# EconS 425 - Bertrand Competition

Eric Dunaway

Washington State University

*eric.dunaway@wsu.edu*

Industrial Organization

# Introduction

- Today, we'll continue our discussion of oligopoly with Bertrand competition, or simply competition in prices.
    - How do Cournot's results compare with Bertrand's?

# Bertrand Competition

- In 1883, 45 years after Antoine Cournot published his work on competition in quantities, Joseph Louis François Bertrand reviewed the results in his own rebuttal, "Book review of theorie mathematique de la rechesse sociale and of recherches sur les principles mathematiques de la theorie des richesses."
  - Translation: "Book review of mathematical theory of social wealth and research on the mathematical principles of wealth theory."
- Bertrand thought it was ridculous that Cournot modelled competition in quantities, since in reality, firms simply choose the price of their own good. He thought economists were relying too much on mathematics, and weren't focusing enough on common sense.
  - Bertrand wouldn't be the last person to make this claim.

# Bertrand Competition

- In Bertrand's model, firms face a total cost curve for producing their good and simply choose the price for their respective goods. Whichever firm sets the lowest price gets all of the consumers, and if the firms set the same price, they both split the market evenly.
  - To make things simple, we'll assume that there are two identical firms that face a constant marginal cost $c$. The firms simultanously choose their prices, $p_1$ and $p_2$, respectively.
- Building a full mathematical model for Bertrand competition would be cumbersome, and honestly, unnecessary.
  - As Bertrand implied, we can derive our best response function by just thinking about how each firm will react to the other.

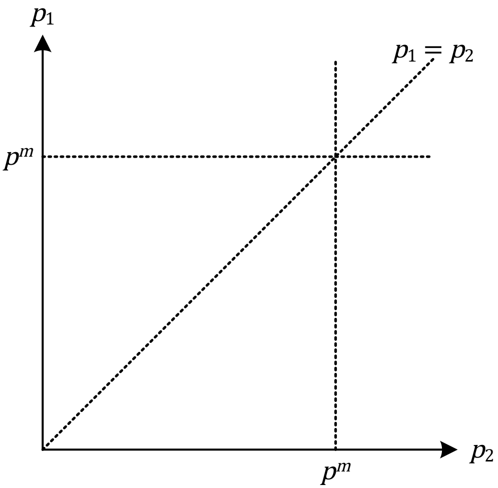
# Bertrand Competition

- Let's look at it from firm 1's perspective (it will be the same for firm 2).
- Firm 1 has three choices:
  - They can set their price above firm 2's price, $p_1 > p_2$.
  - They can match firm 2's price, $p_1 = p_2$.
  - They can set their price below firm 2's price, $p_1 < p_2$.
- Naturally, depending on what firm 2's price is, we could have any of these situations. Let's look at some different conditions.

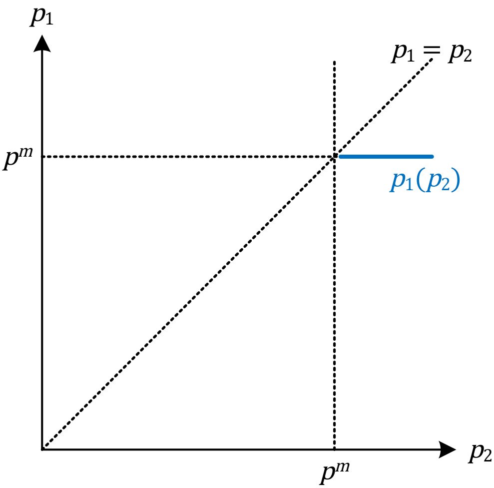
# Bertrand Competition

- Starting with one extreme, suppose $p_2 > p^m$, the monopoly price.
  - If firm 1 chose a price above this, firm 2 would get all of the consumers since their price is lower.
  - If firm 1 matched this price, they would get half of the consumers.
  - If firm 1 set a price lower than this, they would get all of the consumers.
- Naturally, firm 1 would actually want to set $p_1 = p^m$ in this case since that's the price that maximizes their profit level.

# Bertrand Competition



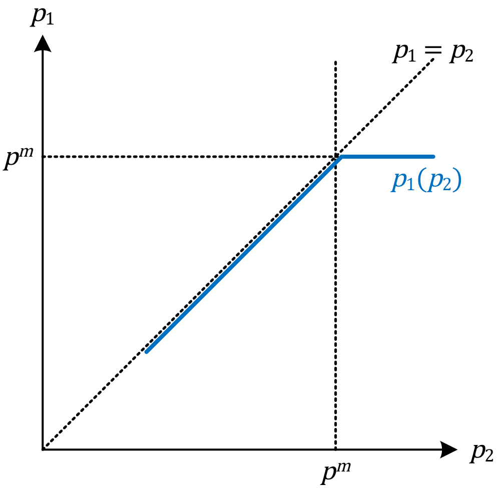
# Bertrand Competition



# Bertrand Competition

- Now, let's suppose firm 2 chose some price that was at most the monopoly price, i.e., $p_2 \leq p^m$. The same results hold.
  - If firm 1 chose a price above this, firm 2 would get all of the consumers since their price is lower.
  - If firm 1 matched this price, they would get half of the consumers.
  - If firm 1 set a price lower than this, they would get all of the consumers.
- It should be obvious that firm 1 wants to set a price lower than firm 2, $p_1 < p_2$. In fact, to maximize profit, firm 1 wants to undercut firm 2 by as little as possible (a single penny). $p_1 = p_2 - \varepsilon$ where $\varepsilon > 0$ is the smallest possible number that firm 1 can pick.
  - That way they get all of the customers while lowering their profit margin by as little as possible.

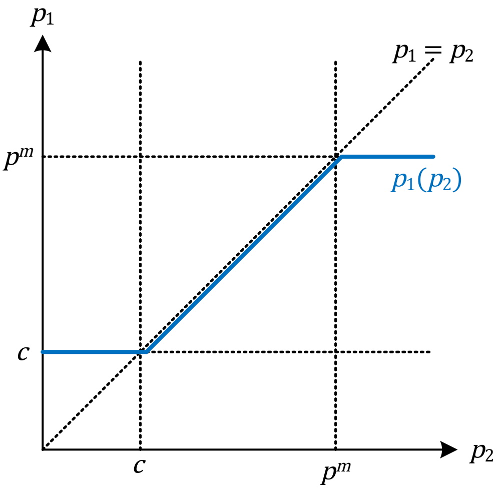
# Bertrand Competition



# Bertrand Competition

- Lastly, suppose firm 2's price were at or below marginal cost, $p_2 \leq c$.
  - If firm 1 chose a price above this, firm 2 would get all of the consumers since their price is lower.
  - If firm 1 matched this price, they would get half of the consumers.
  - If firm 1 set a price lower than this, they would get all of the consumers.
- Pricing below marginal cost would not be an optimal strategy for firm 1. If $p_1 < c$, firm 1 would actually lose money on each unit sold.
  - Thus, the lowest (and only possible) price firm 1 is willing to charge is $p_1 = c$.

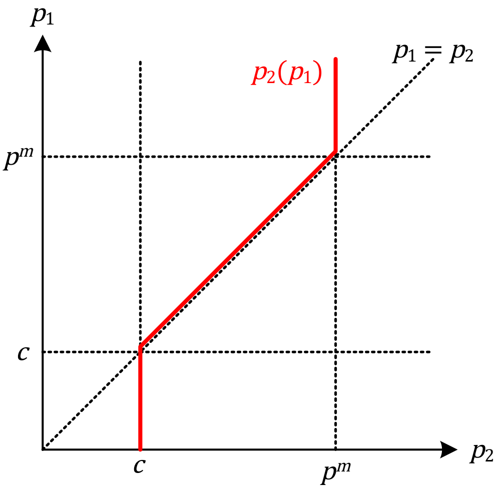
# Bertrand Competition



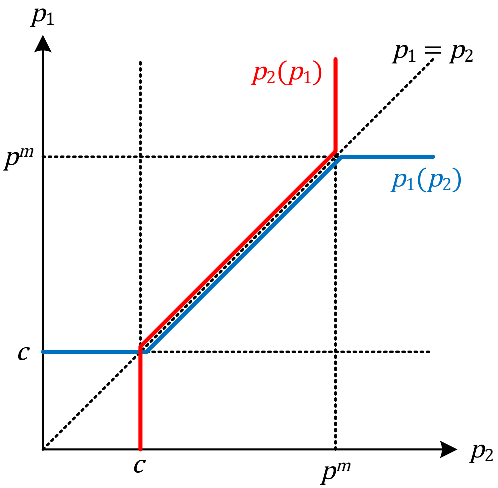
# Bertrand Competition

- The analysis for firm 2 is identical.
  - If firm 1 prices above the monopoly price, $p_2 = p^m$.
  - If firm 1 prices between marginal cost and the monopoly price, $p_2 = p_1 - \varepsilon$, where $\varepsilon > 0$ is the smallest possible number greater than zero.
  - If firm 1 prices at or below marginal cost, $p_2 = c$.

# Bertrand Competition



# Bertrand Competition



# Bertrand Competition

- Returning to our Nash equilibrium solution concept, we know that our equilibrium is when neither firm has any incentive to deviate from their chosen strategy.
  - This occurs where the best response functions intersect.
- There is exactly one intersection point in the previous figure, where $p_1 = p_2 = c$. Thus, both firms price at marginal cost in equilibrium.
  - This should make sense. Each firm wants to undercut the other to claim the whole market. Yet they can't undercut anymore once the price is at marginal cost or they'll suffer losses.
  - Bertrand competition implies that with just two firms, we reach the perfectly competitive equilibrium.
- Since price is set at marginal cost for both firms, economic profit under Bertrand competition equals zero.

# Bertrand Competition

- Bertrand came to a very different conclusion than Cournot.
  - Cournot required an arbitrarily large number of firms to approach the perfectly competitive equilibrium.
  - Bertrand just requires two. (Note: the equilibrium under Bertrand competition doesn't change as we increase the number of firms).
- So who is correct?

# Capacity Constraint

- Bertrand actually makes a very restrictive assumption in his model that Cournot does not.
  - Bertrand assumed that whoever has the lower price can actually supply the whole market.
- For example, suppose at the perfectly competitive price, one million units of the product were demanded.
  - If firm 2 charged the lower price, but could only supply 450,000 units to the market, there would be some residual demand that firm 1 could take advantage of.
  - After the 450,000 customers purchased from firm 2, firm 1 could sell to the other customers at a higher price and actually make some economic profit.

# Capacity Constraint

- Let's relax that assumption made by Bertrand by imposing a capacity constraint.
- Before both firms set their prices, they both have to decide upon their capacity, $k_1$ and $k_2$, respectively.
  - Intuitively, they have to decide how much of the product to stock in advance.
  - They must pay the marginal cost of production in advance before the product is sold.
- Each firm can sell up to their capacity to the market.
- *Note*: The math for this is very complicated. I am going to summarize it here.
  - If you're interested, check out Kreps and Scheinkman (1983). They figured all this out.

# Capacity Constraint

- If either firm sets their capacity too high, they will be left with unsold product.
  - They still have to pay for producing their product, however, so they incur the marginal cost without the associated marginal revenue.
- Thus, we must have that $k_1 + k_2 \leq Q(c)$, the market demand at the perfectly competitive price.
- Furthermore, if both firms are identical, each will produce up to half of perfectly competitive quantity. $k_i \leq \frac{Q(c)}{2}$ for $i = 1, 2$
  - In fact, it may be more profitable to produce less than half of the perfectly competitive quantity..

# Capacity Constraint

- Kreps and Scheinkman (1983) set the matter to rest 100 years after the debate began.
  - In their paper, they showed that in a two-stage game where firms first set their capacities, then their prices, they will choose a capacity equal to the Cournot quantity, and a price equal to the Cournot price.
- Thus, both Cournot and Bertrand competition have the same equilibrium as long as capacity is taking into consideration.
  - In essence, Cournot was correct. Bertrand's model was too restrictive based on its assumption.
  - Math: 1, Bertrand: 0.

# Product Differentiation

- Bertrand competition gets quite a bit more interesting when we differentiate the products a bit.
  - We can model these within the vertical and horizontal differentiation contexts we have looked at.
  - I am going to save these discussions for later though. Let's look at a general form.
- Suppose we had two firms, where the price of one firm impacted the price that the other firm could charge. The demand for the firms are

$$
\begin{aligned}
q_1 &= a - bp_1 + dp_2 \\
q_2 &= a - dp_1 + bp_2
\end{aligned}
$$

where $|d| < b$ denotes how the price of the opposite good affects the demand for the good in question.

  - If $d > 0$, the goods are substitutes. If $d < 0$, the goods are complements.
  - We'll assume that $d > 0$.

# Product Differentiation

- Setting up firm 1's profit maximization problem with constant marginal cost $c$,

$$\max_{p_1} \quad p\left(a - bp_1 + dp_2\right) - c\left(a - bp_1 + dp_2\right)$$

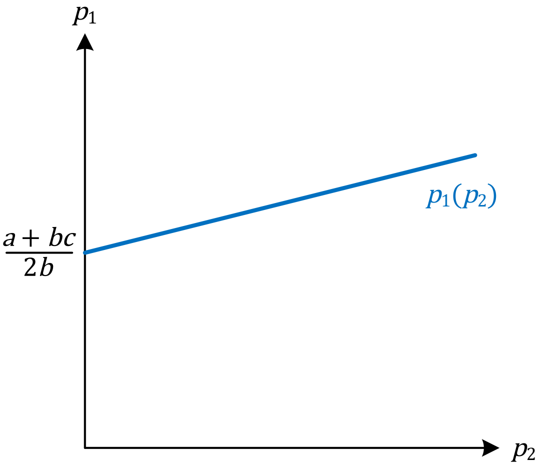
and taking a first-order condition with respect to their price,

$$\frac{\partial \pi_1}{\partial p_1} = a - 2bp_1 + dp_2 + bc = 0$$

Solving this expression for $p_1$ gives us firm 1's best response function,

$$p_1\left(p_2\right) = \frac{a + bc}{2b} + \frac{d}{2b}p_2$$

# Product Differentiation



# Product Differentiation

- Notice that the slope of the best response function, $\frac{d}{2b}$ is bounded between $-\frac{1}{2}$ and $\frac{1}{2}$ since $|d| < b$.
  - The sign of $d$ determines whether the best response function is increasing or decreasing. The strategic behavior of the firms is dependent on whether their goods are substitutes or complements.
  - When $d > 0$ (substitutes), as firm 2 charges a higher price, firm 1 responds by increasing the price of their own good.
  - When $d < 0$ (complements), the opposite relationship holds. Firm 1 lowers their price in response to an increase in price for firm 2.
  - When $d = 0$, the goods to not affect one another, and each firm's best response is the monopoly price.

# Product Differentiation

- Performing the same analysis for firm 2, we first set up their profit maximization problem,

$$\max_{p_2} \quad p_2(a - dp_1 + bp_2) - c(a - dp_1 + bp_2)$$

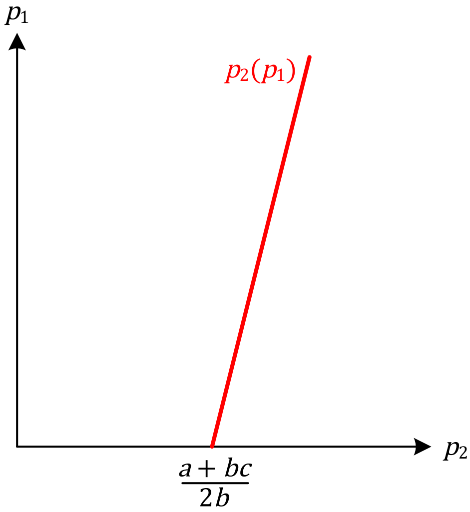
with first-order condition with respect to $p_2$,

$$\frac{\partial \pi_2}{\partial p_2} = a - dp_1 - 2bp_2 + bc = 0$$

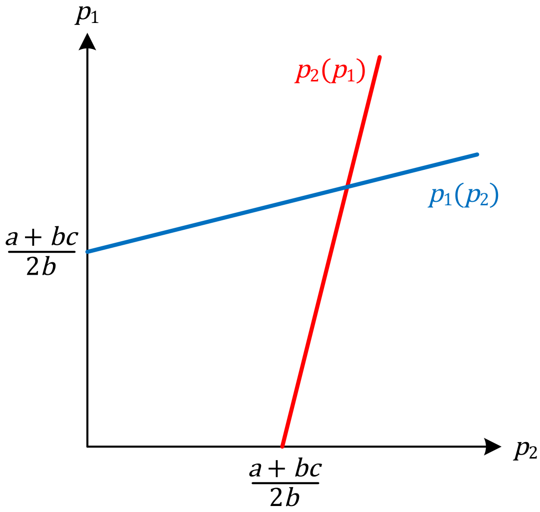
Solving this expression for $p_2$ gives us firm 2's best response function to any price set by firm 1,

$$p_2(p_1) = \frac{a + bc}{2b} + \frac{d}{2b}p_1$$

# Product Differentiation



# Product Differentiation



# Product Differentiation

- As we know, our Nash equilibrium will be where the two best response functions intersect, as neither firm has any incentive to deviate from that strategy. From our two best response functions,

$$
\begin{aligned}
p_1 &= \frac{a+bc}{2b} + \frac{d}{2b}p_2 \\
p_2 &= \frac{a+bc}{2b} + \frac{d}{2b}p_1
\end{aligned}
$$

- Solving these expressions simultaneously (I'll spare you the algebra), we obtain,

$$p_1^* = p_2^* = \frac{a+bc}{2b-d}$$

- From which we can derive the market quantities and profits.

# Product Differentiation

$$p_1^* = p_2^* = \frac{a + bc}{2b - d}$$

- Interestingly, when $d > 0$, each firm charges a price higher than their standard monopoly price.
  - This isn't an error. Since the goods are substitutes, the higher price of their rival's good actually increases the demand for their own good.
- On the other hand, if $d < 0$, each firm prices below their monopoly price.
  - Since the goods are complements, they allow their lower prices to influence the demand of the other product.
- Bertrand competition gets very interesting in the context of product differentiation.

# Price Match Guarantee

- Let's look at one interesting application of the Bertrand model, the price match guarantee.
- Several retailers off a price match guarantee with regard to their competition.
  - If another firm has a lower advertised price, the firm will simply sell their product to a consumer for the same price.
  - Firms claim that this allows consumers to get the best prices on their items.
  - Does it?

# Price Match Guarantee

- Consider two identical firms that compete in prices. Each firm now implements a price match guarantee, where if the opposite firm has a lower price than they do, they will simply sell their good to the consumer at the lowest price.
  - Otherwise, the model is identical to the standard Bertrand model.
- Again, we choose whether to price above, price the same, or price below the opposite firm.
  - The results for when the opposite firm's price is below marginal cost or above the monopoly price are unchanged.
  - Let's look at when the price is between those two extremes.

# Price Match Guarantee

- For firm 1, if firm 2's price is between marginal cost and the monopoly price, $c < p_2 < p^m$, firm 1 has three options:
  - It could price higher than firm 2, $p_1 > p_2$, and then all of firm 1's consumers will simply price match to firm 2's price, and each firm would receive half of the market.
  - It could price the same as firm 2, $p_1 = p_2$, and receive half of the market.
  - It could price lower than firm 2, $p_1 < p_2$, and then all of firm 2's consumers will simply match to firm 1's price, and each firm would receive half of the market.
- Now, no matter what firm 1 does, it will always have half of the market.

# Price Match Guarantee

- Undercutting is now the worst option for firm 1.
  - It lowers the profit margin for both firms, but doesn't actually gain any more consumers.
- In fact, charging a price higher than firm 2's price, $p_1 > p_2$ weakly dominates charging the same price as firm 2.
  - Remember that weak dominance implies that it may not be better, but can never be worse.
  - Taking this to its extreme, charging a price of $p_1 = p^m$ weakly dominates any other price between marginal cost and the monopoly price.

## Price Match Guarantee

- Price match guarantees result in "price creep"
  - Both firms will gradually raise their prices since doing so will not lose them any customers.
  - Eventually both firms will charge the monopoly price, maximizing their own profit levels.
- Thus, price match guarantees do not help consumers at all. They are strictly worse off under them since price competition is essentially removed from the market.

# Summary

- Bertrand competition causes firms to set their prices equal to marginal cost.
  - This is different that Cournot competition, but can be reconciled with a capacity constraint.

## Next Time

- Sequential competition
- Reading: 8.1-8.2 (and maybe more)

# Homework 4-1

- Return to our example on product differentiation, where each firm faced the following demand function,

$$\begin{aligned} q_1 &= a - bp_1 + dp_2 \\ q_2 &= a - dp_1 + bp_2 \end{aligned}$$

1. We already derived the equilibrium prices, but finish the example by deriving the equilibrium quantities and profit level.
2. Are profits increasing or decreasing in $d$? (There is both an intuitive and a mathematical answer to this question)