EconS 425 - Vertical Differentiation

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Industrial Organization

Introduction

- For our next couple of lectures, we are going to see what happens when firms sell products that are not identical.
    - These products can vary in quality, defining characteristics, or something as simple as its color.
- Note: My office hour on Wednesdays is permanently being moved to 2:00 PM.

Product Differentiation

- Firms may want to offer different versions of the same product.
  - By diversifying their product line, they can attract different consumers with varying tastes and (of course) extract more profit from the market.
- At the same time, if there were multiple firms in a market, they may want to differentiate their products to increase their market power.
  - We'll talk about this at a later time (we'll need some game theory).

Product Differentiation

- Product differentiation can also be quite subtle.
  - Is Cougar Gold sold in Pullman the same as Cougar Gold sold in Spokane?
  - No. There are additional costs involved to ship Cougar Gold from one location to another. We can actually treat these two products (while identical in appearance) as separate.

Product Differentiation

- Product Differentiation often leads to price discrimination.
  - Be careful, however. We may see situations where different groups are charged different prices, yet there is not price discrimination going on.
  - For example, suppose gas were being sold in Pullman for \$3 a gallon, while it was being sold for \$2 a gallon in Colfax (ignore taxes, etc.). Now suppose the marginal cost for gas was \$2 in Pullman and \$1 in Colfax. In this case, the prices merely take into consideration the difference in marginal costs.
- To determine if price discrimination is happening, we want to see if the net prices (price less costs) differ across markets.

Product Differentiation

- Product differentiation easily satisfies our necessary conditions for price discrimination.
- By offering different versions of a good, we can design it such that different consumers want to purchase different goods.
  - This will cause consumers to sort themselves into their own groups, even if we can't observe their characteristics.
  - Also, a group will want their product more than the other, reducing the chance of arbitrage.

Product Differentiation

- We classify product differentiation into two broad categories, vertical and horizontal. We separate them based on how consumers view quality.
  - With Vertical Differentiation, consumers can all agree "what quality is," but they have different willingnesses to pay for quality.
    - Examples: Lexus vs. Toyota, Banana Republic vs. GAP vs. Old Navy.
  - With Horizontal Differentiation, consumers can't agree on what quality is, and have different preferences for different characteristics.
    - Example: Different kinds of Colgate toothpaste.

Vertical Differentiation

- Today, we'll focus on vertical differentiation.
  - If a firm can offer different versions of the same product that vary in their quality, how should they price them?
- We will use some technniques that we saw in second-degree price discrimination, namely the participation and incentive compatibility constraints.
  - They're a bit easier in this context.

Vertical Differentiation

- Let's start with a simple model.
- A firm can produce two versions of a good, one of low quality and one of high quality. The marginal costs of producing these goods are c_L and c_H, respectively and $c_L < c_H$.
  - The quality of the goods is predetermined (exogenous). We'll relax this later.
- There are N_1 consumers who value both versions of the good less (the low type), and N_2 consumers who value both versions of the good more (the high type).
  - We are unable to observe any differences between the consumers.
  - Perhaps the high type has more income than the low, or the low type is just a cheapskate.
- Consumers have unit demand (They will only buy one unit, or none at all).

Vertical Differentiation

- The low type consumer has a reservation price of V_1^L for the low quality good, and V_1^H for the high quality good. Likewise, the high type consumer has a reservation price of V_2^L for the low quality good and V_2^H for the high quality good. We can rank the valuations as following.

$$\begin{array}{lll} V_1^H & > & V_1^L \\ V_2^H & > & V_1^H \end{array} \qquad \begin{array}{l} V_2^H > V_2^L \\ V_2^L > V_1^L \end{array}$$

  thus we know that the high type values both versions of the good more than the low type, and both consumers value the high quality good more than the low quality good.
- We'll also assume that all reservation prices are above their respective costs, otherwise it would be inefficient for a consumer to buy the product. We'll also assume

$$V_i^H - V_i^L > c_H - c_L \quad \text{for} \quad i = 1, 2$$

Vertical Differentiation

- If the firm knows everything about the consumers except for which consumer is which, the firm has three viable options:
  - Offer only the high quality good, and sell it to both types of consumer.
  - Offer only the high quality good, and sell it to only the high type.
  - Offer both types of good, and target a different type of consumer with each.

Vertical Differentiation

- If the firm offers only the high quality good, but wants to sell it to both types of consumers, the most they can charge is V_1^H. Anything higher than that, and it would be better for the low type consumer to not buy the good.
- The firm's profit level is

$$\begin{aligned} \pi_1 &= (N_1 + N_2)(p^H - c_H) \\ &= (N_1 + N_2)(V_1^H - c_H) \end{aligned}$$

- Consumer 1 receives no surplus, while consumer 2 receives a surplus of $V_2^H - V_1^H$.

Vertical Differentiation

- Next, the firm could offer only the high quality good, but sell it only to the high type consumer. In this case, it can charge a much higher price, V_2^H, but it loses out on all of the low type consumers.
- The firm's profit level is

$$\begin{aligned} \pi_2 &= N_2(p^H - c_H) \\ &= N_2(V_2^H - c_H) \end{aligned}$$

- Neither consumer receives any surplus in this case. High types pay their whole valuation, while the low types don't buy.

Vertical Differentiation

- Lastly, the firm could offer both types of good, and target the high quality to the high type and the low quality to the low type.
  - We have to be aware, though, that one consumer may want to purchase the product not intended for them. This requires us to use participation and incentive compatibility constraints.
- As before, we'll have two participation constraints and two incentive compatibility constraints, one for each type of consumer. Again, only one participation constraint and one incentive compatibility constraint will bind in equilibrium.
  - I'll spoil it and tell you that the participation constraint for the low type and the incentive compatibility constraint for the high type bind.

Vertical Differentiation

- Our constraints are

$$\underbrace{V_1^L - p^L}_{\text{Consumer 1's surplus}} = 0 \qquad (PC_1)$$

$$\underbrace{V_2^H - p^H}_{\substack{\text{Consumer 2's surplus} \\ \text{if they buy high quality}}} = \underbrace{V_2^L - p^L}_{\substack{\text{Consumer 2's surplus} \\ \text{if they buy low quality}}} \qquad (IC_2)$$

- We can rearrange these to solve for prices,

$$\begin{aligned} p^L &= V_1^L \\ p^H &= V_2^H - V_2^L + \underbrace{V_1^L}_{p^L} \end{aligned}$$

Vertical Differentiation

$$\begin{aligned} p^L &= V_1^L \\ p^H &= V_2^H - V_2^L + V_1^L \end{aligned}$$

- Now, we can calculate profits for the firm,

$$\begin{aligned} \pi_3 &= N_1(p^L - c_L) + N_2(p^H - c_H) \\ &= N_1(V_1^L - c_L) + N_2(V_2^H - V_2^L + V_1^L - c_H) \end{aligned}$$

- Once again, the low type consumer receives no surplus, but the high type receives a surplus of $V_2^L - V_1^L$

Vertical Differentiation

- Which of the three cases should the firm choose?
  - The one that gives the most profits, of course!
- We can compare the profit levels across cases, and see which ones are the best, given what we know about our consumers. Starting with the first two cases, the firm would rather only sell the high quality good to the high type consumer instead of selling to both consumers if

$$\begin{aligned} \pi_2 &> \pi_1 \\ N_2(V_2^H - c_H) &> (N_1 + N_2)(V_1^H - c_H) \end{aligned}$$

Vertical Differentiation

$$N_2(V_2^H - c_H) > (N_1 + N_2)(V_1^H - c_H)$$

- We can rearrange this expression to obtain

$$N_2(V_2^H - V_1^H) > N_1(V_1^H - c_H)$$

- This expression is actually quite intuitive.
  - The left-hand side, $N_2(V_2^H - V_1^H)$, is the amount of extra profit the firm can squeeze out of the high type if the firm sells only to them.
  - The right-hand side, $N_1(V_1^H - c_H)$, is how much profit the firm would get from the low type by including them in the market.

Vertical Differentiation

$$N_2(V_2^H - V_1^H) > N_1(V_1^H - c_H)$$

- This condition is not guaranteed to hold.
  - If N_1 is much larger than N_2, i.e., there are a lot more low types than high types, the condition is unlikely to hold.
  - If the high type has a much larget reservation price than the low type relative to the marginal cost, the condition is likely to hold.
- It is a case by case basis that you will have to analyze.

Vertical Differentiation

- What about the third case where the firm offers both products? It would be better for the firm to do this (assuming $\pi_2 > \pi_1$) if

$$\pi_3 > \pi_2$$

$$N_1(V_1^L - c_L) + N_2(V_2^H - V_2^L + V_1^L - c_H) > N_2(V_2^H - c_H)$$

which rearranges to

$$N_1(V_1^L - c_L) > N_2(V_2^L - V_1^L)$$

Vertical Differentiation

$$N_1(V_1^L - c_L) > N_2(V_2^L - V_1^L)$$

- Again, this also has some great interpretation,
  - The left-hand side, $N_1(V_1^L - c_L)$ is how much profit the firm makes by letting the low type into the market and selling them the low quality good.
  - The right-hand side, $N_2(V_2^L - c_H)$ is how much profit the firm loses to consumer surplus to the high type, since it has to lower the price they charge due to incentive compatibility.
- Naturally, the firm chooses whichever method yields the highest profits (again, it depends on parameters).

Vertical Differentiation

- Let's make the model a little harder. So far we've let quality be determined outside of the model, but now were going to let the firm choose the quality they offer along with the price.
- We need add a few things to the model.
  - The firm can choose the quality, s_j of both of its j products.
  - Quality is costly. To add quality to a product, the firm must pay some cost function $c(s)$ which is increasing (always going up) and convex (getting steeper). Higher quality adds a lot of cost to the firm.
  - The consumer valuation of the product is $\theta_i s^j$, where θ_i is consumer i's preference for quality. Let $\theta_2 > \theta_1$, so the high type values quality more than the low type.

Vertical Differentiation

- Going back to our three cases, if the firm sells a single quality of the good to all consumers, its profit maximization problem is

$$\max_{p,s} \quad (N_1 + N_2)(p - c(s))$$

subject to two participation constraints

$$\begin{aligned} \theta_1 s - p &\geq 0 \qquad (PC_1) \\ \theta_2 s - p &\geq 0 \qquad (PC_2) \end{aligned}$$

of which only one will bind, PC_1.

Vertical Differentiation

$$\theta_1 s - p = 0 \qquad (PC_1)$$

- From this participation constraint, we can solve for price as a function of quality,

$$p = \theta_1 s$$

and substitute this back into our maximization problem

$$\begin{aligned} \max_{p,s} \quad & (N_1 + N_2)(p - c(s)) \\ \max_{s} \quad & (N_1 + N_2)(\theta_1 s - c(s)) \end{aligned}$$

Vertical Differentiation

$$\max_s \quad (N_1 + N_2)(\theta_1 s - c(s))$$

- From here, we take a first-order condition with respect to s to obtain,

$$(N_1 + N_2)(\theta_1 - c'(s)) = 0$$

which reduces to

$$c'(s) = \theta_1$$

- Thus, when the firm sells one quality of good to the whole market, it sets that marginal cost of quality equal to the low type's preference for quality (Their marginal willingness to pay for quality).
  - Again, the low type receives no surplus, while the high type receives a surplus of $(\theta_2 - \theta_1)s^*$.

Vertical Differentiation

- For the case where the firm sells a single quality of good, but only caters to the high type, its maximization problem is

$$\max_{p,s} \quad N_2(p - c(s))$$

subject to a single participation constraint,

$$\theta_2 s - p \geq 0 \qquad (PC_2)$$

which will bind in equilibrium.

Vertical Differentiation

- Our results are very similar to those in the first case. We will find that

$$p = \theta_2 s$$

from our participation constraint, and in equilibrium,

$$c'(s) = \theta_2$$

the marginal cost of quality will now equal the high type's preference for quality (Their marginal willingness to pay).

- No consumer will have any surplus either (since the low type doesn't buy the product).

Vertical Differentiation

- Lastly, we have our third case, where the firm offers more than one quality of the good. To make this case a little bit easier, I'm going to impose $c(s) = \frac{1}{2}s^2$. The firm's profit maximization problem is,

$$\max_{p_1,p_2,s_1,s_2} \quad N_1 \left(p_1 - c(s_1)\right) + N_2 \left(p_2 - c(s_2)\right)$$

$$\max_{p_1,p_2,s_1,s_2} \quad N_1 \left(p_1 - \frac{1}{2}s_1^2\right) + N_2 \left(p_2 - \frac{1}{2}s_2^2\right)$$

  with two participation and two incentive compatibility constraints. Again PC_1 and IC_2 bind.

Vertical Differentiation

- Our binding constraints are

$$\begin{aligned} \theta_1 s_1 - p_1 &= 0 && (PC_1) \\ \theta_2 s_2 - p_2 &= \theta_2 s_1 - p_1 && (IC_2) \end{aligned}$$

  which we can solve for p_1 and p_2 to obtain

$$\begin{aligned} p_1 &= \theta_1 s_1 \\ p_2 &= \theta_2 s_2 - (\theta_2 - \theta_1) s_1 \end{aligned}$$

- Notice that the low type is charged their valuation, while the high type is charged less than their valuation.

Vertical Differentiation

- Substituting these values back into the profit maximization problem,

$$\max_{p_1,p_2,s_1,s_2} \quad N_1\left(p_1 - \frac{1}{2}s_1^2\right) + N_2\left(p_2 - \frac{1}{2}s_2^2\right)$$

$$\max_{s_1,s_2} \quad N_1\left(\theta_1 s_1 - \frac{1}{2}s_1^2\right) + N_2\left(\theta_2 s_2 - (\theta_2 - \theta_1)s_1 - \frac{1}{2}s_2^2\right)$$

and calculate first-order conditions for s_1 and s_2 to obtain,

$$\begin{aligned} N_1(\theta_1 - s_1) - N_2(\theta_2 - \theta_1) &= 0 \\ N_2(\theta_2 - s_2) &= 0 \end{aligned}$$

Vertical Differentiation

$$\begin{aligned} N_1(\theta_1 - s_1) - N_2(\theta_2 - \theta_1) &= 0 \\ N_2(\theta_2 - s_2) &= 0 \end{aligned}$$

- Solving for s_1 and s_2, we have

$$\begin{aligned} s_1^* &= \theta_1 - \frac{N_2}{N_1}(\theta_2 - \theta_1) \\ s_2^* &= \theta_2 \end{aligned}$$

- Again, notice that the low type receives a lower quality than the efficient level ($c'(s) = s$, and $s_i^* = \theta_i$ is the efficient level) while the high type receives the efficient level.

Vertical Differentiation

- The results of vertical differentiation are very similar to those we saw with menu pricing.
  - In menu pricing, the low type was given a smaller quantity of the good, while in vertical differentiation, the low type is simply given a lower quality good.
  - The low type pays their valuation in both cases, while the high type pays a lower price to ensure incentive compatibility.

Vertical Differentiation

- It's really interesting that the firm gives the low type a purposefully lower quality good. The book has a great quote on this from 1849 (well before these ideas were formalized):
  - "It is not because of the few thousand francs which would have to be spent to put a roof over the third-class seats that some company or other has open carriages with wooden benches. What that company is trying to do is prevent the passengers who can pay the second class fare from travelling third class; it hits the poor, not because it wants to hurt them, but to frighten the rich. And it is again for the same reason that the companies, having proved almost cruel to the third-class passengers and mean to the second-class ones, become lavish in dealing with first-class passengers. Having refused the poor what is necessary, they give the rich what is superfluous. (Jules Dupuit, 1849)"

Vertical Differentiation

- What Dupuit was explaining is incentive compatibility.
  - If they improved the third-class seats, many passengers who could afford second-class tickets would likely ride third-class in order to save some money. Thus the quality is kept so low to guarantee that those passengers pay the second-class price.
  - On the other hand, since they can't lower the second-class quality relative to first-class, they have to make first-class extravagant in order to ensure that first-class passengers wouldn't rather ride second-class.
- Basically, it's not because the train operator is cheap, it's because they want to get the most profit out of their passengers as possible.

Vertical Differentiation

- Purposely lowering the quality of a product is actually very common today.
  - If a firm wants to sell to a segment of the market that is excluded due to the price being too high, they can offer a lower quality version of that product.
  - They have to make sure that the low quality product doesn't take from their high type consumers, though.

Vertical Differentiation

- Example: Table vs. Cooking wine.
  - Many people like to use wine while cooking. Wine, however, can be an expensive ingredient for a meal.
  - As a wine firm, if you wanted to offer a cheap type of wine specifically marketed for cooking, you run into the problem where consumers may want to drink that wine instead of the more expensive table wine meant for drinking.
  - How do you solve this problem? You take the table wine, purposely damage it by adding enough salt to it to make it undrinkable, then sell it as cooking wine.
- Other examples: Laser printers, student versions of software, iPhone 5C, etc.

Vertical Differentiation

- What's interesing about the case of cooking wine is that the marginal cost to produce the low quality product is actually higher than the marginal cost to produce the high quality product, i.e., $c_L > c_H$.
  - Intuitively, you can't make the low quality product without starting from the high quality product, so it has to be more expensive.
- Remember from our first example that the firm will want to offer both qualities if

$$N_1(V_1^L - c_L) > N_2(V_2^L - V_1^L)$$

and thus, a "purposely damaged" product like this will be harder to implement in this case.

Vertical Differentiation

$$N_1(V_1^L - c_L) > N_2(V_2^L - V_1^L)$$

- However, if the high type consumer puts a very low valuation on the low quality good (V_2^L, think of a master chef wanting to use cooking wine rather than table wine) relative to the low type's valuation (V_1^L), this constraint can still hold, implying that it might be better for the firm to sell both qualities of the good.
  - The interesting thing is that if purposely damaging a good allows more consumers into the market, it actually reduces deadweight loss.

Summary

- Vertical Differentiation is a useful tool to a firm in order to implement price discrimination.
  - If quality can be varied, the firm can get consumers to self-select into the best product for themselves.

Next Time

- A brief explanation of horizontal differentiation, then budling and tying.
  - I don't like the book's coverage of horizontal differentiation, so skip over section 6.2 for now; we'll cover it in more depth after we do game theory.
- Reading: 6.3.

Homework 3-1

- Consider a firm that wants to offer two different qualities of a good marketed to a low type and a high type consumer. The firm cannot tell the difference between the two consumers, but knows that the high type values the good twice as much as the low type, i.e., $\theta_1 = 1$ and $\theta_2 = 2$. In order to produce quality in their product, the firm faces the cost function $c(s) = \frac{1}{3}s^3$ where s is the quality level. The proportion of low type consumers is γ and the proportion of high type consumers is $1 - \gamma$ where $\gamma \equiv \frac{N_1}{N_1+N_2} = \frac{N_1}{N}$.
  1. Set up the firm's profit maximization problem and list all four constraints.
  2. Assume PC_1 and IC_2 bind. Calculate the equilibrium qualities and prices offered to each type of consumer.
  3. For what values of γ does the firm offer any quality at all to the low type consumer? (*Hint*: look at s_1^*).