deadweight loss.
- By implementing third-degree price discrimination, the firm can get the Idaho subgroup into the market, converting some of the deadweight loss into welfare for both the firm and the Idaho consumers.
  - This has no impact on the Not Idaho subgroup's welfare, so it is certainly welfare improving.
- Note: This is only guaranteed to work if only one firm is in the market under uniform pricing.

Summary

- Block Pricing is a way for firms to get around the arbitrage problems inherent in two-part pricing.
- Third-degree price discrimination allows a firm to increase their profits by segregating the market into identifiable subgroups.

Next Time

- Second-degree price discrimination.
  - Reading: 5.4. Note: This section is tough.

Homework 2-4

- A monopolist with marginal cost of production of 40 sells to two distinct regions. In Region 1, demand is given by: $q_1 = 300 - p_1$. In Region 2, it is given by: $q_2 = 180 - p_2$.
  1. Determine the optimal uniform price and output when discrimination is impossible.
  2. Assume the monopolist can discriminate between regions. What price and quantity will be set for each region?
  3. How does the discriminatory price relate to the price elasticity of demand for each region?