EconS 425 - Horizontal Differentiation

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Industrial Organization

Introduction

- Today we'll finish up our discussion on product differentiation with a brief look at Horizontal Differentiation.
  - Note: This section is not presented in your textbook.

Horizontal Differentiation

- Recall that when a market is horizontally differentiated, consumers can't agree on what quality is.
  - Consumers have some inherent preference about a quality of the good.
  - These different preferences can be substantial, such as the difference between Coke and Pepsi, or they can be minor, like the difference between colors of toothpaste.
  - Sometimes, you just like one thing better than the other.

Horizontal Differentiation

- Most horizontal differentiation analysis is done in the duopoly (2 firms) context.
    - Honestly, this is the much more interesting way to look at horizontal differentiation, but it requires game theory.
- A single firm can offer horizontally differentiated goods, however, and we'll see how that works today.
    - By offering differentiated products, a firm can better serve a diverse market.

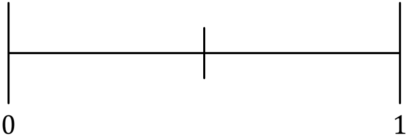
Horizontal Differentiation

- First, we need to figure out how we differentiate our consumers.
- One of the most common methods is the Hotelling line.
  - Traditionally, these were designed for models of spatial discrimination. Imagine a town with a single street, with everyone living somewhere along that street.
  - People had to drive some distance to get to the store in town (which was located somewhere along that road). That drive is costly.
  - The firm must consider that cost when pricing its goods, or it may find itself losing customers.

Horizontal Differentiation

- We can abstract this to talk about differentiated products.
  - Instead of a single street, we can say that consumers are different in their preferences for a good. Where their ideal preference is aligns with where they "lived" in the spatial discrimination model.
  - Put one one extreme on one end of the line, and the other extreme on the other. We can normalize the line such that it ranges from 0 to 1.

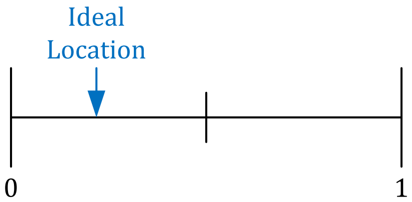
Horizontal Differentiation



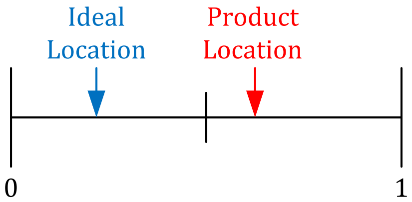
Horizontal Differentiation

- We say our consumers' preferences are distributed along the line by some probability density function, $f(\theta)$.
  - To make it simple, we'll say that they are uniformly distributed, $f(\theta) = 1$.
- The firm also picks a location along the Hotelling line that defines its product.
- The further that a consumer's ideal preference is from the product they are being sold, the worse off they are.
  - They incur a psychic "transportation cost" in terms of their utility.
- Otherwise, the consumers are all identical.

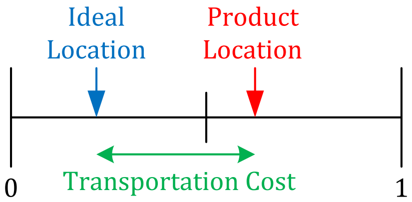
Horizontal Differentiation



Horizontal Differentiation



Horizontal Differentiation



Horizontal Differentiation

- We don't have to use a Hotelling line, or even a line in general.
  - Some models shape consumer preferences as a circle. The Salop Circle model allows for preferences to circle back around to the original point.
- For the most part, we stick to the Hotelling line due to its simplicity.

Horizontal Differentiation

- Model time!
- Starting with consumer i, they have the following surplus,

$$CS_i = K - t(\theta_i, \bar{\theta}) - p$$

where K is some inherent value that the consumer places on the good, and $t(\theta_i, \bar{\theta})$ is the transportation cost as a function of θ_i, the location of the consumer's ideal product, and $\bar{\theta}$, the product's location.

  - $t(\theta_i, \bar{\theta})$ can take many different shapes. We will be using a linear form, $t(\theta_i, \bar{\theta}) = t(\theta_i - \bar{\theta})$ when $\theta_i > \bar{\theta}$ and $t(\theta_i, \bar{\theta}) = t(\bar{\theta} - \theta_i)$ when $\theta_i < \bar{\theta}$. The quadratic form of $t(\theta_i, \bar{\theta}) = (\theta_i - \bar{\theta})^2$ is also frequently used.

Horizontal Differentiation

- The firm has a few choices:
  - Set a single price and try to sell a single product all consumers.
  - Set a single price and try to sell a single product to a certain portion of consumers.
  - Price discriminate based on a consumer's ideal product location (if that is observable).
  - Sell multiple types of their product to different segments of the market.

Horizontal Differentiation

- If the firm wanted to set a single price and try to sell a single product to all consumers, it should be fairly intuitve that the firm's product location should be exactly in the middle of the Hotelling line, $\bar{\theta} = 0.5$.
  - We can make this endogenous, but that would require game theory.
- If the firm wants to make sure everyone buys, it has to set a price such that even those who are at the very ends of the Hotelling line still want to buy. Let $r \equiv \left|\theta_i - \bar{\theta}\right|$, be the distance between consumer i's ideal location and the product location.

Horizontal Differentiation

- In this case, the consumers at the end of the line have $r = 0.5$. To guarantee that they purchase the product, we must have,

$$\begin{aligned} K - tr - p &\geq 0 \\ K - 0.5t - p &\geq 0 \end{aligned}$$

- Since the firm wants to maximize profits, this equation will bind, and solving for p, the price we can charge to the whole market is,

$$p^* = K - 0.5t$$

- *Note*: since price must not be negative, we must have that $0.5t \leq K$ in order for the firm to serve the whole market.

Horizontal Differentiation

- The firm's profits are

$$\pi_1 = \int_0^1 (p - c) f(\theta) d\theta$$

where c is our constant marginal cost of production. For simplicity, we'll assume that $c = 0$. Since our consumers are uniformly distributed, $f(\theta) = 1$, we can substitue in our price and integrate,

$$\begin{aligned} \pi_1 &= \int_0^1 (K - 0.5t) dr \\ &= (K - 0.5t) r|_0^1 \\ &= K - 0.5t \end{aligned}$$

- Thus, as t increases, consumers have more of a distaste from consuming a product different from their ideal, and as a result, the price and profit level fall.

Horizontal Differentiation

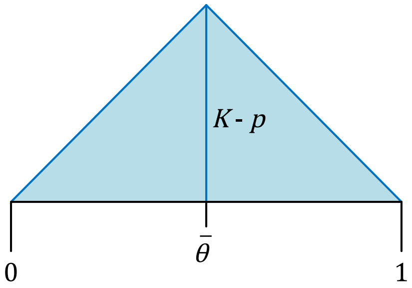
- The consumers at the ends of the Hotelling line receive no consumer surplus, while the consumer at $\theta_i = 0.5$ receives $K - t(0) - (K - 0.5t) = 0.5t$ of consumer surplus.
- We can sum up the consumer surplus of this market as

$$\begin{aligned} CS &= \int_0^{0.5} \left[K - t(\bar{\theta} - \theta_i) - p\right] f(\theta) d\theta \\ &\quad + \int_{0.5}^{1} \left[K - t(\theta_i - \bar{\theta}) - p\right] f(\theta) d\theta \end{aligned}$$

- Since we have a uniform (symmetric) distribution, this simplifies nicely to

$$\begin{aligned} CS &= 2\int_0^{0.5} (0.5t - tr)\, dr \\ &= 2\left(0.5tr - 0.5tr^2\right)\Big|_0^{0.5} \\ &= 0.25t \end{aligned}$$

Horizontal Differentiation



Horizontal Differentiation

- What if the firm wanted to increase its price by leaving out a segment of the market?
  - It could chose a value of $r < 0.5$ along with price to maximize profits.
- In this case, its profit maximization problem becomes

$$\max_{p,r} \int_{\bar{\theta}-r}^{\bar{\theta}+r} (p-c) f(\theta) d\theta$$

Again, imposing $c = 0$ and our uniform distribution, we have

$$\max_{p,r} \int_{\bar{\theta}-r}^{\bar{\theta}+r} p d\theta$$

- This is subject to the constraint that the consumer located r away from the product's location having non-negative surplus,

$$K - tr - p \geq 0$$

Horizontal Differentiation

$$K - tr - p \geq 0$$

- Again, in order to maximize profit, the firm will make sure this expression binds. Solving for p,

$$p = K - tr$$

and substituting this back into our profit maximization problem gives us

$$\begin{aligned} & \max_{p,r} \int_{\bar{\theta}-r}^{\bar{\theta}+r} p d\theta \\ = & \max_{r} \int_{\bar{\theta}-r}^{\bar{\theta}+r} (K - tr)\, d\theta \\ = & \max_{r} (K - tr)\theta|_{\bar{\theta}-r}^{\bar{\theta}+r} \\ = & \max_{r} 2r(K - tr) \end{aligned}$$

Horizontal Differentiation

$$\max_{r} \quad 2r(K - tr)$$

- Taking a first-order condition with respect to r,

$$2\left[K - tr + -tr\right] = 0$$

  and solving for r gives us our equilibrium distance from the product location,

$$r^* = \frac{K}{2t}$$

- Plugging this back into our constraint gives us our equilibrium price

$$p^* = K - tr^* = \frac{K}{2}$$

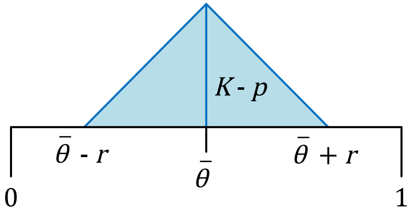
- Our profits are

$$\pi_2 = 2r^*(K - tr^*)$$
$$= 2\left(\frac{K}{2t}\right)\left(K - t\frac{K}{2t}\right) = \frac{K^2}{2t}$$

with consumer surplus,

$$CS = 2\int_0^{r^*}(K - tr - p^*)dr$$
$$= r^*(K - p^*) = \frac{K^2}{4t}$$

Horizontal Differentiation



Horizontal Differentiation

- A couple things to note:
  - We had to assume that $r < 0.5$ for this model. Otherwise, the firm would set $r = 0.5$ and serve the whole market. Thus,

$$r = \frac{K}{2t} < 0.5$$
$$K < t$$

    must hold if we want to implement this method.
  - As long as $K < t$, this method produces higher profits than serving the whole market, i.e., $\pi_2 > \pi_1$.

Horizontal Differentiation

- Suppose now that the firm could identify each consumer's ideal location (or in the case of the spatial model, where they live).
  - If that were the case, and assuming they couldn't resell the good, why not use first-degree price discrimination? The firm could charge each individual consumer their valuation (surplus)

$$K - t\left|\bar{\theta} - \theta_i\right| - p_i \geq 0$$
$$p_i = K - t\left|\bar{\theta} - \theta_i\right|$$

  and extract all of the surplus from the market.

Horizontal Differentiation

- The firm's profits are

$$\pi_3 = \int_0^1 (p_i - c) f(\theta_i) d\theta_i$$

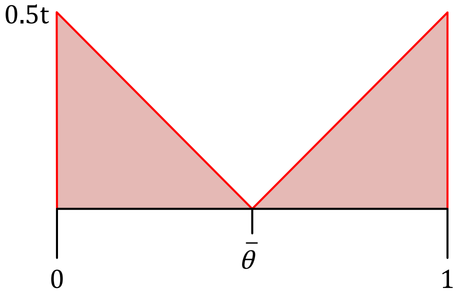
and imposing our simplifications onto this model, we have

$$\begin{aligned}\pi_3 &= 2\int_0^{0.5} (K - tr)\, dr \\ &= 2\left(Kr - \frac{1}{2}tr^2\right)\Big|_0^{0.5} \\ &= K - \frac{t}{4}\end{aligned}$$

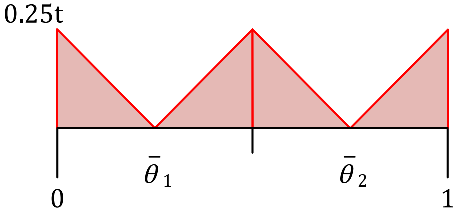
Horizontal Differentiation

- Let's now consider what would happen if the firm could offer two differentiated versions of its product.
  - The firm would have to choose where along the Hotelling line to locate these two versions. We'll assume that $\bar{\theta}_1 = 0.25$ and $\bar{\theta}_2 = 0.75$.
- The total transportation cost among all consumers will fall significantly. This allows the firm to charge a higher price, as it doesn't have to cater to the outskirts of the market.
  - Interestingly, consumers in the middle of the market now have the highest transportation cost, when they had the lowest before.

Horizontal Differentiation



Horizontal Differentiation



Horizontal Differentiation

- Suppose we wanted each consumer in the market to purchase at least one of the goods. We follow the same steps as before from the first example.
- In this case, the consumers with the highest transportation cost are a distance of $r = 0.25$ away from each of the differentiated products.
  - Thus, we must ensure that their surplus is at least zero to get them into the market.
  $$K - tr - p_j \geq 0$$
  where p_j is the price charged for product j. Under profit maximization, we have that
  $$p = p_1 = p_2 = K - 0.25t$$
- All consumers with $\theta_i < 0.5$ buy product 1 and all consumers with $\theta_i > 0.5$ buy product 2.

Horizontal Differentiation

- The firm's profits are the same as in the first case,

$$\pi_4 = \int_0^1 (p - c) f(\theta) d\theta$$

and simplifying,

$$\begin{aligned} \pi_4 &= 4 \int_0^{0.25} (K - 0.25t)\, dr \\ &= 4 (K - 0.25t)\, r|_0^{0.25} \\ &= K - 0.25t \end{aligned}$$

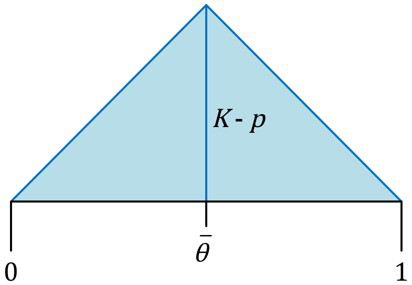
Horizontal Differentiation

- For consumer surplus, we can integrate as before, or just use a triangle formula,

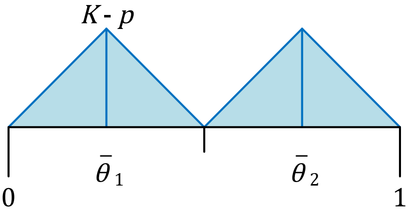
$$
\begin{aligned}
CS &= 2 * \frac{1}{2}(0.5)(K - p^{*}) \\
&= 0.5(0.25t) = 0.125t
\end{aligned}
$$

- Interestingly, by adding a second version of the product, consumer surplus falls,

Horizontal Differentiation



Horizontal Differentiation



Horizontal Differentiation

- We can extend our analysis to the other cases, as well.
  - In general, adding the second type of product increases profits while decreasing consumer surplus, as the firm is able to extract what the consumers were originally spending for transportation.
- Being able to discriminate based on location always yields the highest profit level.

Summary

- Horizontal differentiation allows a firm to design its product with a characteristic some customers may like, while others would prefer something completely different.
  - By offering several versions of the same product, the firm can capture more of the consumer surplus and increase their profits.

Next Time

- Budling and tying. (For real this time)
- Reading: 6.3.

Homework 3-2

- Return to our model of horizontal differentiation, but replace the linear transportation cost with a quadratic one, $t(\theta_i, \bar{\theta}) = t(\theta_i - \bar{\theta})^2 = tr^2$. Everything else remains the same.
  1. If the firm wanted to sell to all consumers, what relationship between K and t must hold?
  2. Suppose $K < 0.75t$ and the firm wanted to sell to only a portion of the market, i.e., $r < 0.5$. Find the equilibrium values for r and p.
  3. How do the results from a quadratic transportation cost compare to those from a linear transportation cost? Does this difference make sense?