EconS 425 - Horizontal Merger Paradox

Eric Dunaway

Washington State University

eric.dunaway@wsu.edu

Industrial Organization

Introduction

- We're moving on to our next unit: contractual relationships between firms.
- These primarily deal with mergers, where two firms combine into a larger firm in order to achieve some kind of benefit.
- We're going to cover several different types of mergers: starting today with horizontal mergers.
  - Horizontal mergers occur when two firms that sell substitutes combine into one larger firm.

Mergers

- As said before, mergers are a powerful tool that firms can use to consolidate market power.
  - They are the closest thing to a legal cartel, as combining large firms in a market with few firms brings the market one step closer to monopoly.
- Naturally, mergers are heavily scrutinized by regulatory agencies, as they have the potential to be damaging to consumers.
  - It is possible, however, that a merger will benefit the consumer: more on that later.

Mergers

- Why would firms want to merge in the first place?
    - Perhaps the merger leads to a larger market share.
    - Perhaps there will be less competition.
    - Perhaps there are cost advantages to merging.
- In the real world, mergers are costly.
    - There is a lot of negotiation as to who runs the newly merged firm and how all of the previous owners are compensated.

Mergers

- In a horizontal merger, the firms seeking to merge sell to the same market and their goods are substitutes.
  - Essentially, these firms are the same as we have seen in Cournot and Bertrand competition.
- We can add mergers to the models we have looked at before.
  - Just have the merging firms act as one larger firm (the same as a cartel).
  - We'll need at least 3 firms to make this work, unless we are having the firms merge into a monopolist.

Horizontal Merger Paradox

- Consider a market with 3 firms that competes in quantities and faces an inverse market demand of

$$p = a - bq_1 - bq_2 - bq_3$$

where firms face constant marginal costs of c.

- We can obtain our equilibrium values if each of this firms operates independently the same way that we have before, finding

$$q_i^* = \frac{a-c}{4b} \qquad p^* = \frac{a+3c}{4} \qquad \pi_i^* = \frac{(a-c)^2}{16b}$$

Horizontal Merger Paradox

- Now, suppose that firms 2 and 3 decided to merge.
  - Now, they represent one large firm Q_2 where $Q_2 = q_2 + q_3$.
- The profit maximization problem for firm 1 becomes

$$\max_{q_1} \; (a - bq_1 - bQ_2)q_1 - cq_1$$

with first-order condition

$$\frac{\partial \pi_1}{\partial q_1} = a - 2bq_1 - bQ_2 - c = 0$$

- Solving this expression for q_1 gives firm 1's best response function

$$q_1(q_2) = \frac{a-c}{2b} - \frac{Q_2}{2}$$

Horizontal Merger Paradox

- We can perform the same analysis for the merged firm, with their profit maximization function,

$$\max_{Q_2} \ (a - bq_1 - bQ_2)Q_2 - cQ_2$$

with first-order condition

$$\frac{\partial \pi_2}{\partial Q_2} = a - bq_1 - 2bQ_2 - c = 0$$

- Solving this expression for Q_2 gives firm 2's best response function

$$Q_2(q_1) = \frac{a-c}{2b} - \frac{q_1}{2}$$

Horizontal Merger Paradox

$$
\begin{aligned}
q_1(q_2) &= \frac{a-c}{2b} - \frac{Q_2}{2} \\
Q_2(q_1) &= \frac{a-c}{2b} - \frac{q_1}{2}
\end{aligned}
$$

- This leaves us with two equations and two unknowns, which we can solve to obtain

$$q_1^* = Q_2^* = \frac{a-c}{3b}$$

and we can plug these values back into our inverse demand function to obtain the price and the profit level,

$$p^* = \frac{a+2c}{3} \qquad \pi_i^* = \frac{(a-c)^2}{9b}$$

Horizontal Merger Paradox

- Let's compare the results for the pre and post mergers,

	Pre-Merger	Post-Merger
q_1^*	$\frac{a-c}{4b}$	$\frac{a-c}{3b}$
q_2^*, q_3^*	$\frac{a-c}{4b}$	$\frac{a-c}{6b}$
Q_2^*	$\frac{a-c}{2b}$	$\frac{a-c}{3b}$
p^*	$\frac{a+3c}{4}$	$\frac{a+2c}{3}$
π_1^*	$\frac{(a-c)^2}{16b}$	$\frac{(a-c)^2}{9b}$
π_2^*, π_3^*	$\frac{(a-c)^2}{16b}$	$\frac{(a-c)^2}{18b}$
Π_2^*	$\frac{(a-c)^2}{8b}$	$\frac{(a-c)^2}{9b}$

Horizontal Merger Paradox

- The first thing to notice is that the merged firm's output level is less than the aggregate output level of the two firms pre merge.
  - This actually makes sense. When the firms merge, they know that their own output level affects the price that not only they get, but their now partner firm gets as well. They have internalized the effect of their output on the other firm.
- The output of the unmerged firm also increases in this situation.
  - Since there are less competitors, and they are reducing their output level, it makes sense that the other firms will increase theirs in response (from the best response function).
- The price also increases.
  - Since the total quantity is reduced, the price increases. This implies that the consumers are worse off under the merger.

Horizontal Merger Paradox

- What's quite strange about these results is that the merged firm's profit level falls to $\frac{(a-c)^2}{9b}$ as opposed to the sum of the individual firm's profits before the merger of $\frac{(a-c)^2}{8b}$.
  - The merged firm actually makes less money than the individual firms made before the merger.
  - This is known as the horizontal merger paradox. Without any cost or market advantage, it's not profitable for two firms to merge unless that merger gives them a monopoly in the market.
- The real winner of this merger is actually firm 1.
  - They don't have to do anything and their profits go up.
  - This would actually further disincentivize firms from merging.

Horizontal Merger Paradox

- A profitable horizontal merger among many firms may exist, however.
  - It's possible that the price could raise high enough to offset the reduction in quantity.
- Suppose there were N firms in a market, and M of them chose to merge. If we performed the same analysis as before, the pre merge values for the individual and aggregate merged firm's profits are

$$\pi_i^* = \frac{(a-c)^2}{(N+1)^2 b} \qquad \Pi_M^* = \frac{M(a-c)^2}{(N+1)^2 b}$$

  whereas, after M firms merge, there are now $N - M + 1$ firms in the market, leading our equilibrium values to

$$\pi_i^* = \frac{(a-c)^2}{(N-M+1)^2 b} \qquad \Pi_M^* = \frac{(a-c)^2}{(N-M+1)^2 b}$$

Horizontal Merger Paradox

- A merger of M firms will only be profitable if the merged firm's profits are higher than the aggregate profits among the unmerged firms, i.e.,

$$\frac{(a-c)^2}{(N-M+1)^2 b} \geq \frac{M(a-c)^2}{(N+1)^2 b}$$

rearranging,

$$(N+1)^2 > M(N-M+1)^2$$

- This is a bit hard to solve for M, but if we define M as $M = \alpha N$, where $\alpha < 1$, we can figure out what the proportion of merged firms has to be of the total firms in order for the merger to be profitable.

Horizontal Merger Paradox

$$(N+1)^2 > \alpha N(N - \alpha N + 1)^2$$

- Solving this expression for α,

$$\alpha > \frac{3 + 2N - \sqrt{5 + 4N}}{2N}$$

and plugging in a few different values of N,

N	α
4	0.802
5	0.8
10	0.815
15	0.831
20	0.845

Horizontal Merger Paradox

- The smalles value α can take is 0.8 at $N = 5$, and it approaches infinity as N approaches infinity.
  - Note: α isn't a function of the demand parameters or marginal costs. Only the number of firms matters.
- At the very least, 80% of the firms in a market must merge in order for the merger to be profitable.
  - This is known as the 80% rule.
- However, this whole process relies on a very strict assumption: that the firms receive no cost or market advantages from merging.
  - We should probably look at those, too.

Horizontal Mergers

- Suppose now that merging imposed some kind of cost savings on a firm.
  - These can take two forms, cost savings in fixed or variable costs.
- Let's assume that a merger causes a reduction in fixed costs paid by the merged firm.
  - This could be explained by less overhead charges, combining middle management, etc.

Horizontal Mergers

- Let's return to our previous example with 3 firms where firms 2 and 3 merge. Let's assume that each firm must pay a fixed cost of F to operate in the market.
- Before the merger, the equilibrium values for the firms are

$$q_i^* = \frac{a-c}{4b} \qquad p^* = \frac{a+3c}{4} \qquad \pi_i^* = \frac{(a-c)^2}{16b} - F$$

where the total profits for firms 2 and 3 are

$$\Pi_M^* = \frac{(a-c)^2}{8b} - 2F$$

Horizontal Mergers

- Suppose that when firms 2 and 3 merge, they receive some reduction in their total fixed costs, such that the fixed cost of the merged firm is αF, where $1 < \alpha < 2$. From the same analysis as before, the equilibrium values after the merge are

$$q_1^* = Q_2^* = \frac{a-c}{3b} \qquad p^* = \frac{a+2c}{3}$$
$$\pi_1^* = \frac{(a-c)^2}{9b} - F \qquad \Pi_2^* = \frac{(a-c)^2}{9b} - \alpha F$$

- The merger will be profitable if the profits after the merger (taking advantage of the cost savings) are greater than the aggregate pre merger profits,

$$\frac{(a-c)^2}{9b} - \alpha F \geq \frac{(a-c)^2}{8b} - 2F$$

Horizontal Mergers

$$\frac{(a-c)^2}{9b} - \alpha F \geq \frac{(a-c)^2}{8b} - 2F$$

- Solving this expressions for α,

$$\alpha \leq 2 - \frac{(a-c)^2}{72bF}$$

which implies that if the fixed costs are large, it's easier for the merger to be profitable.

Horizontal Mergers

- Now suppose that a merger lowered the variable (marginal) cost for the merged firm.
  - As a single firm, there could be more capital to go around, delaying the onset of diminishing marginal returns.
  - There also could be technology shared between the two firms.
- As usual, for our pre merger case, we have our equilibrium values of

$$q_i^* = \frac{a-c}{4b} \qquad p^* = \frac{a+3c}{4} \qquad \pi_i^* = \frac{(a-c)^2}{16b}$$

where the total profits for firms 2 and 3 are

$$\Pi_M^* = \frac{(a-c)^2}{8b}$$

Horizontal Mergers

- Now, suppose that when firms 2 and 3 merge, the marginal cost for the merged firm becomes αc, where $\alpha < 1$ represents the cost reduction of the merged firm.
- In this case, our equilibrium values become

$$\pi_1^* = \frac{(a - 2c + \alpha c)^2}{9b} \qquad \Pi_M^* = \frac{(a - 2\alpha c + c)^2}{9b}$$

and a merger is profitable if the profits of the merged firm are higher than the aggregate profits of the pre merged firms

$$\frac{(a - 2\alpha c + c)^2}{9b} \geq \frac{(a - c)^2}{8b}$$

Horizontal Mergers

$$\frac{(a - 2\alpha c + c)^2}{9b} \geq \frac{(a-c)^2}{8b}$$

- Once again, solving this expression for α, we have

$$\alpha \leq \frac{(\sqrt{8}-3)a + (\sqrt{8}+3)c}{2\sqrt{8}c}$$

which fortunately is between 0 and 1.

  - As a gets larger (i.e., the market demand increases) relative to the marginal cost, c, a larger cost reduction is required to have a profitable merger.

Horizontal Mergers

- Cost savings are an excellent reason for a horizontal merger to occur, but they still have two limitations.
  - Empirically, cost savings from mergers tend to be on the order of 1 to 2 percent. This severely limits our ability for a merger to be profitable.
  - In all of our situations, the firm that doesn't merge has its profits increase. This creates a free rider problem. Why go through a costly merger when you can receive the benefits for free?

Horizontal Mergers

- Rather than cost savings being the product of mergers, what if merging gave the merged firm some kind of advantage in the market?
- Continuing with our previous example, suppose that should firms 2 and 3 merge, they receive the ability to set their output level first, making them a Stackelberg leader.
    - From before, since the merged firm has the capital of two smaller firms, they have a higher output capacity and can reliably commit to a production level.
    - This is fairly reasonable, as larger firms tend to have an advantage over smaller firms.

Horizontal Mergers

- Returning to our example, before the merger, our equilibrium values are
$$q_i^* = \frac{a-c}{4b} \qquad p^* = \frac{a+3c}{4} \qquad \pi_i^* = \frac{(a-c)^2}{16b}$$
where the total profits for firms 2 and 3 are
$$\Pi_M^* = \frac{(a-c)^2}{8b}$$

- Whereas after the merger, our equilibrium values are
$$\begin{aligned} q_1^* &= \frac{a-c}{4b} \qquad Q_2^* = \frac{a-c}{2b} \qquad p^* = \frac{a+3c}{4} \\ \pi_1^* &= \frac{(a-c)^2}{16b} \qquad \Pi_M^* = \frac{(a-c)^2}{8b} \end{aligned}$$

Horizontal Mergers

- Notice that the profits for before and after the merger are identical.
  - This is unfortunately a consequence of only 3 firms. Examples can't always work.
- Suppose we had 4 firms! Before the merger, our equilibrium values are

$$q_i^* = \frac{a-c}{5b} \qquad p^* = \frac{a+4c}{5} \qquad \pi_i^* = \frac{(a-c)^2}{25b}$$

where the total profits for firms 2 and 3 are

$$\Pi_M^* = \frac{2(a-c)^2}{25b}$$

- After the merger, our equilibrium values are (remember the midterm!)

$$\begin{aligned} q_1^* &= q_4^* = \frac{a-c}{6b} \qquad Q_2^* = \frac{a-c}{2b} \qquad p^* = \frac{a+5c}{6} \\ \pi_1^* &= \frac{(a-c)^2}{36b} \qquad \Pi_M^* = \frac{(a-c)^2}{12b} \end{aligned}$$

Horizontal Mergers

- Now with 4 firms, the merger is profitable as long as

$$\frac{(a-c)^2}{12b} \geq \frac{2(a-c)^2}{25b}$$

which reduces to $25 \geq 24$, so it is always profitable for two firms to merge when there are 4 firms.

- These results also hold with $N \geq 5$ firms.
  - The market advantage obtained by the merging firms makes merging the best choice.

Horizontal Mergers

- Giving market advantage to merging firms also has another interesting property: the profits for the non merged firms actually decreases.
  - This is unique in all of the other cases we have seen today.
- In fact, even after one group of firms have already merged, it is always better to find another firm to merge with if it will bestow first mover advantage on to the newly merged firm as well.
  - All of the firms keep merging until every firm has merged with another and the market advantage is mitigated.
  - Do those firms merge again to increase their advantage? Probably.

Horizontal Mergers

- What about consumers?
  - We can directly relate the consumer surplus to the price charged by consumers.
  - With a low level of merged firms, the total output level increases which will lead to the price decreasing. This is good for consumers.
- Suppose we had N firms in the market, L of which are merged firms, and $N - L$ are unmerged. If we allow two more firms to merge, we will now have $L + 1$ merged firms and $N - L - 2$ unmerged firms (since now there are $N - 1$ firms in total).
- Before the merger, the price minus marginal cost charged to consumers is

$$p - c = \frac{a - c}{(L + 1)(N - L + 1)}$$

whereas after the merger, it changes to

$$p - c = \frac{a - c}{(L + 2)(N - L - 1)}$$

Horizontal Mergers

- For the merger to be beneficial to consumers, we must have that adding another merged firm leads to a decrease in price (which should happen for a low number of merged firms), i.e.,

$$\frac{a-c}{(L+2)(N-L-1)} < \frac{a-c}{(L+1)(N-L+1)}$$

and solving this expression for L gives us

$$L < \frac{N}{3} - 1$$

- This implies that as long as less than a third of the firms are merged, it's better for consumers if an additional merge occurs.
  - You can bet that regulators use this rule quite a bit.

Summary

- Mergers are a very powerful tool available to firms.
- Unless there is a cost or market advantage to a merger, it's not profitable unless the vast majority of firms merge together.

Next Time

- More Horizontal Mergers
  - What happens when firms merge sequentially or have differentiated products?

Homework 5-5

- Suppose that there were only two firms in a market and they competed in quantities. Would these firms want to merge? Why or why not? If they merged, would a regulator take issue with the merger?