EconS 425 - Repeated Games

Eric Dunaway

Washington State University

eric.dunaway@wsu.edu

Industrial Organization

Introduction

- Today, we're going to look at what happens when simultaneous move games are repeated, both finitely and infinitely.
  - This allows us to create a model of sustainability for a cartel.

Repeated Games

- Repeated games are simply games that we play over and over again.
  - Sometimes they repeated a fixed number of times, sometimes they repeat forever.
- Under certain circumstances, we can obtain different Nash equilibria than what we saw in just a single round game.

- Let's look at a Bertrand competition game.
  - Remember that if each firm decides to price at the monopoly price, they will each split the monopoly profit, i.e., $\pi_i = \frac{\pi^M}{2}$ for $i = 1, 2$.
  - Each firm could choose to deviate from this and undercut the other firm by $\varepsilon > 0$. If this happens, the firm that undercuts the other firm receives profits of π^M while the other firm receives profits of 0.
  - If both firms decide to undercut one another, price falls to marginal cost, and they each earn a profit level of 0.
- We can express this as a simple normal form game.

Repeated Games

		Firm 2: Cooperate	Firm 2: Undercut
Firm 1	Cooperate	$\frac{1}{2}\pi^M, \frac{1}{2}\pi^M$	$0, \pi^M$
	Undercut	$\pi^M, 0$	$0, 0$

Repeated Games

		Firm 2	
		Cooperate	Undercut
Firm 1	Cooperate	$\frac{1}{2}\pi^M, \frac{1}{2}\pi^M$	$0, \underline{\pi^M}$
	Undercut	$\underline{\pi^M}, 0$	$\underline{0}, \underline{0}$

Repeated Games

- As expected, when this game is played once, it is always better for each player to undercut the other, and our only Nash equilibrium in this game is when both players undercut and neither of them receive any profit.
- This is a pretty terrible outcome for the firms, as they could both do much better by cooperating and receiving half of the monopoly profit.
    - The incentive to undercut, however, is simply too strong, and cooperation cannot be sustained in a "one shot" game.

Repeated Games

- What if they played this game over and over again?
  - This isn't unreasonable at all. If they were competitors in the same market, it's natural to think that they would have many interactions over time.
- Let's assume that our game from above is repeated some fixed number, T times.
  - Suppose there was a known date of their final interaction, perhaps the exhaustion of a common resource that they use, or perhaps they know when the world will end.
- Does this change our results?

Repeated Games

- An important facet of this repetition is that while at the beginning of each round of play, each player can observe what has happened in the previous rounds.
  - Thus, the history of the game is both important, and a great tool to simplify our analysis.
- Let's consider the following strategy for both firms:
  - Cooperate in the first round of the game.
  - If the other firm has cooperated in every previous period up until the one we are in now, then cooperate. Otherwise, undercut.
- In this strategy, both firms will cooperate until one of the firms undercuts the other. Once that happens, they both revert to the Nash equilibrium of their one shot game for the remaining rounds.
  - Can this strategy be an equilibrium?

Repeated Games

- We can essentially do backward induction here.
- Let's start at the Tth round of the game (the very last interaction). Suppose both firms have cooperated up until this point.
  - We want to figure out if both firms still want to cooperate, as their strategy says they should, or if there is a better alternative by undercutting.
- Should the firms cooperate or undercut?
  - Since there are no more interactions after this one, neither firm has any incentive to continue their cooperation.
  - They could cooperate and receive $\frac{\pi^M}{2}$ in profits, or undercut and receive π profits.
  - Thus, both firms find it better to undercut.

Repeated Games

- Cooperation can't be sustained in the Tth period.
  - This is a common result in game theory. In a finitely repeated game with T rounds, the final round must have players selecting a Nash equilibrium from the one shot game.
- From here, we look at period $T-1$, the second-to-last period.
  - Both firms know that the other firm is going to undercut them in the next period, so now this essentially becomes the last period of the game.
  - Since undercutting will happen regardless, now there's no incentive to cooperate in this period either. Both firms undercut once again.
- The strategy suggested before then unravels completely.
  - Without any incentives to cooperate, both players just revert to undercutting every single period of the game.

Repeated Games

- That's unfortunate, but also another general result from game theory.
  - In a finitely repeated game that only has one Nash equilibrium in the stage game, the only subgame perfect Nash equilibrium that exists is when all players choose the stage game Nash equilibrium in every stage of the game.
  - Essentially, if there is only one strategy that works for both players in a single round of the game, they'll always play it in a repeated game.
- Notice, however, that this result is limited to where there is only one Nash equilibrium in the stage game.
  - What happens when we have more?
- Let's look at another game where a firm can choose whether to charge a low, medium, or high price.

Repeated Games

		Firm 2		
		Low	Medium	High
	Low	6, 6	8, 5	9, 4
Firm 1	Medium	5, 8	8.5, 8.5	10, 5
	High	4, 9	5, 10	9.5, 9.5

Repeated Games

- Naturally, the best possible outcome for both firms would be if they priced high, leading to profits of 9.5 for each firm.
  - However, each firm would rather price medium if the other priced high, since they would receive a profit of 10 if they did so.
- Otherwise, there are two Nash equilibria of this one shot game.
  - Both firms price low, and both firms price high.

Repeated Games

		Firm 2		
		Low	Medium	High
	Low	6, 6	8, 5	9, 4
Firm 1	Medium	5, 8	8.5, 8.5	10, 5
	High	4, 9	5, 10	9.5, 9.5

Repeated Games

- Since there are two possible Nash equilibria in the one shot game, there is a bit of uncertainty which one the firms will arrive at in equilibrium.
  - Both firms charging a medium price is much better than both firms charging a low price, but there is nothing that tells us for sure that we will end up there.
  - Both firms would still rather charge a high price though.
- We can use the fact that there are two Nash equilibria in the stage game to induce cooperation between the two firms!
  - This is known as a "Carrot-Stick" strategy.

Repeated Games

- Suppose this game is played twice, and the firms discount their profits in the second stage by δ.
- Consider the following strategy:
  - In the first stage, charge a high price.
  - In the second stage, charge a medium price if the other firm charged a high price in stage 1 (The carrot). Otherwise, charge a low price (The stick).
- Intuitively, if the other firm cooperates with you in the first stage of the game, you respond by picking the better of the two Nash equilibria in the second stage (The carrot). If they don't cooperate with you, you respond by punishing them with the worse Nash equilibria in the second stage (the stick).
  - Since they are both Nash equilibria, the other firm has no choice by to accept the punishment (they can do no better).

- Let's see if cooperation can be sustained with this strategy. Looking at firm 1's payoff (it's identical for firm 2), if they cooperate, they receive a payoff of

$$\pi_1^C = \underbrace{9.5}_{\text{High}} + \underbrace{\delta 8.5}_{\text{Medium}}$$

and if they deviate in the first stage by charging a medium price, they receive a payoff of

$$\pi_1^D = \underbrace{10}_{\text{Medium}} + \underbrace{\delta 6}_{\text{Low}}$$

- Firm 1 will want to cooperate with this strategy as long as

$$\begin{aligned} \pi_1^C &\geq \pi_1^D \\ 9.5 + \delta 8.5 &> 10 + \delta 6 \\ \delta 2.5 &> 0.5 \\ \delta &> 0.2 \end{aligned}$$

Repeated Games

- Thus, as long as firm 1 isn't too impatient, they will cooperate and charge a high price in period 1, knowing it will lead to a medium price in period 2.
  - We were able to induce a strategy that isn't a Nash equilibrium at all and play it in equilibrium!
- This works for two reasons:
  - First of all, there is an outcome that is best for both players.
  - Second, players have a credible way to punish one another if they deviate from the cooperative outcome.

Repeated Games

- Those two concepts are really important as we start talking about cartels.
  - It must be beneficial to be a member of the cartel.
  - Also, the cartel has to have a way to keep unruly members in line.
- How do we solve the problem of there only being one Nash equilibrium in the stage game though?

Repeated Games

- Suppose now that the game repeated infinitely.
  - No, games don't last forever in the real world. However, what's the difference between a game that lasts forever and a game that you don't know when it will end?
  - It's like watching the Simpsons. You know it has to end someday, but you have no idea how long they'll drag it out.
- Without a definite end to the game, we don't have that situation where our strategy can unravel, since cooperation can always remain on the table.

Repeated Games

- Consider the same strategy as in the finite game:
  - Cooperate in the first round of the game.
  - If the other firm has cooperated in every previous period up until the one we are in now, then cooperate. Otherwise, undercut.
- In the infinitely repeated game, this is known as a "Grim Trigger" strategy.
  - Cooperate up until the first time the other firm undercuts you.
  - After that, never trust the other firm again and revert to the stage game Nash equilibrium.

- Let's figure out if firm 1 wants to go along with this strategy.
  - Again, firms discount their payoff in each later period by δ.
  - Also, we'll say that the market exists in the next period with probability p.
- Firm 1's payoff if they cooperate is

$$\begin{aligned} \pi_1^C &= \frac{\pi^M}{2} + p\delta\frac{\pi^M}{2} + (p\delta)^2\frac{\pi^M}{2} + (p\delta)^3\frac{\pi^M}{2} + \ldots \\ & \frac{\pi^M}{2}\left(1 + \rho + \rho^2 + \rho^3 + \ldots\right) \end{aligned}$$

where $\rho = p\delta$ is the probability adjusted discount factor.

Repeated Games

- Remember our trick for infinite series?

$$1 + \rho + \rho^2 + \rho^3 + ... = \sum_{i=0}^{\infty} \rho^i = \frac{1}{1-\rho}$$

and thus,

$$\pi_1^C = \frac{\pi^M}{2}\left(\frac{1}{1-\rho}\right)$$

- If firm 1 instead decides to undercut in some period, the receive the full profit for that period, then 0 forever after as both firms revert to the Nash equilibrium of the stage game

$$\pi_1^D = \pi^M + \rho 0 + \rho^2 0 + \rho^3 0 + ... = \pi^M$$

- As before, firm 1 wants to cooperate as long as

$$\begin{aligned} \pi_1^C &\geq \pi_1^D \\ \frac{\pi^M}{2}\left(\frac{1}{1-\rho}\right) &\geq \pi^M \end{aligned}$$

Rearranging terms,

$$\frac{1}{1-\rho} \geq 2$$

and solving for ρ,

$$\rho \geq \frac{1}{2}$$

Repeated Games

- Thus, as long as firms are sufficiently patient, and the probability that the market will continue on is high enough, cooperation can be sustained.
  - Essentially, the two firms can collude and become a cartel.
- This is the basic requirement that firms have to trust one another to fix prices.
  - We'll look at some variations on this next time.

Repeated Games

- What about a Cournot cartel?
  - We can have those, too.
- Let's remember some Cournot equilibrium values. If firms collude and each produce half of the monopoly quantity, their profits are

$$\pi^M = \frac{(a-c)^2}{8b}$$

and if they compete in a Cournot setting, their profits are

$$\pi^N = \frac{(a-c)^2}{9b}$$

- Lastly, if one firm produces half of the monopoly quantity, the best response from the other firm yields a profit level of

$$\pi^B = \frac{9(a-c)^2}{64b}$$

Repeated Games

- In the one shot version of the game, the result is basically the same as the Bertrand version.
  - Firms will deviate and produce a higher quantity, leading to the Cournot outcome as the Nash equilibrium of the stage game.
- For the infinitely repeated game, if both players adopt the strategy where they cooperate until one of the players deviates, then revert to the Cournot level afterwards, player 1's payoff from cooperating is

$$\pi_1^C = \pi^M + \rho\pi^M + \rho^2\pi^M + \rho^3\pi^M + ... = \pi^M\left(\frac{1}{1-\rho}\right)$$

and their payoff from deviating is

$$\pi_1^D = \pi^B + \rho\pi^N + \rho^2\pi^N + \rho^3\pi^N + ...$$

- It's a little bit harder to compact the payoff from deviating. If we rewrite it as

$$\pi_1^D = \pi^B + \rho\pi^N(1+\rho+\rho^2+\rho^3+...) = \pi^B + \pi^N\left(\frac{\rho}{1-\rho}\right)$$

- Player 1 will want to cooperate if

$$\begin{aligned} \pi_1^C &\geq \pi_1^D \\ \pi^M\left(\frac{1}{1-\rho}\right) &\geq \pi^B + \pi^N\left(\frac{\rho}{1-\rho}\right) \end{aligned}$$

- Rearranging terms,

$$\pi^M \geq \pi^B(1-\rho) + \pi^N\rho$$

$$\pi^M \geq \pi^B(1-\rho) + \pi^N \rho$$

- Doing some more rearranging,

$$\rho(\pi^B - \pi^N) \geq \pi^B - \pi^M$$

$$\rho \geq \frac{\pi^B - \pi^M}{\pi^B - \pi^N} = \frac{\frac{9(a-c)^2}{64b} - \frac{(a-c)^2}{8b}}{\frac{9(a-c)^2}{64b} - \frac{(a-c)^2}{9b}}$$

$$\rho \geq \frac{9}{17} \approx 0.529$$

Repeated Games

- Interestingly, neither the values from the inverse demand function (a and b) nor the marginal cost (c), determine the critical value of ρ.
  - It's all about how patient you are and how likely the market is to continue.
- It's slightly harder to induce cooperation under a Cournot model. Why?
  - Since the firms still earn positive profits after deviating, it's slightly harder to induce cooperation.

Summary

- Repeated games let us iterate single shot games over and over again, and depending on the structure, the results can change.
  - For a finitely repeated game, cooperation cannot be sustained in a game with a single Nash equilibrium in the stage game.
  - For an infinitely repeated game, we can induce cooperation.

Next Time

- Cartels
  - What factors influence the sustainability of a cartel?
  - Reading: 10.4-10.5.

Homework 5-2

- Return to our game with three options for the firm (pricing low, medium, or high). Suppose that this game were infinitely repeated.
  1. If the firms used the strategy where they price high until someone deviates, and then priced medium forever after, what is the minimum value of the probability adjusted discount factor (ρ) that sustains cooperation?
  2. If the firms used the strategy where they price high until someone deviates, and then priced low forever after, what is the minimum value of the probability adjusted discount factor (ρ) that sustains cooperation?
  3. Why is it easier to sustain cooperation in part 2 than it is in part 1?