OPTIMAL INCOME TAXATION WITH ADVERSE SELECTION AND MONOPSONIST POWER IN THE LABOR MARKET

Abstract

We extend the job market signaling game from chapter 2 between a representative firm and three types of workers with heterogeneity in productivity and costs of working and costs of acquiring education. Workers choose to enter the job market or not, and whether to receive a productivity enhancing education or not. Monopsonist firms choose wages to maximize profit. The government chooses linear taxes on wages and profits and rebates the revenue in lump sum transfers to maximize utilitarian social welfare function composed of the utility of the workers. Policy may offset the labor market power of the firm. The optimal tax policy includes effective negative wage tax rates for workers and a positive profit tax rate which is facilitated through positive transfers to workers that includes a "basic income".

1. Introduction

In the optimal taxation literature based on Mirrlees (1971) it is often assumed a perfectly competitive labor market produces taxpayer income outcomes observed by the government. When information between firms and workers about individual worker productivity is symmetric each worker is compensated according to their marginal productivity. When this information is asymmetric, as in Spence (1973), it depends on how the equilibrium concept is defined. Each worker may or may not be compensated according to their marginal productivity.[1] Alternatively, if the assumption on a perfectly competitive market is relaxed then workers will not be compensated according to their marginal productivity. This creates possible inefficiencies and inequities that may be addressed by tax policy.

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[1] For example, under a Rothschild-Stiglitz equilibrium [Rothschild & Stiglitz (1976)] each contract offered in equilibrium must yield zero profit separately. Alternatively, a Miyazaki-Wilson-Spence equilibrium [Miyazaki (1977), Wilson (1977), and Spence (1978)] yields zero profits across the portfolio of contracts allowing some individually to yield positive profits as long as others balance with negative profits.

If firms have market power, the situation a worker faces will be different than under competition where firms don't have market power. Under perfect competition one would expect an unemployed worker to face multiple offers which would allow them to bid up the offers until they were fully compensated for their productivity. Any worker with a job facing a pay cut would have similar prospects if they quit since this allows them to avoid such a pay cut by switching jobs. Manning (2005) acknowledges multiple frictions in the labor market that contribute to switching costs which leads employers to have significant oligopsony if not monopsony power. Further, there is a growing literature[2] demonstrating concern that firms exert labor market power over workers due to locality of labor markets where workers participate in the labor market through narrow geographic and industry constraints. All this calls into question the usefulness of perfect competition as an assumption when analyzing optimal taxation.

In this paper we will incorporate government tax policy into the labor signaling model of chapter two where the policy includes an income tax rate, a profit tax, and transfers. The policy will be chosen by the government. We will assume the firm acts as a monopsonist and chooses wages before workers make their education and labor supply decisions. Anticipating this the government chooses its policy before the monopsonist employer chooses wages.

Stantcheva (2014) assumes a perfectly competitive labor market and mostly limits her analysis to a Miyazaki-Wilson-Spence separating equilibrium analysis.[3] The source of inefficiency in her model stems from adverse selection in the labor market where firms cannot observe output directly and must contract for pay based on effort measured as hours worked. This results in a "rat race" where high productivity workers are induced by firms to work more hours than is optimal to provide a signal that separates them from lower productivity workers. This means a positive tax rate can correct the rat race labor supply

[2] See Azar, Marinescu, and Steinbaum (2020), Benmelech, Bergman, and Kim (2020), Berger, Herkenhoff, and Mongey (2022), Card, Cardoso, Heining, and Kline (2018), and Lamadon, Mogstad, and Setzler (2022).

[3] An analysis of a Rothschild-Stiglitz equilibrium is confined to an online appendix.

distortion in a similar manner to a Pigouvian tax. Further, this dynamic allows for greater redistribution from higher skill to lower skill workers since high skill workers have less of an incentive to mimic low skill workers relative to the Mirrlees baseline with symmetric information.

Bastani et al. (2015) sets up a similar framework with adverse selection in the labor market where firms contract for pay based on hours worked but focuses on the Rothschild-Stiglitz equilibrium concept with allowance for pooling equilibria. This means that while each individual contract does yield zero profit, more than one worker type can self-select the same contract, which requires the wage paid to be equal to the average of the worker's productivities. This is done to emphasize that income taxation can achieve redistribution through two means, one direct and one indirect. The tax-transfer policy can directly change the income distribution through transfers. Second, the policy can indirectly change the income distribution by altering wages. This creates an explicit trade-off between efficiency gained from revelation of information in a separating equilibrium with equity gains from wage pooling. There is no consideration of individuals who do not work in equilibrium in Bastani et al. or Stancheva.

da Costa & Maestri (2019) assume the same underlying framework with adverse selection in the labor market but relax the assumption that the labor market is perfectly competitive. The assumption of market power for firms means that workers may not be paid their marginal product, which is a potential source of inefficiency. The tax policy they study features labor income tax schedules, a tax on firms' profits, and transfers to workers who do not work and stay out of the labor market, which they characterize as "unemployment benefits". While the model is set up explicitly where multiple firms are randomly matched with a finite number of workers and each worker is matched with a single firm, they show each firm acts equivalently to a single monopsonist firm. They show the optimal tax schedule exhibits negative marginal tax rates for most incomes except for the top of the distribution and must include a positive transfer to individuals who do not work.

As in chapter 2 we rely on the Spence (1973) signaling model of education but where education enhances productivity. We assume there are three types of workers who are heterogeneous in their productivity and costs of work and education. We add an assumption on each worker's net benefit calculated by the difference in their productivity and costs of education and working. We explicitly assume that the high-type, medium-type, and low-type maximize their net benefits when they acquire an education and work at the educated wage, do not acquire education and work at the uneducated wage, and do not acquire education and forego work, respectively. We assume the firm acts as a monopsonist and chooses the educated and uneducated wages before individuals make their education and labor market decisions. We assume in the absence of tax policy the monopsonist employer has an incentive to maximize profit by choosing wages that do not sustain the fully informative separating equilibrium where each worker maximizes net benefits. Anticipating this, the government acts before the firm and chooses the income tax rate, the tax rate on profit, and transfers corresponding to income levels.

We find that given the assumption on worker's net benefits of education and working that the separating equilibrium maximizes total social welfare, but the firm prefers to choose wages that sustain other equilibria rather than the separating equilibrium. Increasing certain transfers favors the sustainability of some equilibria over others by affecting the relative profitability of equilibria for the firm in addition to directly changing the income distribution and indirectly affecting the wage distribution. For example, in the absence of tax policy a firm might choose lower wages that only incentivizes the high-type worker to get an education and work. Therefore, increasing the transfer a worker receives while working without an education can incentivize the medium-type worker to work at a lower wage making it profitable from the firm's perspective to hire them. However, this may also incentivize the firm to adjust wages to hire both the high and medium types to work without an education. Thus, increasing the transfer a worker receives while working with an education can incentivize the high-type worker to still work and receive an education. Increasing the transfer an individual receives when not working while increasing the other two transfers one for one simultaneously increases the utility of all workers while maintaining all the same incentives for workers and the firm from before such a change is implemented. Specific numerical solutions are presented where the optimal policy entails a positive profit tax, negative income tax rates for all workers, and the transfer to the individuals who do not work is positive. The profit tax is used to overcome the market power of the monopsonist.

The paper is organized as follows. Section 2 presents the model and government program. In section 3 we identify the results on social welfare and profit of equilibria and resulting conditions on transfers that maximizes social welfare. Section 4 presents numerical examples and discusses our findings and section 5 concludes the paper.

2. Model

There are three types of workers distinguished by their level of productivity: high, medium, and low. The high, medium, and low productivity workers have production values equal to $\overline{r}_e$, $\tilde{r}_e$, and $\underline{r}_e$, respectively, with investment in education and $\overline{r}_{ne}$, $\tilde{r}_{ne}$, and $\underline{r}_{ne}$, respectively, with no investment in education where $\overline{r}_j > \tilde{r}_j > \underline{r}_j$ and $j = ne, e$. Workers move first and each worker can choose to seek work while investing in their human capital through education that enhances their productivity, $r_e > r_{ne}$ for all types, or not. The worker can also choose not to seek work. This decision depends on a comparison of the educated wage with the uneducated wage, which is determined by the labor market, and whether firms decide to hire them or not.

It is assumed the worker's cost of investing in human capital and working is perfectly inversely correlated with their productivity. Let c_e be the cost of education, where this cost is lowest for the high productivity worker and highest for the low productivity worker, i.e., $\underline{c}_e < \tilde{c}_e < \overline{c}_e$. The cost of working for each type of worker high, medium, and low productivity is $\underline{c}_{new}$, $\tilde{c}_{new}$, and $\overline{c}_{new}$, respectively, when not educated and $\underline{c}_{eew}$, $\tilde{c}_{eew}$, and $\overline{c}_{eew}$, respectively, when educated, where $\underline{c}_k < \tilde{c}_k < \overline{c}_k$ and $k = new, eew$. Total cost after receiving an education and working at the educated wage is $c_{eew} = c_e + c_{ew}$ where $\underline{c}_e < \tilde{c}_e < \overline{c}_e$ which means $\underline{c}_{eew} < \tilde{c}_{eew} < \overline{c}_{eew}$ by definition. It can be the case that $c_{new} \leq$

c_{ew} or $c_{new} \geq c_{ew}$ for all types but it is assumed that $c_{new} < c_{eew}$ for all types[4]. Further, it is assumed $\underline{c}_{eew} - \underline{c}_{new} < \tilde{c}_{eew} - \tilde{c}_{new} < \overline{c}_{eew} - \overline{c}_{new}$. Simply put, the increase in cost from working without an education to working with an education must also preserve the inverse relationship between productivity and cost. This makes it such that education and willingness to work act as a perfect signal for productivity.

Nature determines the true probability a worker is of a certain type where the distribution is defined by parameters $\alpha \in [0, 1]$ and $\beta \in [0, 1]$, which represent the probability a worker is high and medium productivity, respectively. Also, define $n_h = \alpha N, n_m = \beta N,$ and $n_l = (1 - \alpha - \beta)N$ where N is the total number of workers so n_h is the total number of high types, n_m is the total number of medium types, and n_l is the total number of low types. Workers who work receive a wage w_e if educated and w_{ne} if not educated. Workers also receive a lump sum transfer respectively denoted by i_n where the amount depends on whether the worker chooses to work with an education or without or not work at all and $n = ew, new, and\ nw$, respectively. The government imposes a tax on wages denoted by $t \in [0, 1]$ and a tax on profit $T \in [0, 1]$, where the revenue is rebated back to the workers through lump sum transfers. The firm forms a belief about the productivity of a worker after observing their educational choice and whether they are willing to work. Let the parameters $\gamma, \delta,$ and ε denote the conditional probability the firm believes a worker is high, medium, or low productivity, respectively, after observing their educational choice. Similarly, the conditional probability distribution over productivity when they observe no education is defined by parameters $\eta, \theta,$ and λ where the parameters denote the probability the firm believes a worker is high, medium, or low productivity respectively where $\gamma, \delta, \varepsilon, \eta, \theta,$ and $\lambda \in$

[4] There is no reason to generally assume that working with an education is less costly than working without an education or vice versa. However, it must be the case that after factoring in the cost of education the summed cost of education and working with an education must be greater than working without an education or else there is no tradeoff. Workers could produce more output at lower cost thereby making it more efficient for all workers to get an education. In that case, there would be no separating equilibrium by definition.

$[0, 1]$, $\gamma + \delta + \varepsilon = 1$, and $\eta + \theta + \lambda = 1$. Based on these beliefs the firm decides to hire a willing worker or they don't hire them.[5]

Workers receive payoffs $i_{ew} + (1 - t)w_e - c_{eew}$ and $i_{new} + (1 - t)w_{ne} - c_{new}$ when working with an education and without an education, respectively. When not working with an education they receive $i_{nw} - c_e$ and when not working without an education they receive i_{nw}. Each cost c corresponds to $\underline{c}, \tilde{c},$ and $\overline{c}$ for each worker type high, medium, and low. The firm receives a payoff of zero if it does not hire a worker, $r_{ne} - w_{ne}$ when hiring a worker at the uneducated wage, and $r_e - w_e$ when hiring a worker at the educated wage. Each revenue r corresponds to $\underline{r}, \tilde{r},$ and $\overline{r}$ for each worker type, low, medium, and high.

We show in chapter 2 there are ten sustainable equilibria of this game and explain how changes in costs and beliefs expand or contract the range of wages that sustain such equilibria. Now we will expand on this game adding two more stages to the above game that will act as the third stage. In the first stage the government sets tax rates t and T and transfers i_{ew}, i_{new}, and i_{nw}. In the second stage the firm takes tax rates t and T and transfers i_{ew}, i_{new}, and i_{nw} as given and chooses wages w_e and w_{ne}. In the third stage workers take wages w_e and w_{ne}, the tax rate t, and transfers i_{ew}, i_{new}, and i_{nw} as given and the high, medium, and low-type individuals choose to seek work while investing in their human capital through education that enhances their productivity or not or choose to not seek work as in chapter 2. This choice determines the transfer and wage combination they receive in addition to the cost and revenue combination they bear and deliver to the firm. This quadruple combination that results from their education and labor supply choice is defined as (i^I, w^I, c^I, r^I) where this denotes the transfer, wage, cost, and revenue quadruple for individual $I = h, m, l$ for the high, medium, and low type individual. Each individual's quadruple is an element of the following sets:

$$(i^h, w^h, c^h, r^h) \in \{(i_{nw}, 0, 0, 0), (i_{new}, w_{ne}, \underline{c}_{new}, \overline{r}_{ne}), (i_{ew}, w_e, \underline{c}_{eew}, \overline{r}_e)\},$$

[5] Belief formation for the firm is carried out as it is in chapter 2.

$$(i^m, w^m, c^m, r^m) \in \{(i_{nw}, 0, 0, 0), (i_{new}, w_{ne}, \tilde{c}_{new}, \tilde{r}_{ne}), (i_{ew}, w_e, \tilde{c}_{eew}, \tilde{r}_e)\}, \text{ and}$$

$$(i^l, w^l, c^l, r^l) \in \{(i_{nw}, 0, 0, 0), (i_{new}, w_{ne}, \overline{c}_{new}, \underline{r}_{ne}), (i_{ew}, w_e, \overline{c}_{eew}, \underline{r}_e)\}.$$

Each quadruple equals the first element of the set if the given individual chooses to not work, equals the second element if they choose to work without an education, and equals the third element if they choose to work with an education. Each individual takes the costs inherent to themselves, tax rate, transfers, and wages as given and chooses the combination that maximizes their own individual payoffs, respectively,

$$i^h + (1-t)w^h - c^h,$$

$$i^m + (1-t)w^m - c^m,$$

and

$$i^l + (1-t)w^l - c^l.$$

There are 10 different combinations of quadruples that individual types can choose that is sustainable as an equilibrium as a function of tax rates, transfers, and wages as shown in chapter 2. The firm knowing the workers will choose the quadruple that individually maximizes their payoff and taking tax rates t and T and transfers i_{ew}, i_{new}, and i_{nw} as given the firm chooses wages w_e and w_{ne} to maximize

$$(1-T)[n_h(r^h - w^h) + n_m(r^m - w^m) + n_l(r^l - w^l)]$$

The government knowing the firm knows the workers will choose the quadruple that individually maximizes their payoff and knowing the firm will choose wages w_e and w_{ne} to maximize profit, the government chooses tax rates t and T and transfers i_{ew}, i_{new}, and i_{nw} to maximize social welfare, the utilitarian sum of the individual utility of the workers,

$$n_h(i^h + (1-t)w^h - c^h) + n_m(i^m + (1-t)w^m - c^m) + n_l(i^l + (1-t)w^l - c^l)$$

subject to its budget constraint, labelled BC, and the non-negative profit condition for the representative firm, labelled profit,

$$BC: t(n_h w^h + n_m w^m + n_l w^l) + T\big(n_h(r^h - w^h) + n_m(r^m - w^m) + n_l(r^l - w^l)\big) \geq n_h i^h + n_m i^m + n_l i^l$$

$$profit: (1-T)[n_h(r^h - w^h) + n_m(r^m - w^m) + n_l(r^l - w^l)] \geq 0.$$

We will assume the government's budget balances. If it were the case that tax revenues exceeded transfers, then "money is being left on the table" and it is possible to increase utility by increasing transfers. If it were the case the tax revenues were less than transfers, then there is additional value being added into the system in the model and we assume there is no outside value other than what is generated by workers working for the firm. The second constraint on private profit recognizes the firm can't operate with negative profit. In addition, cross subsidization across types may occur through wages. Relaxing the restriction for zero profit makes it possible for equilibria that features cross subsidization within a contract to work with or without education as well as across contracts as long as profits are non-negative.

We will also make the following assumptions on the revenue workers generate and their associated costs for the high, medium, and low type, respectively, which determines their relative net return of their labor,

$$\overline{r}_e - \underline{c}_{eew} > \overline{r}_{ne} - \underline{c}_{new} > 0,$$

$$\tilde{r}_{ne} - \tilde{c}_{new} > \tilde{r}_e - \tilde{c}_{eew} > 0,$$

$$\text{and } 0 > \underline{r}_{ne} - \overline{c}_{new} > \underline{r}_e - \overline{c}_{eew}.$$

The net benefit of the high-type worker working at the educated wage is greater than if they worked at the uneducated wage or chose not to work at all. The net benefit of the medium-type worker working at the uneducated wage is greater than if they worked at the educated wage or did not work at all. The net benefit of the low-type worker not working at all is greater than if they worked at the uneducated wage or the educated wage. These assumptions on net return will determine the government's objective and therefore, its maximization program, as we will show next.

3. Results

3.1 Total Social Welfare of Full Separating Equilibrium

We have our first result that the full separating equilibrium maximizes total social welfare. For clarity when referring to proposition 3.x this refers to a proposition in chapter 2.

Proposition 3.1.1: *Given the definitions of payoffs for the firm and workers, total social welfare, and government budget constraint the separating equilibrium in Proposition 3.1 maximizes total social welfare relative to all other equilibria if*

$$\overline{r}_e - \underline{c}_{eew} > \overline{r}_{ne} - \underline{c}_{new} > 0,$$

$$\tilde{r}_{ne} - \tilde{c}_{new} > \tilde{r}_e - \tilde{c}_{eew} > 0,$$

$$0 > \underline{r}_{ne} - \overline{c}_{new} > \underline{r}_e - \overline{c}_{eew}.$$

The proof appears in the appendix but in words this says that if the net benefit individually is greatest from the high type working at educated wage, the medium type working at the uneducated wage, and the low type not working at all then total net benefits are maximized when each type is slotted into that role in equilibrium.[6] This means that if surplus is greatest in the separating equilibrium, then utility is potentially maximized in the separating equilibrium. This implies the goal of optimal tax policy in this context should be to sustain the separating equilibrium and the following will be part of the solution to the government's problem above:

$$(i^h, w^h, c^h, r^h) = (i_{ew}, w_e, \underline{c}_{eew}, \overline{r}_e),$$

$$(i^m, w^m, c^m, r^m) = (i_{new}, w_{ne}, \tilde{c}_{new}, \tilde{r}_{ne}), \text{ and}$$

$$(i^l, w^l, c^l, r^l) = (i_{nw}, 0, 0, 0).$$

The government's problem can be recharacterized as

$$\max_{t,\ T,\ i_{ew},\ i_{new},\ i_{nw}} n_h\left(i_{ew} + (1-t)w_e - \underline{c}_{eew}\right) + n_m(i_{new} + (1-t)w_{ne} - \tilde{c}_{new}) + n_l i_{nw}$$

subject to

$$IC^{hm}: i_{ew} + (1-t)w_e - \underline{c}_{eew} \geq i_{new} + (1-t)w_{ne} - \underline{c}_{new}$$

$$IC^{hl}: i_{ew} + (1-t)w_e - \underline{c}_{eew} \geq i_{nw}$$

$$IC^{mh}: i_{new} + (1-t)w_{ne} - \tilde{c}_{new} \geq i_{ew} + (1-t)w_e - \tilde{c}_{eew}$$

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[6] In the separating equilibrium cross subsidization can still occur through wages since they are not restricted to being equal to marginal productivity.

$$IC^{ml}: i_{new} + (1-t)w_{ne} - \tilde{c}_{new} \geq i_{nw}$$

$$IC^{lh}: i_{nw} \geq i_{ew} + (1-t)w_e - \overline{c}_{eew}$$

$$IC^{lm}: i_{nw} \geq i_{new} + (1-t)w_{ne} - \overline{c}_{new}.$$

where these constraints are incentive compatibility constraints ensuring that each type self-selects into the appropriate transfer, wage, and cost/revenue combination.

$$BC: t(n_h w_e + n_m w_{ne}) + T\big(n_h(\overline{r}_e - w_e) + n_m(\tilde{r}_{ne} - w_{ne})\big) \geq n_h i_{ew} + n_m i_{new} + n_l i_{nw}$$

$$profit: \pi^1 = (1-T)[n_h(\overline{r}_e - w_e) + n_m(\tilde{r}_{ne} - w_{ne})] \geq 0$$

$$\pi^1 > \pi^k \; for \; all \; k = 2, 3, \dots, 10$$

where the constraint labelled BC is the specific government budget constraint for the separating equilibrium and the constraint labelled profit is the specific non-negative profit condition for the representative firm for the separating equilibrium. The profit tax is necessary if all effective income tax rates are negative, without it the government budget would not balance in this case.

Next, we will develop expressions for the above conditions on profit in equilibrium 1 relative to the other equilibria.

3.2 Total Profit of Full Separating Equilibrium

Solved total profits will be maximized within a given equilibrium when wages w_e and w_{ne} are at their minimum values that still sustain that equilibrium. The derivation of these terms using the minimum of wage values taken from the corresponding proposition for each equilibrium from chapter 2 are given in the appendix of this chapter.

Proposition 3.2.1: *In the absence of tax policy, i.e.* $T = t = i_{ew} = i_{new} = i_{nw} = 0$*, total profit in the separating equilibrium in Proposition 3.1 is greater than total profits in the equilibria in propositions 3.3, 3.4, 3.5, 3.6, 3.7, 3.8, and 3.10 if*

$$\overline{r}_e - \underline{c}_{eew} > \overline{r}_{ne} - \underline{c}_{new} > 0,$$

$$\tilde{r}_{ne} - \tilde{c}_{new} > \tilde{r}_e - \tilde{c}_{eew} > 0, and$$

$$0 > \underline{r}_{ne} - \overline{c}_{new} > \underline{r}_e - \overline{c}_{eew}.$$

Proposition 3.2.2: *In the absence of tax policy, i.e.* $T = t = i_{ew} = i_{new} = i_{nw} = 0$*, total profit in the partial pooling equilibrium in Proposition 3.2 is greater than total profit in the separating equilibrium in Proposition 3.1 if*

$$\overline{r}_e - \underline{c}_{eew} > \overline{r}_{ne} - \underline{c}_{new} > 0,$$

$$\tilde{r}_{ne} - \tilde{c}_{new} > \tilde{r}_e - \tilde{c}_{eew} > 0,$$

$$0 > \underline{r}_{ne} - \overline{c}_{new} > \underline{r}_e - \overline{c}_{eew}, and$$

$$n_m(\tilde{r}_{ne} - \tilde{c}_{new}) < n_h(\tilde{c}_{new} - \underline{c}_{new}).$$

That is, in the absence of policy where $T = t = i_{ew} = i_{new} = i_{nw} = 0$, it must be the case that the loss from not hiring the medium worker at the uneducated wage is less than the savings from no information rent paid to the high-type worker for profit in equilibrium 2 to be greater than profit in equilibrium 1.

Proposition 3.2.3: *In the absence of tax policy, i.e.* $T = t = i_{ew} = i_{new} = i_{nw} = 0$*, total profit in the partial pooling equilibrium in Proposition 3.9 is greater than total profit in the separating equilibrium in Proposition 3.1 if*

$$\overline{r}_e - \underline{c}_{eew} > \overline{r}_{ne} - \underline{c}_{new} > 0,$$

$$\tilde{r}_{ne} - \tilde{c}_{new} > \tilde{r}_e - \tilde{c}_{eew} > 0,$$

$$0 > \underline{r}_{ne} - \overline{c}_{new} > \underline{r}_e - \overline{c}_{eew}, and$$

$$n_h(\overline{r}_e - \underline{c}_{eew}) - n_h(\overline{r}_{ne} - \underline{c}_{new}) + n_m(\tilde{r}_{ne} - \tilde{c}_{new}) < n_h(\tilde{c}_{new} - \underline{c}_{new}).$$

That is, in the absence of policy where $T = t = i_{ew} = i_{new} = i_{nw} = 0$, if the loss in net productivity for the high-type and medium-type workers is less than the savings from no information rent paid to the high-type then profit in equilibrium 9 is greater than profit in equilibrium 1.

Lemma 3.2.a: *In the absence of tax policy, i.e.* $T = t = i_{ew} = i_{new} = i_{nw} = 0$*, total profit in the partial pooling equilibrium in Proposition 3.2 is greater than total profits in the partial pooling equilibrium in Proposition 3.9 if*

$$(\overline{r}_e - \underline{c}_{eew}) > (\overline{r}_{ne} - \underline{c}_{new}).$$

Therefore, going forward we will assume the conditions in Propositions 3.2.2 and 3.2.3 hold. Thus, In the absence of tax policy when wages are at their minimums that sustain equilibria 2 and 9 profits will be greater than when wages are at their minimums that sustain equilibrium 1. Also, given the assumptions on revenues workers generate and their associated costs, profit in equilibrium 2 is greater than profit in equilibrium 9. Finally, under the same conditions per proposition 3.2.1, profit in equilibrium 1 is greater than profit in the rest of the equilibria. This implies that in the absence of policy the firm prefers equilibrium 2 to equilibrium 9 which is preferred to equilibrium 1, and which, in turn, is preferred to the rest. The goal of optimal tax policy in this context should be to incentivize the firm to choose wages that sustain equilibrium 1 over equilibria 2 and 9 without subverting the pre-tax policy incentives to prefer equilibrium 1 over equilibria 3, 4, 5, 6, 7, 8, and 10. Next, we will derive conditions for which profit in equilibrium 1 is greater than profit in the other 9 equilibria in the presence of tax policy.

Proposition 3.2.4: *In the presence of tax policy, i.e.* $T > 0, t > 0, i_{ew} > 0, i_{new} > 0,$ and $i_{nw} \geq 0$, *total profit in separating equilibrium 1 is greater than total profit in partial pooling equilibrium 2 when*

$$if\ i_{new} \leq i_{nw} + \underline{c}_{new}\ then\ n_m \left[\tilde{r}_{ne} - \frac{\tilde{c}_{new}}{1-t} + \frac{(i_{new} - i_{nw})}{1-t}\right] > n_h \left(\frac{\tilde{c}_{new} - \underline{c}_{new}}{1-t}\right)$$

$$and\ if\ i_{new} > i_{nw} + \underline{c}_{new}\ then\ n_m \left[\tilde{r}_{ne} - \frac{\tilde{c}_{new}}{1-t} + \frac{(i_{new} - i_{nw})}{1-t}\right] > n_h \left(\frac{\tilde{c}_{new} + (i_{nw} - i_{new})}{1-t}\right).$$

Per proposition 3.2.2 we assume that in the absence of tax policy these conditions do not hold. The above conditions tell us that the left-hand side is increasing in i_{new}, the transfer given to an individual working at the uneducated wage. For sufficiently large i_{new} relative to i_{nw} and the cost of the high-type individual working at the uneducated wage the right-hand side is decreasing in i_{new}. Therefore, to incentivize the firm to choose wages that supports equilibrium 1 relative to equilibrium 2, i_{new} needs to be sufficiently large. The reverse is true for i_{nw}, the transfer given to an individual not working. The left-hand side is decreasing in i_{nw} and for sufficiently large i_{new} the right-hand side is increasing in i_{nw}. Therefore, any increase in i_{nw} must be accompanied by a commensurate increase in i_{new} for the above conditions to hold.

Proposition 3.2.5: *In the presence of tax policy, i.e.* $T > 0, t > 0, i_{ew} > 0, i_{new} > 0,$ and $i_{nw} \geq 0$, *total profit in separating equilibrium 1 is greater than total profit in partial pooling equilibrium 3 when*

$$if\ i_{new} \leq i_{nw} + \tilde{c}_{new}\ then\ n_m \left[\left(\tilde{r}_{ne} - \frac{\tilde{c}_{new}}{1-t}\right) - \left(\tilde{r}_e - \frac{\tilde{c}_{eew}}{1-t}\right) + \frac{(i_{new} - i_{ew})}{1-t}\right] > n_h \left[\frac{\left(\underline{c}_{eew} - \underline{c}_{new}\right)}{1-t} - \frac{(\tilde{c}_{eew} - \tilde{c}_{new})}{1-t}\right]$$

$$and\ if\ i_{new} > i_{nw} + \tilde{c}_{new}\ then\ n_m \left[\left(\tilde{r}_{ne} - \frac{\tilde{c}_{new}}{1-t}\right) - \left(\tilde{r}_e - \frac{\tilde{c}_{eew}}{1-t}\right) + \frac{(i_{new} - i_{nw}) + (i_{new} - i_{ew}) - \tilde{c}_{new}}{1-t}\right] > n_h \left[\frac{\left(\underline{c}_{eew} - \underline{c}_{new}\right)}{1-t} + \frac{(i_{nw} - i_{new}) - \tilde{c}_{eew}}{1-t}\right].$$

Per proposition 3.2.1 given the assumptions on revenues workers generate and their associated costs the above conditions hold in the absence of tax policy. The above conditions tell us that the left-hand side is increasing in i_{new}. For sufficiently large i_{new} relative to i_{nw} and the cost of the medium-type individual working at the uneducated wage the right-hand side is decreasing in i_{new}. Additionally, the left-hand side is decreasing in i_{ew}, the transfer given to an individual working at the educated wage. For sufficiently large i_{new} the left-hand side is decreasing in i_{nw} and the right-hand side is increasing in i_{nw}. Therefore, sufficiently large i_{ew} and i_{nw} must be compensated by increases in i_{new} for the above inequalities to continue to hold. Thus, the firm is incentivized to choose wages that support equilibrium 1 relative to equilibrium 3 if i_{ew} and i_{nw} are not sufficiently larger than i_{new}.

Proposition 3.2.6: *In the presence of tax policy, i.e.* $T > 0, t > 0, i_{ew} > 0, i_{new} > 0,$ and $i_{nw} \geq 0$, *total profit in separating equilibrium 1 is greater than total profit in full pooling equilibrium 4 when*

$$if\ i_{new} \leq i_{nw} + \overline{c}_{new}\ then\ n_m \left[\left(\tilde{r}_{ne} - \frac{\tilde{c}_{new}}{1-t}\right) - \left(\tilde{r}_e - \frac{\overline{c}_{eew}}{1-t}\right) + \frac{(i_{new} - i_{ew})}{1-t}\right]$$

$$> n_h \left[\left(\frac{\left(\underline{c}_{eew} - \underline{c}_{new}\right) - \left(\overline{c}_{eew} - \tilde{c}_{new}\right)}{1-t}\right)\right] + n_l \left[\left(\underline{r}_e - \frac{\overline{c}_{eew}}{1-t}\right) + \frac{(i_{ew} - i_{nw})}{1-t}\right]$$

$$and\ if\ i_{new} > i_{nw}$$

$$+ \overline{c}_{new}\ then\ n_m \left[\left(\tilde{r}_{ne} - \frac{\tilde{c}_{new}}{1-t}\right) - \left(\tilde{r}_e - \frac{\overline{c}_{eew}}{1-t}\right) + \frac{(i_{new} - i_{nw}) + (i_{new} - i_{ew}) - \overline{c}_{new}}{1-t}\right]$$

$$> n_h \left[\left(\frac{\left(\underline{c}_{eew} - \underline{c}_{new}\right) - \left(\overline{c}_{eew} - \tilde{c}_{new}\right)}{1-t}\right) + \frac{(i_{nw} - i_{new})}{1-t}\right]$$

$$+ n_l \left[\underline{r}_e - \left(\frac{\overline{c}_{eew} - \overline{c}_{new}}{1-t}\right) + \frac{(i_{ew} - i_{new})}{1-t}\right].$$

Per proposition 3.2.1 given the assumptions on revenues workers generate and their associated costs the above conditions hold in the absence of tax policy. The above conditions tell us that the left-hand side is increasing in i_{new} and decreasing in i_{ew} and the right-hand side is increasing in i_{ew} and decreasing in i_{nw}. For sufficiently large i_{new} relative to i_{nw} and the cost of the low-type individual working at the uneducated wage the left-hand side is decreasing in i_{nw} and the right-hand side is increasing in i_{nw} and decreasing in i_{new}. Therefore, sufficiently large i_{ew} must be compensated by increases in i_{new} for the above inequalities to continue to hold. For sufficiently large i_{new} relative to i_{nw} sufficiently large i_{nw} must be compensated by increases in i_{new}. Thus, the firm is incentivized to choose wages that supports equilibrium 1 relative to equilibrium 4 if i_{ew} and i_{nw} are not sufficiently large relative to i_{new}.

Proposition 3.2.7: *In the presence of tax policy, i.e.* $T > 0, t > 0, i_{ew} > 0, i_{new} > 0,$ and $i_{nw} \geq 0$, *total profit in separating equilibrium 1 is greater than total profit in partial pooling equilibrium 5 when*

$$n_m\left[\left(\tilde{r}_{ne}-\frac{\tilde{c}_{new}}{1-t}\right)-\left(\tilde{r}_e-\frac{\tilde{c}_{eew}}{1-t}\right)+\left(\frac{\overline{c}_{new}-\tilde{c}_{new}}{1-t}\right)\right]$$

$$>n_h\left[\left(\frac{\underline{c}_{eew}-\underline{c}_{new}}{1-t}\right)-\left(\frac{\tilde{c}_{eew}-\tilde{c}_{new}}{1-t}\right)-\left(\frac{\overline{c}_{new}-\tilde{c}_{new}}{1-t}\right)\right]$$

$$+n_l\left[\left(\underline{r}_{ne}-\frac{\overline{c}_{new}}{1-t}\right)+\frac{(i_{nw}-i_{new})}{1-t}\right].$$

Per proposition 3.2.1 given the assumptions on revenues workers generate and their associated costs the above condition holds in the absence of tax policy. The above condition tells us that the right-hand side is increasing in i_{nw} and decreasing in i_{new}. Thus, sufficiently large i_{nw} must be compensated by increases in i_{new} for the above inequality to continue to hold. Therefore, the firm is incentivized to choose wages that supports equilibrium 1 relative to equilibrium 5 if i_{nw} is not sufficiently large relative to i_{new}.

Proposition 3.2.8: *In the presence of tax policy, i.e.* $T>0, t>0, i_{ew}>0, i_{new}>0,$ and $i_{nw}\geq 0,$ *total profit in separating equilibrium 1 is greater than total profit in partial pooling equilibrium 6 when*

$$n_h\left[\frac{\overline{c}_{new}-\tilde{c}_{new}}{1-t}\right]+n_m\left[\frac{\overline{c}_{new}-\tilde{c}_{new}}{1-t}\right]>n_l\left[\left(\underline{r}_{ne}-\frac{\overline{c}_{new}}{1-t}\right)+\frac{(i_{new}-i_{nw})}{1-t}\right].$$

Per proposition 3.2.1 given the assumptions on revenues workers generate and their associated costs the above condition holds in the absence of tax policy. The above condition tells us that the right-hand side is increasing in i_{new} and decreasing in i_{nw}. Thus, sufficiently large i_{new} must be compensated by increases in i_{nw} for the above inequality to continue to hold. Therefore, the firm is incentivized to choose wages that supports equilibrium 1 relative to equilibrium 6 if i_{new} is not sufficiently large relative to i_{nw}.

Proposition 3.2.9: *In the presence of tax policy, i.e.* $T>0, t>0, i_{ew}>0, i_{new}>0,$ and $i_{nw}\geq 0,$ *total profit in separating equilibrium 1 is greater than total profit in full pooling equilibrium 7 when*

$$if\ i_{ew} \le i_{nw} + \overline{c}_{eew}\ then\ n_h\left[\left(\overline{r}_e - \frac{\underline{c}_{eew}}{1-t}\right) - \left(\overline{r}_{ne} - \frac{\underline{c}_{new}}{1-t}\right) + \frac{(\overline{c}_{new} - \tilde{c}_{new})}{1-t} + \frac{(i_{ew} - i_{new})}{(1-t)}\right]$$

$$+ n_m\left[\frac{(\overline{c}_{new} - \tilde{c}_{new})}{1-t}\right] > n_l\left[\left(\underline{r}_{ne} - \frac{\overline{c}_{new}}{1-t}\right) + \frac{(i_{new} - i_{nw})}{1-t}\right]$$

$$and\ if\ i_{ew} > i_{nw}$$

$$+ \overline{c}_{eew}\ then\ n_h\left[\left(\overline{r}_e - \frac{\underline{c}_{eew}}{1-t}\right) - \left(\overline{r}_{ne} - \frac{\underline{c}_{new}}{1-t}\right) - \frac{\left(\underline{c}_{eew} - \underline{c}_{new} + \tilde{c}_{new}\right)}{1-t}\right.$$

$$\left. + \frac{(i_{ew} - i_{nw})}{1-t} + \frac{(i_{ew} - i_{new})}{1-t}\right]$$

$$> n_m\left[\frac{\left(\underline{c}_{eew} - \underline{c}_{new} + \tilde{c}_{new}\right)}{1-t} + \frac{(i_{nw} - i_{ew})}{1-t}\right]$$

$$+ n_l\left[\left(\underline{r}_{ne} - \frac{\overline{c}_{new}}{1-t}\right) + \left(\frac{(i_{new} - i_{ew}) + \underline{c}_{eew}}{1-t}\right)\right].$$

Per proposition 3.2.1 given the assumptions on revenues workers generate and their associated costs the above conditions hold in the absence of tax policy. The above conditions tell us that the left-hand side is increasing in i_{ew} and decreasing in i_{new} and the right-hand side is increasing in i_{new} and decreasing in i_{nw}. For sufficiently large i_{ew} relative to i_{nw} and the cost of the low-type individual working at the educated wage the left-hand side is decreasing in i_{nw} and the right-hand side is increasing in i_{new} and i_{nw} and decreasing in i_{ew}. Therefore, sufficiently large i_{new} must be compensated by increases in i_{ew} for the above inequalities to continue to hold. For sufficiently large i_{ew} relative to i_{nw} sufficiently large i_{nw} must be compensated by increases in i_{ew}. Thus, the firm is incentivized to choose wages that supports equilibrium 1 relative to equilibrium 7 if i_{new} and i_{nw} are not sufficiently large relative to i_{ew}.

Proposition 3.2.10: *In the presence of tax policy, i.e.* $T > 0, t > 0, i_{ew} > 0, i_{new} > 0,$ and $i_{nw} \ge 0,$ *total profit in separating equilibrium 1 is greater than total profit in the partial pooling equilibrium 8 when*

$$if\ i_{ew} \leq i_{nw} + \tilde{c}_{eew}\ then\ n_h \left[\left(\overline{r}_e - \frac{\underline{c}_{eew}}{1-t}\right) - \left(\overline{r}_{ne} - \frac{\underline{c}_{new}}{1-t}\right) + \left(\frac{(i_{ew} - i_{new})}{(1-t)}\right)\right] > 0$$

$$and\ if\ i_{ew} > i_{nw}$$

$$+ \tilde{c}_{eew}\ then\ n_h \left[\left(\overline{r}_e - \frac{\underline{c}_{eew}}{1-t}\right) - \left(\overline{r}_{ne} - \frac{\underline{c}_{new}}{1-t}\right) - \frac{(\underline{c}_{eew} - \underline{c}_{new} + \tilde{c}_{new})}{1-t} + \frac{(i_{ew} - i_{nw})}{1-t} + \frac{(i_{ew} - i_{new})}{1-t}\right]$$

$$> n_m \left[\left(\frac{\underline{c}_{eew} - \underline{c}_{new} + \tilde{c}_{new}}{1-t}\right) + \frac{(i_{nw} - i_{ew})}{1-t} + \frac{(i_{nw} - i_{new})}{1-t}\right].$$

Per proposition 3.2.1 given the assumptions on revenues workers generate and their associated costs the above conditions hold in the absence of tax policy. The above conditions tell us that the left-hand side is increasing in i_{ew} and decreasing in i_{new}. For sufficiently large i_{ew} relative to i_{nw} and the cost of the medium-type individual working at the educated wage the left-hand side is decreasing in i_{nw} and the right-hand side is increasing in i_{nw} and decreasing in i_{ew} and i_{new}. Therefore, sufficiently large i_{new} must be compensated by increases in i_{ew} for the above inequalities to continue to hold. For sufficiently large i_{ew} relative to i_{nw} sufficiently large i_{nw} must be compensated by increases in i_{ew}. Thus, the firm is incentivized to choose wages that support equilibrium 1 relative to equilibrium 8 if i_{new} and i_{nw} are not sufficiently large relative to i_{ew}.

Proposition 3.2.11: *In the presence of tax policy, i.e.* $T > 0, t > 0, i_{ew} > 0, i_{new} > 0,$ and $i_{nw} \geq 0$, *total profit in separating equilibrium 1 is greater than total profit in the partial pooling equilibrium 9 when*

$$if\ i_{ew} \leq i_{nw} + \underline{c}_{eew}\ then\ n_h \left[\left(\overline{r}_e - \frac{\underline{c}_{eew}}{1-t}\right) - \left(\overline{r}_{ne} - \frac{\underline{c}_{new}}{1-t}\right) + \frac{(i_{ew} - i_{new})}{1-t}\right]$$

$$+ n_m \left[\left(\tilde{r}_{ne} - \frac{\tilde{c}_{new}}{1-t}\right) + \frac{(i_{new} - i_{nw})}{1-t}\right] > n_h \left[\frac{(\tilde{c}_{new} - \underline{c}_{new})}{1-t}\right]$$

$$and\ if\ i_{ew} > i_{nw} + \underline{c}_{eew}\ then\ n_h\left[\left(\overline{r}_e - \frac{\underline{c}_{eew}}{1-t}\right) - \left(\overline{r}_{ne} - \frac{\underline{c}_{new}}{1-t}\right) + \frac{(i_{ew} - i_{nw})}{1-t}\right]$$

$$+ n_m\left[\left(\tilde{r}_{ne} - \frac{\tilde{c}_{new}}{1-t}\right) + \frac{(i_{new} - i_{nw})}{1-t}\right] > n_h\left[\frac{(\underline{c}_{eew} - \underline{c}_{new} + \tilde{c}_{new})}{1-t} + \frac{(i_{new} - i_{ew})}{1-t}\right].$$

Per proposition 3.2.3 we assume that in the absence of tax policy these conditions do not hold. The above conditions tell us that the left-hand side is increasing in i_{ew}, decreasing in i_{nw}, and increasing or decreasing in i_{new} depending on the relative number of high and medium type individuals. For sufficiently large i_{ew} relative to i_{nw} and the cost of the high-type individual working at the educated wage the left-hand side is unambiguously increasing in i_{new} and the right-hand side is increasing in i_{new} and decreasing in i_{ew}. Therefore, to incentivize the firm to choose wages that supports equilibrium 1 relative to equilibrium 9 i_{ew} needs to be sufficiently large. Any increase in i_{nw} must be accompanied by a commensurate increase in i_{ew} for the above conditions to hold.

Proposition 3.2.12: *In the presence of tax policy, i.e.* $T > 0, t > 0, i_{ew} > 0, i_{new} > 0$, and $i_{nw} \geq 0$, *total profit in separating equilibrium 1 is greater than total profit in the partial pooling equilibrium 10 when*

$$n_h\left[\left(\overline{r}_e - \frac{\underline{c}_{eew}}{1-t}\right) - \frac{(\tilde{c}_{new} - \underline{c}_{new})}{1-t} + \frac{(i_{ew} - i_{nw})}{1-t}\right] + n_m\left[\left(\tilde{r}_{ne} - \frac{\tilde{c}_{new}}{1-t}\right) + \frac{(i_{new} - i_{nw})}{1-t}\right] > 0$$

Per proposition 3.2.1 given the assumptions on revenues workers generate and their associated costs the above condition holds in the absence of tax policy. The above condition tells us that the right-hand side is increasing in i_{ew} and i_{new} and decreasing in i_{nw}. Thus, sufficiently large i_{nw} must be compensated by increases in i_{ew} and i_{new} for the above inequality to continue to hold. Therefore, the firm is incentivized to choose wages that supports equilibrium 1 relative to equilibrium 10 if i_{nw} is not sufficiently large relative to i_{ew} and i_{new}.

In summary, starting from an absence of tax policy the firm prefers to choose wages that support equilibrium 2. The government seeking to maximize utility of workers seeks to incentivize them to choose wages that supports equilibrium 1. Proposition 3.2.4 tells us that i_{new} needs to be sufficiently large per the conditions stated. This makes sense given that in equilibrium 2 the firm is only hiring the high-type individual at the educated wage and not the medium-type individual at the uneducated wage and the objective for the government is that the medium-type individual is working. Thus, subsidizing the hiring of an uneducated worker is necessary to incentivize the firm to do so. However, per propositions 3.2.8, 3.2.9, 3.2.10, and 3.2.11 this can incentivize the firm to choose wages that sustains equilibria 6, 7, 8, and 9 over equilibrium 1. Then, following propositions 3.2.9, 3.2.10, and 3.2.11 increasing i_{ew} incentivizes the firm to choose wages that sustains equilibrium 1 over equilibria 7, 8, and 9. However, per propositions 3.2.5 and 3.2.6 this can incentivize the firm to choose wages that sustains equilibria 3 and 4. Therefore the conditions in propositions 3.2.5 and 3.2.6 determine the maximum for i_{ew}. Given that i_{ew} has no effect on incentivizing equilibrium 1 over 6 the condition in proposition 3.2.8 determines the maximum on i_{new}. Given this logic and the above conditions we arrive at the following conditions on i_{new} and i_{ew} that sustains equilibrium 1.

Proposition 3.2.13: *The following separating equilibrium can be sustained:*

The high productivity worker invests in education and works, the medium productivity worker does not invest in education and works, and the low productivity worker does not invest in education and does not work, the firm maximizes non-negative profit, and the government budget balances If

$$t(n_h w_e + n_m w_{ne}) + T\big(n_h(\overline{r}_e - w_e) + n_m(\tilde{r}_{ne} - w_{ne})\big) = n_h i_{ew} + n_m i_{new} + n_l i_{nw}$$

$$for\ i_{new} \leq i_{nw} + \underline{c}_{new}$$

$$i_{new} \in \left(\frac{n_h}{n_m}(\tilde{c}_{new} - \underline{c}_{new}) - (\tilde{r}_{ne}(1-t) - \tilde{c}_{new}) + i_{nw}, \frac{n_m + n_h}{n_l}(\overline{c}_{new} - \tilde{c}_{new}) - \big(\underline{r}_{ne}(1-t) - \overline{c}_{new}\big) + i_{nw}\right)$$

$$for\ i_{nw} + \underline{c}_{new} < \ i_{new} < i_{nw} + \tilde{c}_{new}$$

$$i_{new} \in \left(\frac{n_h}{n_m + n_h}(\tilde{c}_{new}) - \frac{n_m}{n_m + n_h}(\tilde{r}_{ne}(1-t) - \tilde{c}_{new}) + i_{nw}, \frac{n_m + n_h}{n_l}(\overline{c}_{new} - \tilde{c}_{new}) - \big(\underline{r}_{ne}(1-t) - \overline{c}_{new}\big) + i_{nw}\right)$$

$$for\ i_{ew} \leq i_{nw} + \underline{c}_{eew}$$

$$i_{ew} > \max [$$

$$(\tilde{c}_{new} - \underline{c}_{new}) + (\overline{r}_{ne}(1-t) - \underline{c}_{new}) - (\overline{r}_e(1-t) - \underline{c}_{eew}) - \frac{n_m}{n_h}[(\tilde{r}_{ne}(1-t) - \tilde{c}_{new}) + (i_{new} - i_{nw})] + i_{new},$$

$$(\overline{r}_{ne}(1-t) - \underline{c}_{new}) - (\overline{r}_e(1-t) - \underline{c}_{eew}) + i_{new},$$

$$\frac{n_l}{n_h}[(\underline{r}_{ne}(1-t) - \overline{c}_{new}) + (i_{new} - i_{nw})] - \frac{n_m}{n_h}[\overline{c}_{new} - \tilde{c}_{new}] + (\overline{r}_{ne}(1-t) - \underline{c}_{new}) - (\overline{r}_e(1-t) - \underline{c}_{eew})$$

$$- (\overline{c}_{new} - \tilde{c}_{new}) + i_{new}]$$

$$for\ i_{nw} + \underline{c}_{eew} < i_{ew} < i_{nw} + \tilde{c}_{eew}$$

$$i_{ew} \geq \max [$$

$$\frac{1}{2}[(\underline{c}_{eew} - \underline{c}_{new} + \tilde{c}_{new}) - (\overline{r}_e(1-t) - \underline{c}_{eew}) + (\overline{r}_{ne}(1-t) - \underline{c}_{new}) + (i_{new} + i_{nw})]$$

$$- \frac{n_m}{2n_h}[(\tilde{r}_{ne}(1-t) - \tilde{c}_{new}) + (i_{new} - i_{nw})],$$

$$(\overline{r}_{ne}(1-t) - \underline{c}_{new}) - (\overline{r}_e(1-t) - \underline{c}_{eew}) + i_{new},$$

$$\frac{n_l}{n_h}[(\underline{r}_{ne}(1-t) - \overline{c}_{new}) + (i_{new} - i_{nw})] - \frac{n_m}{n_h}[\overline{c}_{new} - \tilde{c}_{new}] + (\overline{r}_{ne}(1-t) - \underline{c}_{new}) - (\overline{r}_e(1-t) - \underline{c}_{eew})$$

$$- (\overline{c}_{new} - \tilde{c}_{new}) + i_{new}]$$

$$for\ i_{new} \leq i_{nw} + \tilde{c}_{new}$$

$$i_{ew} < (\tilde{r}_{ne}(1-t) - \tilde{c}_{new}) - (\tilde{r}_e(1-t) - \tilde{c}_{eew}) - \frac{n_h}{n_m}[(\underline{c}_{eew} - \underline{c}_{new}) - (\tilde{c}_{eew} - \tilde{c}_{new})] + i_{new}$$

For the given ranges on i_{ew}, i_{new}, and i_{nw}[7] these conditions on tax rates and transfers for the government ensures that the firm chooses wages that sustain equilibrium 1 and that workers choose the appropriate wage and corresponding education and work choice that maximizes social welfare while balancing the government budget. The most relevant information to glean from proposition 3.2.13 is that there are specific ranges on i_{ew} and i_{new} that sustains equilibrium 1 over the other 9. The range on i_{new} is a function of the number of each individual types, their individual revenues, costs, and i_{nw}. The range

[7] The reasons for the ranges chosen for the transfers are two-fold. First to simplify the statement of the proposition and second to exclude ranges where the minimums and maximums are not well defined.

unambiguously shifts higher with increases in the number of high types, the income tax rate, and the transfer i_{nw}. The range on i_{ew} is a function of the number of each individual types, their individual revenues, costs, i_{nw}, and i_{new}.

To illustrate how this works four numerical solutions are given with discussion.

4. Numerical Solutions and Discussion

We will now choose values for the number of each worker type and their associated costs and revenues produced for different education levels. They are chosen such that all assumptions on relative sizes from chapter 2 hold. Thus, they are the following,

$$n_h = n_m = n_l = 1$$

$$\overline{r}_{ne} = 2 > \tilde{r}_{ne} = 1.5 > \underline{r}_{ne} = 1$$

$$\overline{r}_e = 2.6 > \tilde{r}_e = 2 > \underline{r}_e = 1.5$$

$$\underline{c}_{new} = 0.5 < \tilde{c}_{new} = 1.1 < \overline{c}_{new} = 1.25$$

$$\underline{c}_{eew} = 1 < \tilde{c}_{eew} = 1.8 < \overline{c}_{eew} = 2.05$$

and therefore

$$\underline{c}_{eew} - \underline{c}_{new} = 0.5 < \tilde{c}_{eew} - \tilde{c}_{new} = 0.7 < \overline{c}_{eew} - \overline{c}_{new} = 0.8.$$

Further the conditions in this chapter that determine relative sizes on social welfare and profit for each equilibrium also hold, which is expressed in the following lemma and is proven in the appendix.

Lemma 4.a: *The conditions for Propositions 3.1.1, 3.2.1, 3.2.2, and 3.2.3 that*

$$\overline{r}_e - \underline{c}_{eew} > \overline{r}_{ne} - \underline{c}_{new} > 0,$$

$$\tilde{r}_{ne} - \tilde{c}_{new} > \tilde{r}_e - \tilde{c}_{eew} > 0,$$

$$0 > \underline{r}_{ne} - \overline{c}_{new} > \underline{r}_e - \overline{c}_{eew},$$

$$n_m(\tilde{r}_{ne} - \tilde{c}_{new}) < n_h(\tilde{c}_{new} - \underline{c}_{new}), \textit{and}$$

$$n_h(\overline{r}_e - \underline{c}_{eew}) - n_h(\overline{r}_{ne} - \underline{c}_{new}) + n_m(\tilde{r}_{ne} - \tilde{c}_{new}) < n_h(\tilde{c}_{new} - \underline{c}_{new})$$

hold if

$$n_h = n_m = n_l = 1$$

$$\underline{c}_{new} = 0.5, \tilde{c}_{new} = 1.1, \overline{c}_{new} = 1.25$$

$$\underline{c}_{eew} = 1, \tilde{c}_{eew} = 1.8, \overline{c}_{eew} = 2.05$$

$$\overline{r}_{ne} = 2, \tilde{r}_{ne} = 1.5, \underline{r}_{ne} = 1$$

$$\overline{r}_e = 2.6, \tilde{r}_e = 2, \underline{r}_e = 1.5.$$

Given these values the calculation of social welfare and profit in the absence of tax policy and their relative size are given in the appendix. With the following four examples we will demonstrate how taxes and transfers incentivize the firm to go from choosing wages that sustain an inefficient partial pooling equilibrium in the absence of tax policy to choosing wages that sustain the efficient separating equilibrium. We then show when transfers redistribute profits from the firm to workers and when they do not. We start with the following proposition in the absence of tax policy.

Proposition 4.1: *In the absence of tax policy when the values of tax rates* $t = 0,\ T = 0$, *transfers* $i_{ew} = 0$, $i_{new} = 0$, *and* $i_{nw} = 0$ *the firm chooses wages* $w_{ne} = 0$ *and* $w_e = 1$ *that maximizes profit given tax policy which sustain the partial pooling equilibrium where the high productivity worker invests in education and works and the medium and low productivity worker does not invest in education and does not work. Firm profit is equal to* 1.6 *and total worker utility is equal to* 0. *The government's budget balances trivially.*

Per propositions 3.2.2 and 3.2.3 and lemmas 3.2.a and 4.a in the absence of tax policy the firm prefers to choose wages that sustain this inefficient partial pooling equilibrium. The loss in production from not hiring the medium-type worker is not sufficiently large to overcome the savings from not paying the high-type worker information rent. The high-type worker receives no surplus since the educated wage is set equal to their marginal cost by the firm. Both the medium and low-type workers receive no surplus since they do not work and $i_{nw} = 0$. The firm captures all the surplus from the high-type worker's labor. From the government's perspective this is not desirable per proposition 3.1.1 since this outcome creates a deadweight loss. Per proposition 3.2.13 the government can choose tax rates t and T and the corresponding minimum values of transfers i_{ew} and i_{new} to correct for the firm's market power while balancing their budget yielding the following proposition.

Proposition 4.2: *The values of tax rates* $t = 0.30,\ T = 0.345$, *transfers* $i_{ew} = 0.73$, $i_{new} = 0.59$, *and* $i_{nw} = 0$ *is a solution to the government's program where the government's budget balances and the firm chooses wages* $w_{ne} = 0.729$ *and* $w_e = 1.245$ *that maximizes profit given tax policy which sustain the separating equilibrium where the high productivity worker invests in education and works, the medium productivity worker does not invest in education and works, and the low productivity worker does not invest in education and does not work. Firm profit is equal to* 1.39253 *and total worker utility is equal to* 0.6018*.*

This outcome achieves the government's objective of incentivizing the firm to choose wages that sustain the separating equilibrium by subsidizing the labor of the medium-type worker and in turn subsidizing the labor of the high-type worker to for example not create a perverse incentive for the firm to hire both at the uneducated wage. Profit for the firm is lower than it receives in the absence of tax policy. Given wages, the income tax rate, and transfers, both the high and medium-type workers face an effective negative income tax rate. The profit tax $T = 0.345$ compensates for this to balance the government budget. The high-type worker goes from receiving no surplus in the absence of tax policy to receiving information rent from their labor. The medium-type worker still receives no surplus since the uneducated wage is set equal to their marginal cost by the firm. The outcome is no different for the low-type worker. Suppose the government seeks to correct for this by increasing i_{ew} and i_{new} to their maximum values per proposition 3.2.13 while maintaining $i_{nw} = 0$. This is shown in the following proposition.

Proposition 4.3: *The values of tax rates* $t = 0.30,\ T = 0.6$, *transfers* $i_{ew} = 1.38$, $i_{new} = 0.84$, *and* $i_{nw} = 0$ *is a solution to the government's program where the government's budget balances and the firm* *chooses wages* $w_{ne} = 0.372$ *and* $w_e = 0.315$ *that maximizes profit given tax policy which sustain the separating equilibrium where the high productivity worker invests in education and works, the medium productivity worker does not invest in education and works, and the low productivity worker does not invest in education and does not work. Firm profit is equal to* 1.3652 *and total worker utility is equal to* 0.6019.

This outcome still achieves the government's objective of incentivizing the firm to choose wages that sustain the separating equilibrium but does nothing to affect the distribution of surplus between the firm and workers. The only effect increasing i_{ew} and i_{new} relative to the previous outcome is that the firm can lower wages while still attracting both the high and medium-type workers to work at the educated and uneducated wages, respectively. There is no impact on firm profit save for the government capturing slightly more surplus due to the specific value chosen for the profit tax T for rounding concerns. The high-type worker receives the same surplus from information rent as before and both the medium and low-types still receive no surplus. Suppose now the government seeks to correct for this by increasing i_{nw} and thus i_{ew} and i_{new} simultaneously. This is described in the following proposition.

Proposition 4.4: *The values of tax rates* $t = 0.30,\ T = 0.98$, *transfers* $i_{ew} = 1.18$, $i_{new} = 1.04$, *and* $i_{nw} = 0.45$ *is a solution to the government's program that maximizes worker utility where the government's budget balances and the firm chooses wages* $w_{ne} = 0.729$ *and* $w_e = 1.245$ *that maximizes profit given tax policy which sustain the separating equilibrium where the high productivity worker invests in education and works, the medium productivity worker does not invest in education and works, and the low productivity worker does not invest in education and does not work. Firm profit is equal to* 0.04252 *and total worker utility is equal to* 1.9518.

This outcome still achieves the government's objective of incentivizing the firm to choose wages that sustain the separating equilibrium and now it maximizes worker utility while leaving some positive profit for the firm. Profit is lower for the firm than the previous three outcomes. Surplus for all three workers is increased by the amount $i_{nw} = 0.45$. The high-type worker now receives this surplus in addition to the information rent from their labor while the medium and low-types receive i_{nw} in surplus rather than zero. All worker outcomes are improved while maintaining all incentives for the firm to choose wages that sustain the separating equilibrium while balancing the government budget. Since the amount $i_{nw} = 0.45$ is received by all three workers relative to the equilibrium in proposition 4.3 i_{nw} could be characterized as a “basic income”.

5. Conclusion

This paper extends the Spence (1973) job market signaling game from chapter 2 where there are three worker types, high productivity, medium productivity, and low productivity, who choose to reveal information in their choice of investing in productivity enhancing education and whether to work or not and a wage tax and lump sum transfer are imposed by the government. A profit tax is introduced, and two additional stages are added to the game before workers make their education and labor supply choices. In the first stage the government chooses a wage and profit tax and transfers to maximize utilitarian total social welfare of workers knowing the firm acts as a monopsonist and will choose wages to maximize profit. In the second stage the firm takes taxes and transfers as given and choose wages to maximize profit knowing workers will choose to get an education or not and work or not to maximize their own individual utility. We assume in the absence of tax policy the firm has an incentive to choose wages that results in an inefficient equilibrium where they receive monopsonist rent. The government chooses transfers to subsidize both educated and uneducated labor to incentivize the firm to choose wages that result in the efficient equilibrium. The government then transfers a “basic income” to all workers that still maintains all the same incentives and redistributes profit to workers.

This analysis can be extended and generalized to include an arbitrary number of types or a continuum of types. In addition, following Stancheva (2014) work effort can be included as a choice variable for workers. Firms could observe output at a cost and not effort and contract workers to work at a given wage for a given output. This will create the possibility for interactions between productivity enhancing education and effort which may result in a “rat race” for workers at the top of the income distribution when firms have market power.

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APPENDIX

Definitions of payoffs for the firm and workers, total social welfare, and government budget constraint:

The total payoff for the firm in each equilibrium is:

1. $(1-T)\left(n_h(\overline{r}_e - w_e) + \ n_m(\tilde{r}_{ne} - w_{ne})\right)$
2. $(1-T)\left(n_h(\overline{r}_e - w_e)\right)$
3. $(1-T)\left(n_h(\overline{r}_e - w_e) + n_m(\tilde{r}_e - w_e)\right)$
4. $(1-T)\left(n_h(\overline{r}_e - w_e) + n_m(\tilde{r}_e - w_e) + n_l(\underline{r}_e - w_e)\right)$
5. $(1-T)\left(n_h(\overline{r}_e - w_e) + n_m(\tilde{r}_e - w_e) + n_l(\underline{r}_{ne} - w_{ne})\right)$
6. $(1-T)\left(n_h(\overline{r}_e - w_e) + n_m(\tilde{r}_{ne} - w_{ne}) + n_l(\underline{r}_{ne} - w_{ne})\right)$
7. $(1-T)\left(n_h(\overline{r}_{ne} - w_{ne}) + n_m(\tilde{r}_{ne} - w_{ne}) + n_l(\underline{r}_{ne} - w_{ne})\right)$
8. $(1-T)\left(n_h(\overline{r}_{ne} - w_{ne}) + n_m(\tilde{r}_{ne} - w_{ne})\right)$
9. $(1-T)\left(n_h(\overline{r}_{ne} - w_{ne})\right)$
10. $(1-T)0$

The sum of payoffs for the workers in each equilibrium is:

1. $n_h\left(i_{ew} + (1-t)w_e - \underline{c}_{eew}\right) + n_m(i_{new} + (1-t)w_{ne} - \tilde{c}_{new}) + n_l i_{nw}$
2. $n_h\left(i_{ew} + (1-t)w_e - \underline{c}_{eew}\right) + n_m i_{nw} + n_l i_{nw}$
3. $n_h\left(i_{ew} + (1-t)w_e - \underline{c}_{eew}\right) + n_m(i_{ew} + (1-t)w_e - \tilde{c}_{eew}) + n_l i_{nw}$
4. $n_h\left(i_{ew} + (1-t)w_e - \underline{c}_{eew}\right) + n_m(i_{ew} + (1-t)w_e - \tilde{c}_{eew}) + n_l(i_{ew} + (1-t)w_e - \overline{c}_{eew})$
5. $n_h\left(i_{ew} + (1-t)w_e - \underline{c}_{eew}\right) + n_m(i_{ew} + (1-t)w_e - \tilde{c}_{eew}) + n_l(i_{new} + (1-t)w_{ne} - \overline{c}_{new}) +$
6. $n_h\left(i_{ew} + (1-t)w_e - \underline{c}_{eew}\right) + n_m(i_{new} + (1-t)w_{ne} - \tilde{c}_{new}) + n_l(i_{new} + (1-t)w_{ne} - \overline{c}_{new})$
7. $n_h\left(i_{new} + (1-t)w_{ne} - \underline{c}_{new}\right) + n_m(i_{new} + (1-t)w_{ne} - \tilde{c}_{new}) + n_l(i_{new} + (1-t)w_{ne} - \overline{c}_{new})$
8. $n_h\left(i_{new} + (1-t)w_{ne} - \underline{c}_{new}\right) + n_m(i_{new} + (1-t)w_{ne} - \tilde{c}_{new}) + n_l i_{nw}$

9. $n_h\left(i_{new} + (1-t)w_{ne} - \underline{c}_{new}\right) + n_m i_{nw} + n_l i_{nw}$

10. $n_h i_{nw} + n_m i_{nw} + n_l i_{nw}$

Putting these together we get that total social welfare in each equilibrium is the sum of realized payoffs for each type of worker and the firm which are the following:

1. $n_h\left(i_{ew} + (1-t)w_e - \underline{c}_{eew}\right) + n_m(i_{new} + (1-t)w_{ne} - \tilde{c}_{new}) + n_l i_{nw} + (1-T)\left(n_h(\overline{r}_e - w_e) + n_m(\tilde{r}_{ne} - w_{ne})\right)$

2. $n_h\left(i_{ew} + (1-t)w_e - \underline{c}_{eew}\right) + n_m i_{nw} + n_l i_{nw} + (1-T)\left(n_h(\overline{r}_e - w_e)\right)$

3. $n_h\left(i_{ew} + (1-t)w_e - \underline{c}_{eew}\right) + n_m(i_{ew} + (1-t)w_e - \tilde{c}_{eew}) + n_l i_{nw} + (1-T)\left(n_h(\overline{r}_e - w_e) + n_m(\tilde{r}_e - w_e)\right)$

4. $n_h\left(i_{ew} + (1-t)w_e - \underline{c}_{eew}\right) + n_m(i_{ew} + (1-t)w_e - \tilde{c}_{eew}) + n_l(i_{ew} + (1-t)w_e - \overline{c}_{eew}) + (1-T)\left(n_h(\overline{r}_e - w_e) + n_m(\tilde{r}_e - w_e) + n_l(\underline{r}_e - w_e)\right)$

5. $n_h\left(i_{ew} + (1-t)w_e - \underline{c}_{eew}\right) + n_m(i_{ew} + (1-t)w_e - \tilde{c}_{eew}) + n_l(i_{new} + (1-t)w_{ne} - \overline{c}_{new}) + (1-T)\left(n_h(\overline{r}_e - w_e) + n_m(\tilde{r}_e - w_e) + n_l(\underline{r}_{ne} - w_{ne})\right)$

6. $n_h\left(i_{ew} + (1-t)w_e - \underline{c}_{eew}\right) + n_m(i_{new} + (1-t)w_{ne} - \tilde{c}_{new}) + n_l(i_{new} + (1-t)w_{ne} - \overline{c}_{new}) + (1-T)\left(n_h(\overline{r}_e - w_e) + n_m(\tilde{r}_{ne} - w_{ne}) + n_l(\underline{r}_{ne} - w_{ne})\right)$

7. $n_h\left(i_{new} + (1-t)w_{ne} - \underline{c}_{new}\right) + n_m(i_{new} + (1-t)w_{ne} - \tilde{c}_{new}) + n_l(i_{new} + (1-t)w_{ne} - \overline{c}_{new}) + (1-T)\left(n_h(\overline{r}_{ne} - w_{ne}) + n_m(\tilde{r}_{ne} - w_{ne}) + n_l(\underline{r}_{ne} - w_{ne})\right)$

8. $n_h\left(i_{new} + (1-t)w_{ne} - \underline{c}_{new}\right) + n_m(i_{new} + (1-t)w_{ne} - \tilde{c}_{new}) + n_l i_{nw} + (1-T)\left(n_h(\overline{r}_{ne} - w_{ne}) + n_m(\tilde{r}_{ne} - w_{ne})\right)$

9. $n_h\left(i_{new} + (1-t)w_{ne} - \underline{c}_{new}\right) + n_m i_{nw} + n_l i_{nw} + (1-T)\left(n_h(\overline{r}_{ne} - w_{ne})\right)$

10. $n_h i_{nw} + n_m i_{nw} + n_l i_{nw} + (1-T)0$

Given the non-linear government budget constraint is:

1. $n_h i_{ew} + n_m i_{new} + n_l i_{nw} = n_h t w_e + n_m t w_{ne} + T\left(n_h(\overline{r}_e - w_e) + n_m(\tilde{r}_{ne} - w_{ne})\right)$

2. $n_h i_{ew} + (n_m + n_l) i_{nw} = n_h t w_e + T\big(n_h(\overline{r}_e - w_e)\big)$

3. $(n_h + n_m) i_{ew} + n_l i_{nw} = (n_h + n_m) t w_e + T\big(n_h(\overline{r}_e - w_e) + n_m(\tilde{r}_e - w_e)\big)$

4. $(n_h + n_m + n_l) i_{ew} = (n_h + n_m + n_l) t w_e + T\left(n_h(\overline{r}_e - w_e) + n_m(\tilde{r}_e - w_e) + n_l(\underline{r}_e - w_e)\right)$

5. $(n_h + n_m) i_{ew} + n_l i_{new} = (n_h + n_m) t w_e + n_l t w_{ne} + T\Big(n_h(\overline{r}_e - w_e) + n_m(\tilde{r}_e - w_e) +$
$n_l(\underline{r}_{ne} - w_{ne})\Big)$

6. $n_h i_{ew} + (n_m + n_l) i_{new} = n_h t w_e + (n_m + n_l) t w_{ne} + T\Big(n_h(\overline{r}_e - w_e) + n_m(\tilde{r}_{ne} - w_{ne}) +$
$n_l(\underline{r}_{ne} - w_{ne})\Big)$

7. $(n_h + n_m + n_l) i_{new} = (n_h + n_m + n_l) t w_{ne} + T\Big(n_h(\overline{r}_{ne} - w_{ne}) + n_m(\tilde{r}_{ne} - w_{ne}) +$
$n_l(\underline{r}_{ne} - w_{ne})\Big)$

8. $(n_h + n_m) i_{new} + n_l i_{nw} = (n_h + n_m) t w_{ne} + T\big(n_h(\overline{r}_{ne} - w_{ne}) + n_m(\tilde{r}_{ne} - w_{ne})\big)$

9. $n_h i_{new} + (n_m + n_l) i_{nw} = n_h t w_{ne} + T\big(n_h(\overline{r}_{ne} - w_{ne})\big)$

10. $(n_h + n_m + n_l) i_{nw} = T0,$

then after canceling terms given the previous definitions, we get total social welfare in each equilibrium is

1. $n_h(\overline{r}_e - \underline{c}_{eew}) + n_m(\tilde{r}_{ne} - \tilde{c}_{new})$
2. $n_h(\overline{r}_e - \underline{c}_{eew})$
3. $n_h(\overline{r}_e - \underline{c}_{eew}) + n_m(\tilde{r}_e - \tilde{c}_{eew})$
4. $n_h(\overline{r}_e - \underline{c}_{eew}) + n_m(\tilde{r}_e - \tilde{c}_{eew}) + n_l(\underline{r}_e - \overline{c}_{eew})$
5. $n_h(\overline{r}_e - \underline{c}_{eew}) + n_m(\tilde{r}_e - \tilde{c}_{eew}) + n_l(\underline{r}_{ne} - \overline{c}_{new})$
6. $n_h(\overline{r}_e - \underline{c}_{eew}) + n_m(\tilde{r}_{ne} - \tilde{c}_{new}) + n_l(\underline{r}_{ne} - \overline{c}_{new})$
7. $n_h(\overline{r}_{ne} - \underline{c}_{new}) + n_m(\tilde{r}_{ne} - \tilde{c}_{new}) + n_l(\underline{r}_{ne} - \overline{c}_{new})$
8. $n_h(\overline{r}_{ne} - \underline{c}_{new}) + n_m(\tilde{r}_{ne} - \tilde{c}_{new})$
9. $n_h(\overline{r}_{ne} - \underline{c}_{new})$
10. 0.

Given that

$$n_h = n_m = n_l = 1$$

$$\underline{c}_{new} = 0.5, \tilde{c}_{new} = 1.1, \overline{c}_{new} = 1.25$$

$$\underline{c}_{eew} = 1, \tilde{c}_{eew} = 1.8, \overline{c}_{eew} = 2.05$$

$$\overline{r}_{ne} = 2, \tilde{r}_{ne} = 1.5, \underline{r}_{ne} = 1$$

$$\overline{r}_{e} = 2.6, \tilde{r}_{e} = 2, \underline{r}_{e} = 1.5.$$

Total social welfare in each equilibrium is

1. $n_h(\overline{r}_e - \underline{c}_{eew}) + n_m(\tilde{r}_{ne} - \tilde{c}_{new}) = 1.6 + 0.4 = 2$
2. $n_h(\overline{r}_e - \underline{c}_{eew}) = 1.6$
3. $n_h(\overline{r}_e - \underline{c}_{eew}) + n_m(\tilde{r}_e - \tilde{c}_{eew}) = 1.6 + 0.2 = 1.8$
4. $n_h(\overline{r}_e - \underline{c}_{eew}) + n_m(\tilde{r}_e - \tilde{c}_{eew}) + n_l(\underline{r}_e - \overline{c}_{eew}) = 1.6 + 0.2 - 0.55 = 1.25$
5. $n_h(\overline{r}_e - \underline{c}_{eew}) + n_m(\tilde{r}_e - \tilde{c}_{eew}) + n_l(\underline{r}_{ne} - \overline{c}_{new}) = 1.6 + 0.2 - 0.25 = 1.55$
6. $n_h(\overline{r}_e - \underline{c}_{eew}) + n_m(\tilde{r}_{ne} - \tilde{c}_{new}) + n_l(\underline{r}_{ne} - \overline{c}_{new}) = 1.6 + 0.4 - 0.25 = 1.75$
7. $n_h(\overline{r}_{ne} - \underline{c}_{new}) + n_m(\tilde{r}_{ne} - \tilde{c}_{new}) + n_l(\underline{r}_{ne} - \overline{c}_{new}) = 1.5 + 0.4 - 0.25 = 1.65$
8. $n_h(\overline{r}_{ne} - \underline{c}_{new}) + n_m(\tilde{r}_{ne} - \tilde{c}_{new}) = 1.5 + 0.4 = 1.9$
9. $n_h(\overline{r}_{ne} - \underline{c}_{new}) = 1.5$
10. 0

The ranking of each equilibrium in terms of social welfare is $1 > 8 > 3 > 6 > 7 > 2 > 5 > 9 > 4 > 10$.

In the absence of tax policy, i.e. $\mathrm{T} = \mathrm{t} = \mathrm{i}_{\mathrm{ew}} = \mathrm{i}_{\mathrm{new}} = \mathrm{i}_{\mathrm{nw}} = 0$, total profit in each equilibrium is

1. $n_h[\overline{r}_e - (\underline{c}_{eew} - \underline{c}_{new} + \tilde{c}_{new})] + n_m(\tilde{r}_{ne} - \tilde{c}_{new}) = 1 + 0.4 = 1.4$
2. $n_h[\overline{r}_e - \underline{c}_{eew}] = 1.6$
3. $n_h[\overline{r}_e - \tilde{c}_{eew}] + n_m[\tilde{r}_e - \tilde{c}_{eew}] = 0.8 + 0.2 = 1$
4. $n_h[\overline{r}_e - \overline{c}_{eew}] + n_m[\tilde{r}_e - \overline{c}_{eew}] + n_l[\underline{r}_e - \overline{c}_{eew}] = 0.55 - 0.05 - 0.55 = -0.05$

5. $n_h[\overline{r}_e - (\tilde{c}_{eew} - \tilde{c}_{new} + \overline{c}_{new})] + n_m[\tilde{r}_e - (\tilde{c}_{eew} - \tilde{c}_{new} + \overline{c}_{new})] + n_l[\underline{r}_{ne} - \overline{c}_{new}] = 0.65 + 0.05 - 0.25 = 0.45$

6. $n_h[\overline{r}_e - (\underline{c}_{eew} - \underline{c}_{new} + \overline{c}_{new})] + n_m[\tilde{r}_{ne} - \overline{c}_{new}] + n_l[\underline{r}_{ne} - \overline{c}_{new}] = 0.85 + 0.25 - 0.25 = 0.85$

7. $n_h[\overline{r}_{ne} - \overline{c}_{new}] + n_m[\tilde{r}_{ne} - \overline{c}_{new}] + n_l[\underline{r}_{ne} - \overline{c}_{new}] = 0.75 + 0.25 - 0.25 = 0.75$

8. $n_h[\overline{r}_{ne} - \tilde{c}_{new}] + n_m[\tilde{r}_{ne} - \tilde{c}_{new}] = 0.9 + 0.4 = 1.3$

9. $n_h[\overline{r}_{ne} - \underline{c}_{new}] = 1.5$

10. 0

The ranking of each equilibrium in terms of profit for the firm in the absence of tax policy is $2 > 9 > 1 > 8 > 3 > 6 > 7 > 5 > 4 > 10$.

Derivation of solved firm profit for all equilibria:

Profit in equilibrium 1 is

$$(1-T)\left(n_h(\overline{r}_e - w_e) + n_m(\tilde{r}_{ne} - w_{ne})\right).$$

Profit in equilibrium 1 is maximized when wages are at their minimum values that still sustain the equilibrium. Given from Proposition 3.1

$$w_e \in \left(\frac{\underline{c}_{eew} - \underline{c}_{new} + (i_{new} - i_{ew})}{1-t} + w_{ne}, min\left(\frac{\overline{c}_{eew} + (i_{nw} - i_{ew})}{1-t}, \frac{\tilde{c}_{eew} - \tilde{c}_{new} + (i_{new} - i_{ew})}{1-t} + w_{ne}\right)\right)$$

$$\text{and } w_{ne} \in \left(\frac{\tilde{c}_{new} + (i_{nw} - i_{new})}{1-t}, \frac{\overline{c}_{new} + (i_{nw} - i_{new})}{1-t}\right), \text{then}$$

$$w_{ne} = \frac{\tilde{c}_{new} + (i_{nw} - i_{new})}{1-t} \;\; and \;\; w_e = \frac{\underline{c}_{eew} - \underline{c}_{new} + (i_{new} - i_{ew})}{1-t} + \frac{\tilde{c}_{new} + (i_{nw} - i_{new})}{1-t}$$

$$= \frac{\underline{c}_{eew} - \underline{c}_{new} + \tilde{c}_{new} + (i_{nw} - i_{ew})}{1-t}$$

maximizes profits and respectively profits are

$$(1-T)\left(n_h\left[\overline{r}_e - \left(\frac{\underline{c}_{eew} - \underline{c}_{new} + \tilde{c}_{new} + (i_{nw} - i_{ew})}{1-t}\right)\right] + n_m\left[\tilde{r}_{ne} - \left(\frac{\tilde{c}_{new} + (i_{nw} - i_{new})}{1-t}\right)\right]\right).$$

When $T = t = i_{ew} = i_{new} = i_{nw} = 0$ then profit in equilibrium 1 is

$$n_h[\overline{r}_e - (\underline{c}_{eew} - \underline{c}_{new} + \tilde{c}_{new})] + n_m(\tilde{r}_{ne} - \tilde{c}_{new}).$$

Profit in equilibrium 2 is

$$(1-T)n_h(\overline{r}_e - w_e).$$

Profit in equilibrium 2 is maximized when wages are at their minimum values that still sustain the equilibrium. Given from Proposition 3.2

$$w_e \in \left(max\left(\frac{\underline{c}_{eew} + (i_{nw} - i_{ew})}{(1-t)}, \frac{\underline{c}_{eew} - \underline{c}_{new} + (i_{new} - i_{ew})}{1-t} + w_{ne} \right), \frac{\tilde{c}_{eew} + (i_{nw} - i_{ew})}{(1-t)} \right)$$

$$\text{and } w_{ne} < \frac{\tilde{c}_{new} + (i_{nw} - i_{new})}{(1-t)}, \text{then}$$

$$w_{ne} = 0 \text{ and } if\ i_{nw} \geq i_{new} - \underline{c}_{new} \text{ then } w_e = \frac{\underline{c}_{eew} + (i_{nw} - i_{ew})}{(1-t)} \text{ or}$$

$$\text{if } i_{nw} < i_{new} - \underline{c}_{new} \text{ then } w_e = \frac{\underline{c}_{eew} - \underline{c}_{new} + (i_{new} - i_{ew})}{1-t}$$

maximizes profits and respectively profits are

$$(1-T)n_h \left[\overline{r}_e - \left(\frac{\underline{c}_{eew} + (i_{nw} - i_{ew})}{(1-t)} \right) \right] \ or\ (1-T)n_h \left[\overline{r}_e - \left(\frac{\underline{c}_{eew} - \underline{c}_{new} + (i_{new} - i_{ew})}{1-t} \right) \right].$$

When $T = t = i_{ew} = i_{new} = i_{nw} = 0$ then profit in equilibrium 2 is

$$n_h[\overline{r}_e - \underline{c}_{eew}].$$

Profit in equilibrium 3 is

$$(1-T)\big(n_h(\overline{r}_e - w_e) + n_m(\tilde{r}_e - w_e)\big).$$

Profit in equilibrium 3 is maximized when wages are at their minimum values that still sustain the equilibrium. Given from Proposition 3.3

$$w_e \in \left(\max\left(\frac{\tilde{c}_{eew} + (i_{nw} - i_{ew})}{(1-t)}, \frac{\tilde{c}_{eew} - \tilde{c}_{new} + (i_{new} - i_{ew})}{1-t} + w_{ne} \right), \frac{\overline{c}_{eew} + (i_{nw} - i_{ew})}{(1-t)} \right)$$

$$and\ w_{ne} < \frac{\overline{c}_{new} + (i_{nw} - i_{new})}{(1-t)}, \text{then}$$

$$w_{ne} = 0 \text{ and } if\ i_{nw} \geq i_{new} - \tilde{c}_{new} \text{ then } w_e = \frac{\tilde{c}_{eew} + (i_{nw} - i_{ew})}{(1-t)} \text{ or}$$

$$\text{if } i_{nw} < i_{new} - \tilde{c}_{new} \text{ then } w_e = \frac{\tilde{c}_{eew} - \tilde{c}_{new} + (i_{new} - i_{ew})}{1-t}$$

maximizes profits and respectively profits are

$$(1-T)\left(n_h\left[\overline{r}_e-\left(\frac{\tilde{c}_{eew}+(i_{nw}-i_{ew})}{(1-t)}\right)\right]+n_m\left[\tilde{r}_e-\left(\frac{\tilde{c}_{eew}+(i_{nw}-i_{ew})}{(1-t)}\right)\right]\right) \text{ or}$$

$$(1-T)\left(n_h\left[\overline{r}_e-\left(\frac{\tilde{c}_{eew}-\tilde{c}_{new}+(i_{new}-i_{ew})}{1-t}\right)\right]+n_m\left[\tilde{r}_e-\left(\frac{\tilde{c}_{eew}-\tilde{c}_{new}+(i_{new}-i_{ew})}{1-t}\right)\right]\right).$$

When $T=t=\ i_{ew}=\ i_{new}=i_{nw}=0$ then profit in equilibrium 3 is

$$n_h[\overline{r}_e-\tilde{c}_{eew}]+n_m[\tilde{r}_e-\tilde{c}_{eew}].$$

Profit in equilibrium 4 is

$$(1-T)\left(n_h(\overline{r}_e-w_e)+n_m(\tilde{r}_e-w_e)+n_l(\underline{r}_e-w_e)\right).$$

Profit in equilibrium 4 is maximized when wages are at their minimum values that still sustain the equilibrium. Given from Proposition 3.4

$$w_e>\max\left(\frac{\overline{c}_{eew}+(i_{nw}-i_{ew})}{(1-t)},\frac{\overline{c}_{eew}-\overline{c}_{new}+(i_{new}-i_{ew})}{(1-t)}+w_{ne}\right), \text{then}$$

$$w_{ne}=0 \text{ and } if\ i_{nw}\geq i_{new}-\overline{c}_{new} \text{ then } w_e=\frac{\overline{c}_{eew}+(i_{nw}-i_{ew})}{(1-t)} \text{ or}$$

$$\text{if } i_{nw}<i_{new}-\overline{c}_{new} \text{ then } w_e=\frac{\overline{c}_{eew}-\overline{c}_{new}+(i_{new}-i_{ew})}{1-t}$$

maximizes profits and respectively profits are

$$(1-T)\left(n_h\left[\overline{r}_e-\left(\frac{\overline{c}_{eew}+(i_{nw}-i_{ew})}{(1-t)}\right)\right]+n_m\left[\tilde{r}_e-\left(\frac{\overline{c}_{eew}+(i_{nw}-i_{ew})}{(1-t)}\right)\right]\right.$$
$$\left.+n_l\left[\underline{r}_e-\left(\frac{\overline{c}_{eew}+(i_{nw}-i_{ew})}{(1-t)}\right)\right]\right) \text{ or}$$
$$(1-T)\left(n_h\left[\overline{r}_e-\left(\frac{\overline{c}_{eew}-\overline{c}_{new}+(i_{new}-i_{ew})}{1-t}\right)\right]+n_m\left[\tilde{r}_e-\left(\frac{\overline{c}_{eew}-\overline{c}_{new}+(i_{new}-i_{ew})}{1-t}\right)\right]\right.$$
$$\left.+n_l\left[\underline{r}_e-\left(\frac{\overline{c}_{eew}-\overline{c}_{new}+(i_{new}-i_{ew})}{(1-t)}\right)\right]\right).$$

When $T=t=\ i_{ew}=\ i_{new}=i_{nw}=0$ then profit in equilibrium 4 is

$$n_h[\overline{r}_e-\overline{c}_{eew}]+n_m[\tilde{r}_e-\overline{c}_{eew}]+n_l[\underline{r}_e-\overline{c}_{eew}].$$

Profit in equilibrium 5 is

$$(1-T)\left(n_h(\overline{r}_e-w_e)+n_m(\tilde{r}_e-w_e)+n_l(\underline{r}_{ne}-w_{ne})\right).$$

Profit in equilibrium 5 is maximized when wages are at their minimum values that still sustain the equilibrium. Given from Proposition 3.5

$$w_e \in \left(\frac{\tilde{c}_{eew} - \tilde{c}_{new} + (i_{new} - i_{ew})}{1-t} + w_{ne}, \frac{\overline{c}_{eew} - \overline{c}_{new} + (i_{new} - i_{ew})}{(1-t)} + w_{ne}\right)$$

$$and\ w_{ne} > \frac{\overline{c}_{new} + (i_{nw} - i_{new})}{(1-t)}, \text{then}$$

$$w_{ne} = \frac{\overline{c}_{new} + (i_{nw} - i_{new})}{(1-t)}\ and\ w_e = \frac{\tilde{c}_{eew} - \tilde{c}_{new} + (i_{new} - i_{ew})}{1-t} + \frac{\overline{c}_{new} + (i_{nw} - i_{new})}{(1-t)}$$

$$= \frac{\tilde{c}_{eew} - \tilde{c}_{new} + \overline{c}_{new} + (i_{nw} - i_{ew})}{1-t}$$

maximizes profits and profits are

$$(1-T)\left(n_h\left[\overline{r}_e - \left(\frac{\tilde{c}_{eew} - \tilde{c}_{new} + \overline{c}_{new} + (i_{nw} - i_{ew})}{1-t}\right)\right] + n_m\left[\tilde{r}_e - \left(\frac{\tilde{c}_{eew} - \tilde{c}_{new} + \overline{c}_{new} + (i_{nw} - i_{ew})}{1-t}\right)\right]\right.$$
$$\left. + n_l\left[\underline{r}_{ne} - \left(\frac{\overline{c}_{new} + (i_{nw} - i_{new})}{(1-t)}\right)\right]\right).$$

When $T = t = i_{ew} = i_{new} = i_{nw} = 0$ then profit in equilibrium 5 is

$$n_h[\overline{r}_e - (\tilde{c}_{eew} - \tilde{c}_{new} + \overline{c}_{new})] + n_m[\tilde{r}_e - (\tilde{c}_{eew} - \tilde{c}_{new} + \overline{c}_{new})] + n_l[\underline{r}_{ne} - \overline{c}_{new}].$$

Profit in equilibrium 6 is

$$(1-T)\left(n_h(\overline{r}_e - w_e) + n_m(\tilde{r}_{ne} - w_{ne}) + n_l(\underline{r}_{ne} - w_{ne})\right).$$

Profit in equilibrium 6 is maximized when wages are at their minimum values that still sustain the equilibrium. Given from Proposition 3.6

$$w_e \in \left(\frac{\underline{c}_{eew} - \underline{c}_{new} + (i_{new} - i_{ew})}{1-t} + w_{ne}, \frac{\tilde{c}_{eew} - \tilde{c}_{new} + (i_{new} - i_{ew})}{1-t} + w_{ne}\right)$$

$$\text{and } w_{ne} > \frac{\overline{c}_{new} + (i_{nw} - i_{new})}{(1-t)}, \text{then}$$

$$w_{ne} = \frac{\overline{c}_{new} + (i_{nw} - i_{new})}{(1-t)} \text{ and } w_e = \frac{\underline{c}_{eew} - \underline{c}_{new} + (i_{new} - i_{ew})}{1-t} + \frac{\overline{c}_{new} + (i_{nw} - i_{new})}{(1-t)}$$

$$= \frac{\underline{c}_{eew} - \underline{c}_{new} + \overline{c}_{new} + (i_{nw} - i_{ew})}{1-t}$$

maximizes profits and profits are

$$(1-T)\left(n_h\left[\overline{r}_e-\left(\frac{\underline{c}_{eew}-\underline{c}_{new}+\overline{c}_{new}+(i_{nw}-i_{ew})}{1-t}\right)\right]+n_m\left[\tilde{r}_{ne}-\left(\frac{\overline{c}_{new}+(i_{nw}-i_{new})}{(1-t)}\right)\right]\right.$$
$$\left.+n_l\left[\underline{r}_{ne}-\left(\frac{\overline{c}_{new}+(i_{nw}-i_{new})}{(1-t)}\right)\right]\right).$$

When $T=t=\ i_{ew}=\ i_{new}=i_{nw}=0$ then profit in equilibrium 6 is

$$n_h[\overline{r}_e-(\underline{c}_{eew}-\underline{c}_{new}+\overline{c}_{new})]+n_m[\tilde{r}_{ne}-\overline{c}_{new}]+n_l[\underline{r}_{ne}-\overline{c}_{new}].$$

Profit in equilibrium 7 is

$$(1-T)\left(n_h(\overline{r}_{ne}-w_{ne})+n_m(\tilde{r}_{ne}-w_{ne})+n_l(\underline{r}_{ne}-w_{ne})\right).$$

Profit in equilibrium 7 is maximized when wages are at their minimum values that still sustain the equilibrium. Given from Proposition 3.7

$$w_{ne}>\max\left(\frac{\overline{c}_{new}+(i_{nw}-i_{new})}{(1-t)},\frac{(\underline{c}_{new}-\underline{c}_{eew})+(i_{ew}-i_{new})}{(1-t)}+w_e\right),\text{then}$$

$$w_e=0\text{ and if }\overline{c}_{new}+i_{nw}\geq\underline{c}_{new}-\underline{c}_{eew}+i_{ew}\text{ then }w_{ne}=\frac{\overline{c}_{new}+(i_{nw}-i_{new})}{(1-t)}\ or$$

$$\overline{c}_{new}+i_{nw}<\underline{c}_{new}-\underline{c}_{eew}+i_{ew}\text{ then }w_{ne}=\frac{(\underline{c}_{new}-\underline{c}_{eew})+(i_{ew}-i_{new})}{(1-t)}$$

maximizes profits and respectively profits are

$$(1-T)\left(n_h\left[\overline{r}_{ne}-\left(\frac{\overline{c}_{new}+(i_{nw}-i_{new})}{(1-t)}\right)\right]+n_m\left[\tilde{r}_{ne}-\left(\frac{\overline{c}_{new}+(i_{nw}-i_{new})}{(1-t)}\right)\right]\right.$$
$$\left.+n_l\left[\underline{r}_{ne}-\left(\frac{\overline{c}_{new}+(i_{nw}-i_{new})}{(1-t)}\right)\right]\right)\text{ or}$$

$$(1-T)\left(n_h\left[\overline{r}_{ne}-\left(\frac{(\underline{c}_{new}-\underline{c}_{eew})+(i_{ew}-i_{new})}{(1-t)}\right)\right]+n_m\left[\tilde{r}_{ne}-\left(\frac{(\underline{c}_{new}-\underline{c}_{eew})+(i_{ew}-i_{new})}{(1-t)}\right)\right]\right.$$
$$\left.+n_l\left[\underline{r}_{ne}-\left(\frac{(\underline{c}_{new}-\underline{c}_{eew})+(i_{ew}-i_{new})}{(1-t)}\right)\right]\right).$$

When $T=t=\ i_{ew}=\ i_{new}=i_{nw}=0$ then profit in equilibrium 7 is

$$n_h[\overline{r}_{ne}-\overline{c}_{new}]+n_m[\tilde{r}_{ne}-\overline{c}_{new}]+n_l[\underline{r}_{ne}-\overline{c}_{new}].$$

Profit in equilibrium 8 is

$$(1-T)\big(n_h(\overline{r}_{ne}-w_{ne})+n_m(\tilde{r}_{ne}-w_{ne})\big).$$

Profit in equilibrium 8 is maximized when wages are at their minimum values that still sustain the equilibrium. Given from Proposition 3.8

$$w_{ne}\in\left[\max\left(\frac{\tilde{c}_{new}+(i_{nw}-i_{new})}{(1-t)},\frac{\big(\underline{c}_{new}-\underline{c}_{eew}\big)+(i_{ew}-i_{new})}{(1-t)}+w_e\right),\frac{\overline{c}_{new}+(i_{nw}-i_{new})}{(1-t)}\right]\text{ and }w_e<\frac{\overline{c}_{eew}+(i_{nw}-i_{ew})}{(1-t)},\text{ then}$$

$$w_e=0\text{ and if }\tilde{c}_{new}+i_{nw}\geq\underline{c}_{new}-\underline{c}_{eew}+i_{ew}\text{ then }w_{ne}=\frac{\tilde{c}_{new}+(i_{nw}-i_{new})}{(1-t)}\ or$$

$$if\ \tilde{c}_{new}+i_{nw}<\underline{c}_{new}-\underline{c}_{eew}+i_{ew}\text{ then }w_{ne}=\frac{\big(\underline{c}_{new}-\underline{c}_{eew}\big)+(i_{ew}-i_{new})}{(1-t)}$$

maximizes profits and respectively profits are

$$(1-T)\left(n_h\left[\overline{r}_{ne}-\left(\frac{\tilde{c}_{new}+(i_{nw}-i_{new})}{(1-t)}\right)\right]+n_m\left[\tilde{r}_{ne}-\left(\frac{\tilde{c}_{new}+(i_{nw}-i_{new})}{(1-t)}\right)\right]\right)\text{ or}$$

$$(1-T)\left(n_h\left[\overline{r}_{ne}-\left(\frac{\big(\underline{c}_{new}-\underline{c}_{eew}\big)+(i_{ew}-i_{new})}{(1-t)}\right)\right]+n_m\left[\tilde{r}_{ne}-\left(\frac{\big(\underline{c}_{new}-\underline{c}_{eew}\big)+(i_{ew}-i_{new})}{(1-t)}\right)\right]\right).$$

When $T=t=\ i_{ew}=\ i_{new}=i_{nw}=0$ then profit in equilibrium 8 is

$$n_h[\overline{r}_{ne}-\tilde{c}_{new}]+n_m[\tilde{r}_{ne}-\tilde{c}_{new}].$$

Profit in equilibrium 9 is

$$(1-T)n_h(\overline{r}_{ne}-w_{ne}).$$

Profit in equilibrium 9 is maximized when wages are at their minimum values that still sustain the equilibrium. Given from Proposition 3.9

$$w_{ne}\in\left[\max\left(\frac{\underline{c}_{new}+(i_{nw}-i_{new})}{(1-t)},\frac{\big(\underline{c}_{new}-\underline{c}_{eew}\big)+(i_{ew}-i_{new})}{(1-t)}+w_e\right),\frac{\tilde{c}_{new}+(i_{nw}-i_{new})}{(1-t)}\right]\text{ and}$$

$$w_e<\frac{\tilde{c}_{eew}+(i_{nw}-i_{ew})}{(1-t)},\text{ then}$$

$$w_e=0\text{ and if }i_{nw}\geq i_{ew}-\underline{c}_{eew}\text{ then }w_{ne}=\frac{\underline{c}_{new}+(i_{nw}-i_{new})}{(1-t)}\ or$$

$$if\ i_{nw}<i_{ew}-\underline{c}_{eew}\text{ then }w_{ne}=\frac{\big(\underline{c}_{new}-\underline{c}_{eew}\big)+(i_{ew}-i_{new})}{(1-t)}$$

maximizes profits and profits are

$$(1-T)n_h\left[\overline{r}_{ne}-\left(\frac{\underline{c}_{new}+(i_{nw}-i_{new})}{(1-t)}\right)\right] \text{ or}$$

$$(1-T)n_h\left[\overline{r}_{ne}-\left(\frac{\left(\underline{c}_{new}-\underline{c}_{eew}\right)+(i_{ew}-i_{new})}{(1-t)}\right)\right].$$

When $T=t=\ i_{ew}=\ i_{new}=i_{nw}=0$ then profit in equilibrium 9 is

$$n_h\left[\overline{r}_{ne}-\underline{c}_{new}\right].$$

Profit in equilibrium 3.10 is

$$(1-T)0.$$

Therefore, profit in equilibrium 10 is 0.

Proof of Proposition 3.1.1:

For total social welfare in equilibrium 1, $\boldsymbol{TSW_1}$, to be greater than total social welfare in equilibrium 2, $\boldsymbol{TSW_2}$ it must be the case that

$$n_h\left(\overline{r}_e-\underline{c}_{eew}\right)+n_m(\tilde{r}_{ne}-\tilde{c}_{new})>n_h\left(\overline{r}_e-\underline{c}_{eew}\right), i.e.\ TSW_1>TSW_2.$$

Since

$$\tilde{r}_{ne}-\tilde{c}_{new}>0, \text{then } TSW_1>TSW_2.$$

For total social welfare in equilibrium 1, $\boldsymbol{TSW_1}$, to be greater than total social welfare in equilibrium 3, $\boldsymbol{TSW_3}$ it must be the case that

$$n_h\left(\overline{r}_e-\underline{c}_{eew}\right)+n_m(\tilde{r}_{ne}-\tilde{c}_{new})>n_h\left(\overline{r}_e-\underline{c}_{eew}\right)+n_m(\tilde{r}_e-\tilde{c}_{eew}), i.e.\ TSW_1>TSW_3.$$

Since

$$\tilde{r}_{ne}-\tilde{c}_{new}>\tilde{r}_e-\tilde{c}_{eew}, \text{then } TSW_1>TSW_3.$$

For total social welfare in equilibrium 1, $\boldsymbol{TSW_1}$, to be greater than total social welfare in equilibrium 4, $\boldsymbol{TSW_4}$ it must be the case that

$$n_h\left(\overline{r}_e-\underline{c}_{eew}\right)+n_m(\tilde{r}_{ne}-\tilde{c}_{new})>n_h\left(\overline{r}_e-\underline{c}_{eew}\right)+n_m(\tilde{r}_e-\tilde{c}_{eew})+n_l\left(\underline{r}_{\,ne}-\overline{c}_{new}\right), i.e.\ TSW_1>TSW_4.$$

Since

$$\tilde{r}_{ne}-\tilde{c}_{new}>\tilde{r}_e-\tilde{c}_{ew} \text{ and } \underline{r}_{\,e}-\overline{c}_{ew}<0, \text{then } TSW_1>TSW_4.$$

For total social welfare in equilibrium 1, TSW_1, to be greater than total social welfare in equilibrium 5, TSW_5 it must be the case that

$$n_h\left(\overline{r}_e - \underline{c}_{eew}\right) + n_m(\tilde{r}_{ne} - \tilde{c}_{new}) > n_h\left(\overline{r}_e - \underline{c}_{eew}\right) + n_m(\tilde{r}_e - \tilde{c}_{ew}) + n_l\left(\underline{r}_{ne} - \overline{c}_{new}\right), i.e.\ TSW_1 > TSW_5.$$

Since

$$\tilde{r}_{ne} - \tilde{c}_{new} > \tilde{r}_e - \tilde{c}_{ew} \text{ and } \underline{r}_{ne} - \overline{c}_{new} < 0, \text{then } TSW_1 > TSW_5.$$

For total social welfare in equilibrium 1, TSW_1, to be greater than total social welfare in equilibrium 6, TSW_6 it must be the case that

$$n_h\left(\overline{r}_e - \underline{c}_{eew}\right) + n_m(\tilde{r}_{ne} - \tilde{c}_{new}) > n_h\left(\overline{r}_e - \underline{c}_{eew}\right) + n_m(\tilde{r}_{ne} - \tilde{c}_{new}) + n_l\left(\underline{r}_{ne} - \overline{c}_{new}\right), i.e.\ TSW_1 > TSW_6.$$

Since

$$\underline{r}_{ne} - \overline{c}_{new} < 0, \text{then } TSW_1 > TSW_6.$$

For total social welfare in equilibrium 1, TSW_1, to be greater than total social welfare in equilibrium 7, TSW_7 it must be the case that

$$n_h\left(\overline{r}_e - \underline{c}_{eew}\right) + n_m(\tilde{r}_{ne} - \tilde{c}_{new}) > n_h\left(\overline{r}_{ne} - \underline{c}_{new}\right) + n_m(\tilde{r}_{ne} - \tilde{c}_{new}) + n_l\left(\underline{r}_{ne} - \overline{c}_{new}\right), i.e.\ TSW_1 > TSW_7.$$

Since

$$\overline{r}_e - \underline{c}_{ew} > \overline{r}_{ne} - \underline{c}_{new} \text{ and } \underline{r}_{ne} - \overline{c}_{new} < 0, \text{then } TSW_1 > TSW_7.$$

For total social welfare in equilibrium 1, TSW_1, to be greater than total social welfare in equilibrium 8, TSW_8 it must be the case that

$$n_h\left(\overline{r}_e - \underline{c}_{eew}\right) + n_m(\tilde{r}_{ne} - \tilde{c}_{new}) > n_h\left(\overline{r}_{ne} - \underline{c}_{new}\right) + n_m(\tilde{r}_{ne} - \tilde{c}_{new}), i.e.\ TSW_1 > TSW_8.$$

Since

$$\overline{r}_e - \underline{c}_{ew} > \overline{r}_{ne} - \underline{c}_{new}, \text{then } TSW_1 > TSW_8.$$

For total social welfare in equilibrium 1, TSW_1, to be greater than total social welfare in equilibrium 9, TSW_9 it must be the case that

$$n_h\left(\overline{r}_e - \underline{c}_{eew}\right) + n_m(\tilde{r}_{ne} - \tilde{c}_{new}) > n_h\left(\overline{r}_{ne} - \underline{c}_{new}\right), i.e.\ TSW_1 > TSW_9.$$

Since

$$\overline{r}_e - \underline{c}_{ew} > \overline{r}_{ne} - \underline{c}_{new} \text{ and } \tilde{r}_{ne} - \tilde{c}_{new} > 0, \text{then } TSW_1 > TSW_9.$$

For total social welfare in equilibrium 1, TSW_1, to be greater than total social welfare in equilibrium 10, TSW_{10} it must be the case that

$$n_h(\overline{r}_e - \underline{c}_{eew}) + n_m(\tilde{r}_{ne} - \tilde{c}_{new}) > 0, i.e.\ TSW_1 > TSW_{10}.$$

Since

$$\overline{r}_e - \underline{c}_{ew} > 0 \text{ and } \tilde{r}_{ne} - \tilde{c}_{new} > 0, \text{then } TSW_1 > TSW_{10}.$$

Proof of Proposition 3.2.1:

When $T = t = i_{ew} = i_{new} = i_{nw} = 0$ for profit in equilibrium 1 to be less than profit in equilibrium 3 it must be the case that

$$n_h(\overline{r}_e - \underline{c}_{eew}) + n_m(\tilde{r}_{ne} - \tilde{c}_{new}) - n_h(\tilde{c}_{new} - \underline{c}_{new}) < n_h(\overline{r}_e - \tilde{c}_{eew}) + n_m(\tilde{r}_e - \tilde{c}_{eew}) \text{ or}$$

$$n_m(\tilde{r}_{ne} - \tilde{c}_{new}) - n_m(\tilde{r}_e - \tilde{c}_{eew}) < n_h(\underline{c}_{eew} - \underline{c}_{new}) - n_h(\tilde{c}_{eew} - \tilde{c}_{new}).$$

The term on the left is positive since $\tilde{r}_{ne} - \tilde{c}_{new} > \tilde{r}_e - \tilde{c}_{eew}$ and the term on the right is negative since $\underline{c}_{eew} - \underline{c}_{new} < \tilde{c}_{eew} - \tilde{c}_{new}$. Therefore, this expression is a contradiction and profit in equilibrium 1 is never less than profit in equilibrium 3 when $T = t = i_{ew} = i_{new} = i_{nw} = 0$.

When $T = t = i_{ew} = i_{new} = i_{nw} = 0$ for profit in equilibrium 1 to be less than profit in equilibrium 4 it must be the case that

$$n_h(\overline{r}_e - \underline{c}_{eew}) + n_m(\tilde{r}_{ne} - \tilde{c}_{new}) - n_h(\tilde{c}_{new} - \underline{c}_{new}) < n_h(\overline{r}_e - \overline{c}_{eew}) + n_m(\tilde{r}_e - \overline{c}_{eew}) + n_l(\underline{r}_e - \overline{c}_{eew}) \text{ or}$$

$$n_m(\tilde{r}_{ne} - \tilde{c}_{new}) - n_m(\tilde{r}_e - \overline{c}_{eew}) < n_h(\underline{c}_{eew} - \underline{c}_{new}) - n_h(\overline{c}_{eew} - \tilde{c}_{new}) + n_l(\underline{r}_e - \overline{c}_{eew})$$

The term on the left is positive since $\tilde{r}_{ne} - \tilde{c}_{new} > \tilde{r}_e - \tilde{c}_{eew}$. The term on the right is negative because by definition $\underline{c}_{eew} - \underline{c}_{new} < \overline{c}_{eew} - \overline{c}_{new} < \overline{c}_{eew} - \tilde{c}_{new}$ since $\tilde{c}_{new} < \overline{c}_{new}$ and $\underline{r}_e - \overline{c}_{eew} < 0$ by assumption. Therefore, this expression is a contradiction and profit in equilibrium 1 is never less than profit in equilibrium 4 when $T = t = i_{ew} = i_{new} = i_{nw} = 0$.

When $T = t = i_{ew} = i_{new} = i_{nw} = 0$ for profit in equilibrium 1 to be less than profit in equilibrium 5 it must be the case that

$$n_h(\overline{r}_e - \underline{c}_{eew}) + n_m(\tilde{r}_{ne} - \tilde{c}_{new}) - n_h(\tilde{c}_{new} - \underline{c}_{new})$$

$$< n_h(\overline{r}_e - \tilde{c}_{eew}) + n_m(\tilde{r}_e - \tilde{c}_{eew}) + n_l(\underline{r}_{ne} - \overline{c}_{new}) - n_h(\overline{c}_{new} - \tilde{c}_{new}) - n_m(\overline{c}_{new} - \tilde{c}_{new}) \text{ or}$$

$$n_m(\tilde{r}_{ne} - \tilde{c}_{new}) - n_m(\tilde{r}_e - \tilde{c}_{eew}) + n_m(\overline{c}_{new} - \tilde{c}_{new})$$

$$< n_h(\underline{c}_{eew} - \underline{c}_{new}) - n_h(\tilde{c}_{eew} - \tilde{c}_{new}) - n_h(\overline{c}_{new} - \tilde{c}_{new}) + n_l(\underline{r}_{ne} - \overline{c}_{new}).$$

The term on the left is positive since $\tilde{r}_{ne} - \tilde{c}_{new} > \tilde{r}_e - \tilde{c}_{eew}$ and $\tilde{c}_{new} > \underline{c}_{new}$. The term on the right is negative because by definition $\underline{c}_{eew} - \underline{c}_{new} < \tilde{c}_{eew} - \tilde{c}_{new}$ and $\overline{c}_{new} > \tilde{c}_{new}$ and by assumption $\underline{r}_{ne} - \overline{c}_{new}$. Therefore, this expression is a contradiction and profit in equilibrium 1 is never less than profit in equilibrium 5 when $T = t = i_{ew} = i_{new} = i_{nw} = 0$.

When $T = t = i_{ew} = i_{new} = i_{nw} = 0$ for profit in equilibrium 1 to be less than profit in equilibrium 6 it must be the case that

$$n_h(\overline{r}_e - \underline{c}_{eew}) + n_m(\tilde{r}_{ne} - \tilde{c}_{new}) - n_h(\tilde{c}_{new} - \underline{c}_{new})$$

$$< n_h(\overline{r}_e - \underline{c}_{eew}) + n_m(\tilde{r}_{ne} - \overline{c}_{new}) + n_l(\underline{r}_{ne} - \overline{c}_{new}) - n_h(\overline{c}_{new} - \underline{c}_{new}) \text{ or}$$

$$n_m(\overline{c}_{new} - \tilde{c}_{new}) < n_h(\tilde{c}_{new} - \overline{c}_{new}) + n_l(\underline{r}_{ne} - \overline{c}_{new}).$$

The term on the left is positive since $\overline{c}_{new} > \tilde{c}_{new}$. The term on the right is negative because by definition $\tilde{c}_{new} < \overline{c}_{new}$ and $\underline{r}_{ne} - \overline{c}_{new}$ by assumption. Therefore, this expression is a contradiction and profit in equilibrium 1 is never less than profit in equilibrium 6 when $T = t = i_{ew} = i_{new} = i_{nw} = 0$.

When $T = t = i_{ew} = i_{new} = i_{nw} = 0$ for profit in equilibrium 1 to be less than profit in equilibrium 7 it must be the case that

$$n_h(\overline{r}_e - \underline{c}_{eew}) + n_m(\tilde{r}_{ne} - \tilde{c}_{new}) - n_h(\tilde{c}_{new} - \underline{c}_{new}) < n_h(\overline{r}_{ne} - \overline{c}_{new}) + n_m(\tilde{r}_{ne} - \overline{c}_{new}) + n_l(\underline{r}_{ne} - \overline{c}_{new}) \text{ or}$$

$$n_m(\overline{c}_{new} - \tilde{c}_{new}) < n_h(\overline{r}_{ne} - \underline{c}_{new}) - n_h(\overline{r}_e - \underline{c}_{eew}) + n_h(\tilde{c}_{new} - \overline{c}_{new}) + n_l(\underline{r}_{ne} - \overline{c}_{new})$$

The term on the left is positive since $\overline{c}_{new} > \tilde{c}_{new}$. The term on the right is negative because by assumption $\overline{r}_{ne} - \underline{c}_{new} < \overline{r}_e - \underline{c}_{eew}$ and $\underline{r}_{ne} - \overline{c}_{new} < 0$ and $\tilde{c}_{new} < \overline{c}_{new}$ by definition. Therefore, this expression is a contradiction and profit in equilibrium 1 is never less than profit in equilibrium 7 when $T = t = i_{ew} = i_{new} = i_{nw} = 0$.

When $T = t = i_{ew} = i_{new} = i_{nw} = 0$ for profit in equilibrium 1 to be less than profit in equilibrium 8 it must be the case that

$$n_h(\overline{r}_e - \underline{c}_{eew}) + n_m(\tilde{r}_{ne} - \tilde{c}_{new}) - n_h(\tilde{c}_{new} - \underline{c}_{new}) < n_h(\overline{r}_{ne} - \tilde{c}_{new}) + n_m(\tilde{r}_{ne} - \tilde{c}_{new}) \text{ or}$$

$$n_h(\overline{r}_e - \underline{c}_{eew}) < n_h(\overline{r}_{ne} - \underline{c}_{new})$$

The term on the left is greater than the term on the right by assumption. Therefore, this expression is a contradiction and profit in equilibrium 1 is never less than profit in equilibrium 8 when $T = t = i_{ew} = i_{new} = i_{nw} = 0$.

When $T = t = i_{ew} = i_{new} = i_{nw} = 0$ for profit in equilibrium 1 to be less than profit in equilibrium 10 it must be the case that

$$n_h(\overline{r}_e - \underline{c}_{eew}) + n_m(\tilde{r}_{ne} - \tilde{c}_{new}) < n_h(\tilde{c}_{new} - \underline{c}_{new}) \text{ or}$$

$$n_h\overline{r}_{ne} - n_h\tilde{r}_{ne} + n_h(\overline{r}_e - \underline{c}_{eew}) + n_m(\tilde{r}_{ne} - \tilde{c}_{new}) < n_h\overline{r}_{ne} - n_h\tilde{r}_{ne} + n_h(\tilde{c}_{new} - \underline{c}_{new}), \text{ which is}$$

$$n_h\overline{r}_{ne} - n_h\tilde{r}_{ne} + n_h(\overline{r}_e - \underline{c}_{eew}) - n_h(\overline{r}_{ne} - \underline{c}_{new}) + n_m(\tilde{r}_{ne} - \tilde{c}_{new}) < -n_h(\tilde{r}_{ne} - \tilde{c}_{new}).$$

The term on the left is positive since $\overline{r}_{ne} > \tilde{r}_{ne}, \overline{r}_e - \underline{c}_{eew} > \overline{r}_{ne} - \underline{c}_{new}$, and $\tilde{r}_{ne} - \tilde{c}_{new} > 0$. The term on the right is negative since $\tilde{r}_{ne} - \tilde{c}_{new} > 0$. Therefore, this expression is a contradiction and profit in equilibrium 1 is never less than profit in equilibrium 10 when $T = t = i_{ew} = i_{new} = i_{nw} = 0$.

Proof of Proposition 3.2.2:

When $T = t = i_{ew} = i_{new} = i_{nw} = 0$ for profit in equilibrium 1 to be less than profit in equilibrium 2 it must be the case that

$$n_h(\overline{r}_e - \underline{c}_{eew}) + n_m(\tilde{r}_{ne} - \tilde{c}_{new}) - n_h(\tilde{c}_{new} - \underline{c}_{new}) < n_h(\overline{r}_e - \underline{c}_{eew}) \text{ or}$$

$$n_m(\tilde{r}_{ne} - \tilde{c}_{new}) < n_h(\tilde{c}_{new} - \underline{c}_{new})$$

That is, the benefit from hiring the medium worker at the uneducated wage is less than the information rent paid to the high-type worker for profit in equilibrium 1 to be less than profit in equilibrium 2 when $T = t = i_{ew} = i_{new} = i_{nw} = 0$.

Proof of Proposition 3.2.3:

When $T = t = i_{ew} = i_{new} = i_{nw} = 0$ for profit in equilibrium 1 to be less than profit in equilibrium 9 it must be the case that

$$n_h(\overline{r}_e - \underline{c}_{eew}) + n_m(\tilde{r}_{ne} - \tilde{c}_{new}) - n_h(\tilde{c}_{new} - \underline{c}_{new}) < n_h(\overline{r}_{ne} - \underline{c}_{new}) \text{ or}$$

$$n_h(\overline{r}_e - \underline{c}_{eew}) - n_h(\overline{r}_{ne} - \underline{c}_{new}) + n_m(\tilde{r}_{ne} - \tilde{c}_{new}) < n_h(\tilde{c}_{new} - \underline{c}_{new}).$$

That is, if the loss in net productivity from the high-type and medium-type workers is less than the savings from no information rents then when $T = t = i_{ew} = i_{new} = i_{nw} = 0$ profit in equilibrium 1 is less than profit in equilibrium 9.

Proof of Lemma 3.2.a:

When $T = t = i_{ew} = i_{new} = i_{nw} = 0$ for profit in equilibrium 2 to be less than profit in equilibrium 9 it must be the case that

$$n_h(\overline{r}_e - \underline{c}_{eew}) < n_h(\overline{r}_{ne} - \underline{c}_{new}).$$

Since $(\overline{r}_e - \underline{c}_{eew}) > (\overline{r}_{ne} - \underline{c}_{new})$ by assumption then the above statement is a contradiction and profit in equilibrium 2 is never less than profit in equilibrium 9 when $T = t = i_{ew} = i_{new} = i_{nw} = 0$.

Proof of Proposition 3.2.4:

When $T > 0, t > 0, i_{ew} > 0, i_{new} > 0,$ and $i_{nw} \geq 0$ and if $i_{nw} \geq i_{new} - \underline{c}_{new}$ for profit in equilibrium 1 to be higher than profit in equilibrium 2 assuming wages are at their minimum it must be the case that

$$(1-T)\left(n_h\left[\overline{r}_e - \left(\frac{\underline{c}_{eew} - \underline{c}_{new} + (i_{nw} - i_{ew})}{1-t}\right) - \left(\frac{\tilde{c}_{new}}{1-t}\right)\right] + n_m\left[\tilde{r}_{ne} - \left(\frac{\tilde{c}_{new} + (i_{nw} - i_{new})}{1-t}\right)\right]\right)$$
$$> (1-T)n_h\left[\overline{r}_e - \left(\frac{\underline{c}_{eew} + (i_{nw} - i_{ew})}{(1-t)}\right)\right],$$

which is

$$n_m\left[\tilde{r}_{ne} - \left(\frac{\tilde{c}_{new} + (i_{nw} - i_{new})}{1-t}\right)\right] > n_h\left(\frac{\tilde{c}_{new} - \underline{c}_{new}}{1-t}\right).$$

When $T > 0, t > 0, i_{ew} > 0, i_{new} > 0,$ and $i_{nw} \geq 0$ and if $i_{nw} < i_{new} - \underline{c}_{new}$ for profit in equilibrium 1 to be greater than profit in equilibrium 2 assuming wages are at their minimum it must be the case that

$$(1-T)\left(n_h\left[\overline{r}_e-\left(\frac{\underline{c}_{eew}-\underline{c}_{new}+(i_{nw}-i_{ew})}{1-t}\right)-\left(\frac{\tilde{c}_{new}}{1-t}\right)\right]+n_m\left[\tilde{r}_{ne}-\left(\frac{\tilde{c}_{new}+(i_{nw}-i_{new})}{1-t}\right)\right]\right)$$

$$>(1-T)n_h\left[\overline{r}_e-\left(\frac{\underline{c}_{eew}-\underline{c}_{new}+(i_{new}-i_{ew})}{1-t}\right)\right],$$

which is

$$n_m\left[\tilde{r}_{ne}-\left(\frac{\tilde{c}_{new}+(i_{nw}-i_{new})}{1-t}\right)\right]>n_h\left(\frac{\tilde{c}_{new}+(i_{nw}-i_{new})}{1-t}\right).$$

That is, the net benefit from the medium worker switching from not working to working at the uneducated wage must exceed the increase in cost for the firm for high-type worker. Thus, the above condition is necessary for profit in equilibrium 1 to exceed profit in equilibrium 2 in this case.

Proof of Proposition 3.2.5:

When $T>0, t>0, i_{ew}>0, i_{new}>0,$ and $i_{nw}\geq 0$ and if $i_{nw}\geq i_{new}-\tilde{c}_{new}$ for profit in equilibrium 1 to be higher than profit in equilibrium 3 assuming wages are at their minimum it must be the case that For profit in equilibrium 1 to be higher than profit in equilibrium 3 it must be the case that

$$(1-T)\left(n_h\left[\overline{r}_e-\left(\frac{\underline{c}_{eew}-\underline{c}_{new}+(i_{nw}-i_{ew})}{1-t}\right)-\left(\frac{\tilde{c}_{new}}{1-t}\right)\right]+n_m\left[\tilde{r}_{ne}-\left(\frac{\tilde{c}_{new}+(i_{nw}-i_{new})}{1-t}\right)\right]\right)$$

$$>(1-T)\left(n_h\left[\overline{r}_e-\left(\frac{\tilde{c}_{eew}+(i_{nw}-i_{ew})}{(1-t)}\right)\right]+n_m\left[\tilde{r}_e-\left(\frac{\tilde{c}_{eew}+(i_{nw}-i_{ew})}{(1-t)}\right)\right]\right)$$

which is

$$n_m\left[\left(\tilde{r}_{ne}-\frac{\tilde{c}_{new}}{1-t}\right)-\left(\tilde{r}_e-\frac{\tilde{c}_{eew}}{(1-t)}\right)+\frac{(i_{new}-i_{ew})}{1-t}\right]>n_h\left[\frac{(\underline{c}_{eew}-\underline{c}_{new})}{1-t}-\frac{(\tilde{c}_{eew}-\tilde{c}_{new})}{(1-t)}\right].$$

When $T>0, t>0, i_{ew}>0, i_{new}>0,$ and $i_{nw}\geq 0$ and if $i_{nw}<i_{new}-\tilde{c}_{new}$ for profit in equilibrium 1 to be higher than profit in equilibrium 3 assuming wages are at their minimum it must be the case that

$$(1-T)\left(n_h\left[\overline{r}_e-\left(\frac{\underline{c}_{eew}-\underline{c}_{new}+(i_{nw}-i_{ew})}{1-t}\right)-\left(\frac{\tilde{c}_{new}}{1-t}\right)\right]+n_m\left[\tilde{r}_{ne}-\left(\frac{\tilde{c}_{new}+(i_{nw}-i_{new})}{1-t}\right)\right]\right)$$

$$>(1-T)\left(n_h\left[\overline{r}_e-\left(\frac{\tilde{c}_{eew}-\tilde{c}_{new}+(i_{new}-i_{ew})}{1-t}\right)\right]\right.$$

$$\left.+n_m\left[\tilde{r}_e-\left(\frac{\tilde{c}_{eew}-\tilde{c}_{new}+(i_{new}-i_{ew})}{1-t}\right)\right]\right)$$

which is

$$n_m\left[\left(\tilde{r}_{ne}-\frac{\tilde{c}_{new}}{1-t}\right)-\left(\tilde{r}_e-\frac{\tilde{c}_{eew}}{(1-t)}\right)+\frac{(i_{new}-i_{nw})+(i_{new}-i_{ew})-\tilde{c}_{new}}{1-t}\right]$$

$$>n_h\left[\frac{\left(\underline{c}_{eew}-\underline{c}_{new}\right)}{1-t}-\frac{\left(\tilde{c}_{eew}+(i_{new}-i_{nw})\right)}{(1-t)}\right].$$

That is, the net benefit of the medium worker switching from working at the educated wage to working at the uneducated wage must exceed the change in cost for the high-type worker. It is always true that

$$n_m\left[\left(\tilde{r}_{ne}-\frac{\tilde{c}_{new}}{1-t}\right)-\left(\tilde{r}_e-\frac{\tilde{c}_{eew}}{(1-t)}\right)\right]>n_h\left[\frac{\left(\underline{c}_{eew}-\underline{c}_{new}\right)}{1-t}-\frac{(\tilde{c}_{eew}-\tilde{c}_{new})}{(1-t)}\right]$$

since

$$\frac{\left(\underline{c}_{eew}-\underline{c}_{new}\right)}{1-t}-\frac{(\tilde{c}_{eew}-\tilde{c}_{new})}{(1-t)}<0\ and\ \left(\tilde{r}_{ne}-\frac{\tilde{c}_{new}}{1-t}\right)-\left(\tilde{r}_e-\frac{\tilde{c}_{eew}}{(1-t)}\right)>0.$$

Therefore, as long as i_{ew} is not sufficiently greater than i_{new} such that the above condition holds then profit in equilibrium 1 exceeds the profit in equilibrium 3.

Proof of Proposition 3.2.6:

When $T>0, t>0, i_{ew}>0, i_{new}>0,$ and $i_{nw}\geq 0$ and if $i_{nw}\geq i_{new}-\overline{c}_{new}$ for profit in equilibrium 1 to be higher than profit in equilibrium 4 assuming wages are at their minimum it must be the case that

$$(1-T)\left(n_h\left[\overline{r}_e-\left(\frac{\underline{c}_{eew}-\underline{c}_{new}+\tilde{c}_{new}+(i_{nw}-i_{ew})}{1-t}\right)\right]+n_m\left[\tilde{r}_{ne}-\left(\frac{\tilde{c}_{new}+(i_{nw}-i_{new})}{1-t}\right)\right]\right)$$

$$>(1-T)\left(n_h\left[\overline{r}_e-\left(\frac{\overline{c}_{eew}+(i_{nw}-i_{ew})}{(1-t)}\right)\right]+n_m\left[\tilde{r}_e-\left(\frac{\overline{c}_{eew}+(i_{nw}-i_{ew})}{(1-t)}\right)\right]\right.$$

$$\left.+n_l\left[\underline{r}_e-\left(\frac{\overline{c}_{eew}+(i_{nw}-i_{ew})}{(1-t)}\right)\right]\right),$$

which is

$$n_m\left[\left(\tilde{r}_{ne}-\frac{\tilde{c}_{new}}{1-t}\right)-\left(\tilde{r}_e-\frac{\overline{c}_{eew}}{(1-t)}\right)+\frac{(i_{new}-i_{ew})}{1-t}\right]-n_l\left[\underline{r}_e-\left(\frac{\overline{c}_{eew}+(i_{nw}-i_{ew})}{(1-t)}\right)\right]$$

$$>n_h\left[\left(\frac{\underline{c}_{eew}-\underline{c}_{new}-(\overline{c}_{eew}-\tilde{c}_{new})}{1-t}\right)\right].$$

That is, the net benefit of the medium working switching from working at the educated wage to working at the uneducated wage and the low-type worker switching from not working to working at the educated wage must exceed the change in cost for the high-type worker. Per earlier when $t = i_{ew} = i_{new} = i_{nw} = 0$ the left side is greater than the right side so as long as $\frac{(i_{new}-i_{ew})}{1-t}$ is not sufficiently less than $\frac{(i_{nw}-i_{ew})}{1-t}$ then profit in equilibrium 1 exceeds the profit in equilibrium 4.

When $T > 0, t > 0, i_{ew} > 0, i_{new} > 0,$ and $i_{nw} \geq 0$ and if $i_{nw} < i_{new} - \bar{c}_{new}$ for profit in equilibrium 1 to be higher than profit in equilibrium 4 assuming wages are at their minimum it must be the case that

$$(1-T)\left(n_h\left[\bar{r}_e - \left(\frac{\underline{c}_{eew} - \underline{c}_{new} + \tilde{c}_{new} + (i_{nw} - i_{ew})}{1-t}\right)\right] + n_m\left[\tilde{r}_{ne} - \left(\frac{\tilde{c}_{new} + (i_{nw} - i_{new})}{1-t}\right)\right]\right)$$

$$> (1-T)\left(n_h\left[\bar{r}_e - \left(\frac{\bar{c}_{eew} - \bar{c}_{new} + (i_{new} - i_{ew})}{(1-t)}\right)\right] + n_m\left[\tilde{r}_e - \left(\frac{\bar{c}_{eew} - \bar{c}_{new} + (i_{new} - i_{ew})}{(1-t)}\right)\right]\right.$$

$$\left. + n_l\left[\underline{r}_e - \left(\frac{\bar{c}_{eew} - \bar{c}_{new} + (i_{new} - i_{ew})}{(1-t)}\right)\right]\right),$$

which is

$$n_m\left[\left(\tilde{r}_{ne} - \frac{\tilde{c}_{new}}{1-t}\right) - \left(\tilde{r}_e - \frac{\bar{c}_{eew}}{(1-t)}\right) + \frac{(i_{new} - i_{nw}) + (i_{new} - i_{ew}) - \bar{c}_{new}}{1-t}\right]$$

$$- n_l\left[\underline{r}_e - \left(\frac{\bar{c}_{eew} - \bar{c}_{new} + (i_{new} - i_{ew})}{(1-t)}\right)\right]$$

$$> n_h\left[\left(\frac{\underline{c}_{eew} - \underline{c}_{new} - (\bar{c}_{eew} - \tilde{c}_{new}) - (i_{new} - i_{nw})}{1-t}\right)\right]$$

Proof of Proposition 3.2.7:

When $T > 0, t > 0, i_{ew} > 0, i_{new} > 0,$ and $i_{nw} \geq 0$ for profit in equilibrium 1 to be higher than profit in equilibrium 5 assuming wages are at their minimum it must be the case that

$$(1-T)\left(n_h\left[\overline{r}_e-\left(\frac{\underline{c}_{eew}-\underline{c}_{new}+(i_{nw}-i_{ew})}{1-t}\right)-\left(\frac{\tilde{c}_{new}}{1-t}\right)\right]+n_m\left[\tilde{r}_{ne}-\left(\frac{\tilde{c}_{new}+(i_{nw}-i_{new})}{1-t}\right)\right]\right)$$

$$>(1-T)\left(n_h\left[\overline{r}_e-\left(\frac{\tilde{c}_{eew}-\tilde{c}_{new}+(i_{nw}-i_{ew})}{1-t}\right)-\left(\frac{\overline{c}_{new}}{(1-t)}\right)\right]\right.$$

$$\left.+n_m\left[\tilde{r}_e-\left(\frac{\tilde{c}_{eew}-\tilde{c}_{new}+(i_{nw}-i_{ew})}{1-t}\right)-\left(\frac{\overline{c}_{new}}{(1-t)}\right)\right]+n_l\left[\underline{r}_{ne}-\frac{\overline{c}_{new}+(i_{nw}-i_{new})}{(1-t)}\right]\right)$$

which is

$$n_m\left[\tilde{r}_{ne}-\frac{\tilde{c}_{new}}{1-t}-\left(\tilde{r}_e-\frac{\tilde{c}_{eew}}{1-t}\right)+\left(\frac{\overline{c}_{new}-\tilde{c}_{new}}{1-t}\right)\right]-n_l\left[\underline{r}_{ne}-\frac{\overline{c}_{new}}{1-t}-\frac{(i_{new}-i_{nw})}{(1-t)}\right]$$

$$>n_h\left[\left(\frac{\underline{c}_{eew}-\underline{c}_{new}}{1-t}\right)-\left(\frac{\tilde{c}_{eew}-\tilde{c}_{new}}{1-t}\right)-\left(\frac{\overline{c}_{new}-\tilde{c}_{new}}{(1-t)}\right)\right].$$

That is, the net benefit of the medium-type worker switching from working at the educated wage to working at the uneducated wage and the low type switching from working at the uneducated wage to not working must exceed the change in cost for the high-type worker. This again always holds as long as i_{new} is not sufficiently greater than i_{nw} such that the above condition holds then profit in equilibrium 1 exceeds the profit in equilibrium 5.

Proof of Proposition 3.2.8:

When $T>0, t>0, i_{ew}>0, i_{new}>0,$ and $i_{nw}\geq 0$ for profit in equilibrium 1 to be higher than profit in equilibrium 6 assuming wages are at their minimum it must be the case that

$$(1-T)\left(n_h\left[\overline{r}_e-\left(\frac{\underline{c}_{eew}-\underline{c}_{new}+(i_{nw}-i_{ew})}{1-t}\right)-\left(\frac{\tilde{c}_{new}}{1-t}\right)\right]+n_m\left[\tilde{r}_{ne}-\left(\frac{\tilde{c}_{new}+(i_{nw}-i_{new})}{1-t}\right)\right]\right)$$

$$>(1-T)\left(n_h\left[\overline{r}_e-\left(\frac{\underline{c}_{eew}-\underline{c}_{new}+(i_{nw}-i_{ew})}{1-t}\right)-\left(\frac{\overline{c}_{new}}{(1-t)}\right)\right]\right.$$

$$\left.+n_m\left[\tilde{r}_{ne}-\frac{\overline{c}_{new}+(i_{nw}-i_{new})}{(1-t)}\right]+n_l\left[\underline{r}_{ne}-\frac{\overline{c}_{new}+(i_{nw}-i_{new})}{(1-t)}\right]\right),$$

which is

$$n_l\left[\frac{\overline{c}_{new}}{1-t}-\underline{r}_{ne}-\frac{(i_{new}-i_{nw})}{1-t}\right]>n_h\left[\left(\frac{\tilde{c}_{new}}{1-t}\right)-\left(\frac{\overline{c}_{new}}{(1-t)}\right)\right]+n_m\left[\left(\frac{\tilde{c}_{new}}{1-t}\right)-\frac{\overline{c}_{new}}{(1-t)}\right].$$

That is, the net benefit of the low-type worker switching from working at the uneducated wage to not working must exceed the change in cost for the high and medium type workers. This again always holds as long as i_{new} is not sufficiently greater than i_{nw} such that the above condition holds then profit in equilibrium 1 exceeds the profit in equilibrium 6.

Proof of Proposition 3.2.9:

When $T > 0, t > 0, i_{ew} > 0, i_{new} > 0,$ and $i_{nw} \geq 0$ and if $i_{nw} \geq i_{ew} - \overline{c}_{eew}$ for profit in equilibrium 1 to be higher than profit in equilibrium 7 assuming wages are at their minimum it must be the case that

$$(1-T)\left(n_h\left[\overline{r}_e - \left(\frac{\underline{c}_{eew} - \underline{c}_{new} + (i_{nw} - i_{ew})}{1-t}\right) - \left(\frac{\tilde{c}_{new}}{1-t}\right)\right] + n_m\left[\tilde{r}_{ne} - \left(\frac{\tilde{c}_{new} + (i_{nw} - i_{new})}{1-t}\right)\right]\right)$$

$$> (1-T)\left(n_h\left[\overline{r}_{ne} - \left(\frac{\overline{c}_{new} + (i_{nw} - i_{new})}{(1-t)}\right)\right] + n_m\left[\tilde{r}_{ne} - \left(\frac{\overline{c}_{new} + (i_{nw} - i_{new})}{(1-t)}\right)\right]\right.$$

$$\left. + n_l\left[\underline{r}_{ne} - \left(\frac{\overline{c}_{new} + (i_{nw} - i_{new})}{(1-t)}\right)\right]\right),$$

which is

$$n_h\left[\left(\overline{r}_e - \frac{\underline{c}_{eew}}{1-t}\right) - \left(\overline{r}_{ne} - \frac{\underline{c}_{new}}{1-t}\right) + \left(\frac{\overline{c}_{new} - \tilde{c}_{new}}{1-t}\right) + \left(\frac{(i_{ew} - i_{new})}{(1-t)}\right)\right]$$

$$> n_m\left[\left(\frac{\tilde{c}_{new} - \overline{c}_{new}}{1-t}\right)\right] + n_l\left[\underline{r}_{ne} - \left(\frac{\overline{c}_{new} + (i_{nw} - i_{new})}{(1-t)}\right)\right].$$

That is, the net benefit of the high-type worker switching from working at the uneducated wage to working at the educated wage must exceed the change in cost for the medium-type worker and the net change from the low-type worker switching from working at the uneducated wage to not working. Per earlier when $T = t = i_{ew} = i_{new} = i_{nw} = 0$ the left side is greater than the right side so if $\frac{(i_{ew} - i_{new})}{(1-t)}$ is not sufficiently small then profit in equilibrium 1 exceeds the profit in equilibrium 7.

When $T > 0, t > 0, i_{ew} > 0, i_{new} > 0,$ and $i_{nw} \geq 0$ and if $i_{nw} < i_{ew} - \overline{c}_{eew}$ for profit in equilibrium 1 to be higher than profit in equilibrium 7 assuming wages are at their minimum it must be the case that

$$(1-T)\left(n_h\left[\overline{r}_e-\left(\frac{\underline{c}_{eew}-\underline{c}_{new}+(i_{nw}-i_{ew})}{1-t}\right)-\left(\frac{\tilde{c}_{new}}{1-t}\right)\right]+n_m\left[\tilde{r}_{ne}-\left(\frac{\tilde{c}_{new}+(i_{nw}-i_{new})}{1-t}\right)\right]\right)$$

$$>(1-T)\left(n_h\left[\overline{r}_{ne}-\left(\frac{\left(\underline{c}_{new}-\underline{c}_{eew}\right)+(i_{ew}-i_{new})}{(1-t)}\right)\right]\right.$$

$$+n_m\left[\tilde{r}_{ne}-\left(\frac{\left(\underline{c}_{new}-\underline{c}_{eew}\right)+(i_{ew}-i_{new})}{(1-t)}\right)\right]$$

$$\left.+n_l\left[\underline{r}_{ne}-\left(\frac{\left(\underline{c}_{new}-\underline{c}_{eew}\right)+(i_{ew}-i_{new})}{(1-t)}\right)\right]\right),$$

which is

$$n_h\left[\left(\overline{r}_e-\frac{\underline{c}_{eew}}{1-t}\right)-\left(\overline{r}_{ne}-\frac{\underline{c}_{new}}{1-t}\right)-\left(\frac{\underline{c}_{eew}-\underline{c}_{new}+\tilde{c}_{new}}{1-t}\right)+\frac{(i_{ew}-i_{nw})}{1-t}+\frac{(i_{ew}-i_{new})}{1-t}\right]$$

$$>n_m\left[\left(\frac{\left(\underline{c}_{eew}-\underline{c}_{new}\right)+\tilde{c}_{new}-(i_{ew}-i_{nw})}{(1-t)}\right)\right]+n_l\left[\underline{r}_{ne}-\left(\frac{\left(\underline{c}_{new}-\underline{c}_{eew}\right)+(i_{ew}-i_{new})}{(1-t)}\right)\right]$$

Proof of Proposition 3.2.10:

When $T>0, t>0, i_{ew}>0, i_{new}>0,$ and $i_{nw}\geq 0$ and if $i_{nw}\geq i_{ew}-\tilde{c}_{eew}$ for profit in equilibrium 1 to be higher than profit in equilibrium 8 assuming wages are at their minimum it must be the case that

$$(1-T)\left(n_h\left[\overline{r}_e-\left(\frac{\underline{c}_{eew}-\underline{c}_{new}+(i_{nw}-i_{ew})}{1-t}\right)-\left(\frac{\tilde{c}_{new}}{1-t}\right)\right]+n_m\left[\tilde{r}_{ne}-\left(\frac{\tilde{c}_{new}+(i_{nw}-i_{new})}{1-t}\right)\right]\right)$$

$$>(1-T)\left(n_h\left[\overline{r}_{ne}-\left(\frac{\tilde{c}_{new}+(i_{nw}-i_{new})}{(1-t)}\right)\right]+n_m\left[\tilde{r}_{ne}-\left(\frac{\tilde{c}_{new}+(i_{nw}-i_{new})}{(1-t)}\right)\right]\right),$$

which is

$$n_h\left[\left(\overline{r}_e-\frac{\underline{c}_{eew}}{1-t}\right)-\left(\overline{r}_{ne}-\frac{\underline{c}_{new}}{1-t}\right)+\left(\frac{(i_{ew}-i_{new})}{(1-t)}\right)\right]>0.$$

That is, the net benefit of the high-type worker switching from working at the uneducated wage to working at the educated wage must exceed the change in cost for the medium-type worker, which is zero. This again always holds if i_{new} is not sufficiently greater than i_{nw} such that the above condition holds then profit in equilibrium 1 exceeds the profit in equilibrium 8.

When $T > 0, t > 0, i_{ew} > 0, i_{new} > 0,$ and $i_{nw} \geq 0$ and if $i_{nw} < i_{ew} - \tilde{c}_{eew}$ for profit in equilibrium 1 to be higher than profit in equilibrium 8 assuming wages are at their minimum it must be the case that

$$(1-T)\left(n_h\left[\bar{r}_e - \left(\frac{\underline{c}_{eew} - \underline{c}_{new} + (i_{nw} - i_{ew})}{1-t}\right) - \left(\frac{\tilde{c}_{new}}{1-t}\right)\right] + n_m\left[\tilde{r}_{ne} - \left(\frac{\tilde{c}_{new} + (i_{nw} - i_{new})}{1-t}\right)\right]\right)$$
$$> (1-T)\left(n_h\left[\bar{r}_{ne} - \left(\frac{(\underline{c}_{new} - \underline{c}_{eew}) + (i_{ew} - i_{new})}{(1-t)}\right)\right]\right.$$
$$\left. + n_m\left[\tilde{r}_{ne} - \left(\frac{(\underline{c}_{new} - \underline{c}_{eew}) + (i_{ew} - i_{new})}{(1-t)}\right)\right]\right),$$

which is

$$n_h\left[\left(\bar{r}_e - \frac{\underline{c}_{eew}}{1-t}\right) - \left(\bar{r}_{ne} - \frac{\underline{c}_{new}}{1-t}\right) - \left(\frac{\underline{c}_{eew} - \underline{c}_{new} + \tilde{c}_{new}}{1-t}\right) + \frac{(i_{ew} - i_{nw})}{1-t} + \frac{(i_{ew} - i_{new})}{1-t}\right]$$
$$> n_m\left[\left(\frac{\underline{c}_{eew} - \underline{c}_{new} + \tilde{c}_{new}}{1-t}\right) - \frac{(i_{ew} - i_{nw})}{1-t} - \frac{(i_{new} - i_{nw})}{1-t}\right].$$

Proof of Proposition 3.2.11:

When $T > 0, t > 0, i_{ew} > 0, i_{new} > 0,$ and $i_{nw} \geq 0$ and if $i_{nw} \geq i_{ew} - \underline{c}_{eew}$ for profit in equilibrium 1 to be higher than profit in equilibrium 9 assuming wages are at their minimum it must be the case that

$$(1-T)\left(n_h\left[\bar{r}_e - \left(\frac{\underline{c}_{eew} - \underline{c}_{new} + \tilde{c}_{new} + (i_{nw} - i_{ew})}{1-t}\right)\right] + n_m\left[\tilde{r}_{ne} - \left(\frac{\tilde{c}_{new} + (i_{nw} - i_{new})}{1-t}\right)\right]\right)$$
$$> (1-T)n_h\left[\bar{r}_{ne} - \left(\frac{\underline{c}_{new} + (i_{nw} - i_{new})}{(1-t)}\right)\right]$$

which is

$$n_h\left[\left(\bar{r}_e - \frac{\underline{c}_{eew}}{1-t}\right) - \left(\bar{r}_{ne} - \frac{\underline{c}_{new}}{1-t}\right) + \frac{(i_{ew} - i_{new})}{1-t}\right] + n_m\left[\tilde{r}_{ne} - \left(\frac{\tilde{c}_{new} + (i_{nw} - i_{new})}{1-t}\right)\right] > n_h\left[\frac{\tilde{c}_{new} - \underline{c}_{new}}{1-t}\right].$$

When $T > 0, t > 0, i_{ew} > 0, i_{new} > 0,$ and $i_{nw} \geq 0$ and if $i_{nw} < i_{ew} - \underline{c}_{eew}$ for profit in equilibrium 1 to be higher than profit in equilibrium 9 assuming wages are at their minimum it must be the case that

$$(1-T)\left(n_h\left[\overline{r}_e-\left(\frac{\underline{c}_{eew}-\underline{c}_{new}+\tilde{c}_{new}+(i_{nw}-i_{ew})}{1-t}\right)\right]+n_m\left[\tilde{r}_{ne}-\left(\frac{\tilde{c}_{new}+(i_{nw}-i_{new})}{1-t}\right)\right]\right)$$

$$>(1-T)n_h\left[\overline{r}_{ne}-\left(\frac{(\underline{c}_{new}-\underline{c}_{eew})+(i_{ew}-i_{new})}{(1-t)}\right)\right]$$

which is

$$n_h\left[\left(\overline{r}_e-\frac{\underline{c}_{eew}}{1-t}\right)-\left(\overline{r}_{ne}-\frac{\underline{c}_{new}}{1-t}\right)+\frac{(i_{ew}-i_{nw})}{1-t}\right]-n_h\left[\frac{\underline{c}_{eew}-\underline{c}_{new}+\tilde{c}_{new}-(i_{ew}-i_{new})}{1-t}\right]$$

$$+n_m\left[\tilde{r}_{ne}-\left(\frac{\tilde{c}_{new}+(i_{nw}-i_{new})}{1-t}\right)\right]>0.$$

That is, the net benefit of the high-type worker switching from working at the uneducated wage to working at the educated wage and the medium-type worker switching from not working to working at the uneducated wage must be positive.

Proof of Proposition 3.2.12:

When $T>0, t>0, i_{ew}>0, i_{new}>0,$ and $i_{nw}\geq 0$ for profit in equilibrium 1 to be higher than profit in equilibrium 10 assuming wages are at their minimum it must be the case that

$$(1-T)\left(n_h\left[\overline{r}_e-\left(\frac{\underline{c}_{eew}-\underline{c}_{new}+(i_{nw}-i_{ew})}{1-t}\right)-\left(\frac{\tilde{c}_{new}}{1-t}\right)\right]+n_m\left[\tilde{r}_{ne}-\left(\frac{\tilde{c}_{new}+(i_{nw}-i_{new})}{1-t}\right)\right]\right)>0$$

Proof of Proposition 3.2.13:

In the absence of tax policy, i.e. when $T=t=i_{ew}=i_{new}=i_{nw}=0$, it has been shown that given the assumptions on revenues and costs across worker types profit is greater in equilibrium 2 which is greater than profit in equilibrium 9 which is greater than profit in equilibrium 1 which is greater than profit than the other 7 equilibria. From proposition 3.2.4 we know that,

In the presence of tax policy, i.e. $T>0, t>0, i_{ew}>0, i_{new}>0,$ and $i_{nw}\geq 0$, *total profit in separating equilibrium 1 is greater than total profit in partial pooling equilibrium 2 when*

$$if\ i_{new}\leq i_{nw}+\underline{c}_{new}\ then\ n_m\left[\tilde{r}_{ne}-\frac{\tilde{c}_{new}}{1-t}+\frac{(i_{new}-i_{nw})}{1-t}\right]>n_h\left(\frac{\tilde{c}_{new}-\underline{c}_{new}}{1-t}\right)$$

$$and\ if\ i_{new} > i_{nw} + \underline{c}_{new}\ then\ n_m\left[\tilde{r}_{ne} - \frac{\tilde{c}_{new}}{1-t} + \frac{(i_{new} - i_{nw})}{1-t}\right] > n_h\left(\frac{\tilde{c}_{new} + (i_{nw} - i_{new})}{1-t}\right).$$

Solving these equations for i_{new} as a function of t, i_{nw}, revenues, and costs we get the following minimums for i_{new}

$$for\ i_{new} \leq i_{nw} + \underline{c}_{new}$$

$$i_{new} > \frac{n_h}{n_m}\left(\tilde{c}_{new} - \underline{c}_{new}\right) - \left(\tilde{r}_{ne}(1-t) - \tilde{c}_{new}\right) + i_{nw}$$

$$i_{nw} + \underline{c}_{new} < i_{new}$$

$$i_{new} > \frac{n_h}{n_m + n_h}\left(\tilde{c}_{new}\right) - \frac{n_m}{n_m + n_h}\left(\tilde{r}_{ne}(1-t) - \tilde{c}_{new}\right) + i_{nw}.$$

However, as we know from propositions 3.2.8, 3.2.9, 3.2.10, and 3.2.11 increasing i_{new} can favor equilibria 6, 7, 8, and 9, respectively, over equilibrium 1. But we also know from these propositions that,

In the presence of tax policy, i.e. $T > 0, t > 0, i_{ew} > 0, i_{new} > 0,$ and $i_{nw} \geq 0$, *total profit in separating equilibrium 1 is greater than total profit in partial pooling equilibrium 6 when*

$$n_h\left[\frac{\overline{c}_{new} - \tilde{c}_{new}}{1-t}\right] + n_m\left[\frac{\overline{c}_{new} - \tilde{c}_{new}}{1-t}\right] > n_l\left[\left(\underline{r}_{ne} - \frac{\overline{c}_{new}}{1-t}\right) + \frac{(i_{new} - i_{nw})}{1-t}\right],$$

In the presence of tax policy, i.e. $T > 0, t > 0, i_{ew} > 0, i_{new} > 0,$ and $i_{nw} \geq 0$, *total profit in separating equilibrium 1 is greater than total profit in full pooling equilibrium 7 when*

$$if\ i_{ew} \leq i_{nw} + \overline{c}_{eew}\ then\ n_h\left[\left(\overline{r}_e - \frac{\underline{c}_{eew}}{1-t}\right) - \left(\overline{r}_{ne} - \frac{\underline{c}_{new}}{1-t}\right) + \frac{(\overline{c}_{new} - \tilde{c}_{new})}{1-t} + \frac{(i_{ew} - i_{new})}{(1-t)}\right] + n_m\left[\frac{(\overline{c}_{new} - \tilde{c}_{new})}{1-t}\right]$$

$$> n_l\left[\left(\underline{r}_{ne} - \frac{\overline{c}_{new}}{1-t}\right) + \frac{(i_{new} - i_{nw})}{1-t}\right]$$

$$and\ if\ i_{ew} > i_{nw} + \overline{c}_{eew}\ then\ n_h\left[\left(\overline{r}_e - \frac{\underline{c}_{eew}}{1-t}\right) - \left(\overline{r}_{ne} - \frac{\underline{c}_{new}}{1-t}\right) - \frac{\left(\underline{c}_{eew} - \underline{c}_{new} + \tilde{c}_{new}\right)}{1-t} + \frac{(i_{ew} - i_{nw})}{1-t} + \frac{(i_{ew} - i_{new})}{1-t}\right]$$

$$> n_m\left[\frac{\left(\underline{c}_{eew} - \underline{c}_{new} + \tilde{c}_{new}\right)}{1-t} + \frac{(i_{nw} - i_{ew})}{1-t}\right] + n_l\left[\left(\underline{r}_{ne} - \frac{\overline{c}_{new}}{1-t}\right) + \left(\frac{(i_{new} - i_{ew}) + \underline{c}_{eew}}{1-t}\right)\right]$$

In the presence of tax policy, i.e. $T > 0, t > 0, i_{ew} > 0, i_{new} > 0,$ and $i_{nw} \geq 0$, *total profit in separating equilibrium 1 is greater than total profit in the partial pooling equilibrium 8 when*

$$if\ i_{ew} \leq i_{nw} + \tilde{c}_{eew}\ then\ n_h \left[\left(\overline{r}_e - \frac{\underline{c}_{eew}}{1-t} \right) - \left(\overline{r}_{ne} - \frac{\underline{c}_{new}}{1-t} \right) + \left(\frac{(i_{ew} - i_{new})}{(1-t)} \right) \right] > 0$$

$$and\ if\ i_{ew} > i_{nw} + \tilde{c}_{eew}\ then\ n_h \left[\left(\overline{r}_e - \frac{\underline{c}_{eew}}{1-t} \right) - \left(\overline{r}_{ne} - \frac{\underline{c}_{new}}{1-t} \right) - \frac{\left(\underline{c}_{eew} - \underline{c}_{new} + \tilde{c}_{new} \right)}{1-t} + \frac{(i_{ew} - i_{nw})}{1-t} \right.$$

$$\left. + \frac{(i_{ew} - i_{new})}{1-t} \right] > n_m \left[\left(\frac{\underline{c}_{eew} - \underline{c}_{new} + \tilde{c}_{new}}{1-t} \right) + \frac{(i_{nw} - i_{ew})}{1-t} + \frac{(i_{nw} - i_{new})}{1-t} \right]$$

In the presence of tax policy, i.e. $T > 0, t > 0, i_{ew} > 0, i_{new} > 0,$ and $i_{nw} \geq 0$, *total profit in separating equilibrium 1 is greater than total profit in the partial pooling equilibrium 9 when*

$$if\ i_{ew} \leq i_{nw} + \underline{c}_{eew}\ then\ n_h \left[\left(\overline{r}_e - \frac{\underline{c}_{eew}}{1-t} \right) - \left(\overline{r}_{ne} - \frac{\underline{c}_{new}}{1-t} \right) + \frac{(i_{ew} - i_{new})}{1-t} \right]$$

$$+ n_m \left[\left(\tilde{r}_{ne} - \frac{\tilde{c}_{new}}{1-t} \right) + \frac{(i_{new} - i_{nw})}{1-t} \right] > n_h \left[\frac{\left(\tilde{c}_{new} - \underline{c}_{new} \right)}{1-t} \right]$$

$$and\ if\ i_{ew} > i_{nw} + \underline{c}_{eew}\ then\ n_h \left[\left(\overline{r}_e - \frac{\underline{c}_{eew}}{1-t} \right) - \left(\overline{r}_{ne} - \frac{\underline{c}_{new}}{1-t} \right) + \frac{(i_{ew} - i_{nw})}{1-t} \right]$$

$$+ n_m \left[\left(\tilde{r}_{ne} - \frac{\tilde{c}_{new}}{1-t} \right) + \frac{(i_{new} - i_{nw})}{1-t} \right] > n_h \left[\frac{\left(\underline{c}_{eew} - \underline{c}_{new} + \tilde{c}_{new} \right)}{1-t} + \frac{(i_{new} - i_{ew})}{1-t} \right].$$

For equilibrium 6 there is no way to compensate for increases in i_{new} therefore the condition in proposition 3.2.8 establishes the maximum i_{new} where profit in equilibrium 1 is still greater than profit in equilibrium 6. Solving for i_{new} we get

$$for\ \ i_{new} < i_{nw} + \tilde{c}_{new}$$

$$i_{new} < \frac{n_m + n_h}{n_l} \left(\overline{c}_{new} - \tilde{c}_{new} \right) - \left(\underline{r}_{ne}(1-t) - \overline{c}_{new} \right) + i_{nw}$$

For equilibria 7, 8, and 9 increases in i_{new} can be compensated by increases in i_{ew}. Therefore, the conditions in propositions 3.2.9, 3.2.10, and 3.2.11 establish the minimums for i_{ew} such that profit in equilibrium 1 is greater than profit in equilibria 7, 8, and 9. Solving each, respectively for i_{ew} yields

$$for\ i_{ew} \leq i_{nw} + \underline{c}_{eew}$$

$$i_{ew} > \frac{n_l}{n_h} \left[\left(\underline{r}_{ne}(1-t) - \overline{c}_{new} \right) + (i_{new} - i_{nw}) \right] - \frac{n_m}{n_h} \left[\overline{c}_{new} - \tilde{c}_{new} \right] + \left(\overline{r}_{ne}(1-t) - \underline{c}_{new} \right) - \left(\overline{r}_e(1-t) - \underline{c}_{eew} \right) - \left(\overline{c}_{new} - \tilde{c}_{new} \right)$$

$$+ i_{new}],$$

$$i_{ew} > \left(\overline{r}_{ne}(1-t) - \underline{c}_{new}\right) - \left(\overline{r}_{e}(1-t) - \underline{c}_{eew}\right) + i_{new},$$

and

$$i_{ew} > \left(\tilde{c}_{new} - \underline{c}_{new}\right) + \left(\overline{r}_{ne}(1-t) - \underline{c}_{new}\right) - \left(\overline{r}_{e}(1-t) - \underline{c}_{eew}\right) - \frac{n_m}{n_h}\left[(\tilde{r}_{ne}(1-t) - \tilde{c}_{new}) + (i_{new} - i_{nw})\right]$$

$$+ i_{new}.$$

However, as we know from propositions 3.2.5 and 3.2.6 increasing i_{ew} can favor equilibria 3 and 4 over equilibrium 1 in terms of profit. But profit in equilibrium 3 is always greater than profit in equilibrium 4 therefore the conditions in proposition 3.2.5 informs the maximum on i_{ew} for which profit in equilibrium 1 is still greater than profit in equilibria 3 and 4. Solving for i_{ew} yields

$$for\ i_{new} \leq i_{nw} + \tilde{c}_{new}$$

$$i_{ew} < (\tilde{r}_{ne}(1-t) - \tilde{c}_{new}) - (\tilde{r}_{e}(1-t) - \tilde{c}_{eew}) - \frac{n_h}{n_m}\left[\left(\underline{c}_{eew} - \underline{c}_{new}\right) - (\tilde{c}_{eew} - \tilde{c}_{new})\right] + i_{new}.$$

Proof of Lemma 4.a:

Assume the following values for given variables:

$$n_h = n_m = n_l = 1$$

$$\underline{c}_{new} = 0.5, \tilde{c}_{new} = 1.1, \overline{c}_{new} = 1.25$$

$$\underline{c}_{eew} = 1, \tilde{c}_{eew} = 1.8, \overline{c}_{eew} = 2.05$$

$$\overline{r}_{ne} = 2, \tilde{r}_{ne} = 1.5, \underline{r}_{ne} = 1$$

$$\overline{r}_{e} = 2.6, \tilde{r}_{e} = 2, \underline{r}_{e} = 1.5.$$

Note that given these values the following is true

$$\overline{r}_{e} - \underline{c}_{eew} > \overline{r}_{ne} - \underline{c}_{new} > 0,$$

$$\tilde{r}_{ne} - \tilde{c}_{new} > \tilde{r}_{e} - \tilde{c}_{eew} > 0, \text{and}$$

$$0 > \underline{r}_{ne} - \overline{c}_{new} > \underline{r}_{e} - \overline{c}_{eew}$$

where,

$$1.6 > 1.5 > 0,$$

$$0.4 > 0.2 > 0, \text{and}$$

$$0 > -0.25 > -0.55.$$

Also, the following conditions from propositions 3.2.2 and 3.2.3 are true

$$n_m(\tilde{r}_{ne} - \tilde{c}_{new}) < n_h(\tilde{c}_{new} - \underline{c}_{new})\ and$$

$$n_h(\overline{r}_e - \underline{c}_{eew}) - n_h(\overline{r}_{ne} - \underline{c}_{new}) + n_m(\tilde{r}_{ne} - \tilde{c}_{new}) < n_h(\tilde{c}_{new} - \underline{c}_{new})$$

where,

$$0.4 < 0.6 \text{ and}$$

$$1.6 - 1.5 + 0.4 = 0.5 < 0.6.$$

Proof of Proposition 4.1:

Choose tax rates $t = 0$, $T = 0$, and transfers $i_{ew} = 0$, $i_{new} = 0$, and $i_{nw} = 0$.

Given the tax rates $t = 0$, $T = 0$, and transfers $i_{ew} = 0$, $i_{new} = 0$, and $i_{nw} = 0$ the firm chooses the minimum wages that sustain equilibrium 2

$$w_{ne} = 0 \text{ and } w_e = \frac{\underline{c}_{eew} + (i_{nw} - i_{ew})}{(1-t)}$$

which are,

$$w_{ne} = 0 \text{ and } w_e = \frac{1 + (0 - 0)}{1} = 1.$$

The individual types self-select into the appropriate education and work combinations as long the following incentive compatibility constraints hold.

$$IC^{hm}: i_{ew} + (1-t)w_e - \underline{c}_{eew} \geq i_{new} + (1-t)w_{ne} - \underline{c}_{new}$$

$$IC^{hl}: i_{ew} + (1-t)w_e - \underline{c}_{eew} \geq i_{nw}$$

$$IC^{mh}: i_{nw} \geq i_{ew} + (1-t)w_e - \tilde{c}_{eew}$$

$$IC^{ml}: i_{nw} \geq i_{new} + (1-t)w_{ne} - \tilde{c}_{new}$$

$$IC^{lh}: i_{nw} \geq i_{ew} + (1-t)w_e - \overline{c}_{eew}$$

$$IC^{lm}: i_{nw} \geq i_{new} + (1-t)w_{ne} - \overline{c}_{new}.$$

which are,

$$IC^{hm}: 0 + (1)1 - 1 \geq 0 + (1)0 - 0.5\ or\ 0 \geq -0.5$$

$$IC^{hl}: 0 + (1)1 - 1 \geq 0 \; or \; 0 \geq 0$$

$$IC^{mh}: 0 \geq 0 + (1)1 - 1.8 \; or \; 0 \geq -0.8$$

$$IC^{ml}: 0 \geq 0 + (1)0 - 1.1 \; or \; 0 \geq -1.1$$

$$IC^{lh}: 0 \geq 0 + (1)1 - 2.05 \; or \; 0 \geq -1.05$$

$$IC^{lm}: 0 \geq 0 + (1)0 - 1.25 \; or \; 0 \geq -1.25.$$

Therefore, the high-type chooses to get and education and work at the educated wage and the medium and low-type chooses to not work. Individuals enjoy utility

$$i_{ew} + (1-t)w_e - \underline{c}_{eew} + i_{nw} + i_{nw}$$

which is,

$$0 + (1)1 - 1 + 0 + 0 = 0.$$

The government budget constraint is

$$BC: t(n_h w_e + n_m w_{ne}) + T\big(n_h(\overline{r}_e - w_e)\big) \geq n_h i_{ew} + n_m i_{nw} + n_l i_{nw}$$

which is,

$$BC: 0(1+0) + 0(2.6-1) \geq 0 + 0 + 0 \;\; or \; 0 \geq 0$$

Firm profits are

$$\pi^2 = (1-T)[n_h(\overline{r}_e - w_e)]$$

which is

$$\pi^2 = (1)(2.6-1) = 1.6.$$

All other possible profits given the tax rates $t = 0$, $T = 0$, and transfers $i_{ew} = 0$, $i_{new} = 0$, and $i_{nw} = 0$ given the firm chooses the minimum wages that sustains each equilibrium are

$$\pi^1 = (1)[(2.6-1.6) + (1.5-1.1)] = 1.4$$

$$\pi^3 = (1)[(2.6-1.8) + (2-1.8)] = 1$$

$$\pi^4 = (1)[(2.6-2.05) + (2-2.05) + (1.5-2.05)] = -0.05$$

$$\pi^5 = (1)[(2.6-1.95) + (2-1.95) + (1-1.25)] = 0.45$$

$$\pi^6 = (1)[(2.6-1.75) + (1.5-1.25) + (1-1.25)] = 0.85$$

$$\pi^7 = (1)[(2-1.25) + (1.5-1.25) + (1-1.25)] = 0.75$$

$$\pi^8 = (1)[(2 - 1.1) + (1.5 - 1.1)] = 1.3$$

$$\pi^9 = (1)[(2 - 0.5)] = 1.5$$

$$\pi^{10} = 0$$

Therefore $\pi^2 > \pi^k \ for \ k = 1, 3, \ldots., 10$.

Thus, in the absence of tax policy when the tax rates $t = 0$, $T = 0$, and transfers $i_{ew} = 0$, $i_{new} = 0$, and $i_{nw} = 0$ the firm maximizes profit by choosing wages that sustain partial pooling equilibrium 2.

Proof of Proposition 4.2:

Choose tax rates $t = 0.30$, $T = 0.345$, and transfer $i_{nw} = 0$ to determine i_{new} per proposition 3.2.13

$$for\ i_{new} \leq i_{nw} + \underline{c}_{new}$$

$$i_{new} > \frac{n_h}{n_m}\left(\tilde{c}_{new} - \underline{c}_{new}\right) - (\tilde{r}_{ne}(1-t) - \tilde{c}_{new}) + i_{nw}$$

which is,

$$i_{new} > (1.1 - 0.5) - (1.5(0.7) - 1.1) + 0 = 0.65$$

but $i_{new} \leq i_{nw} + \underline{c}_{new}$ does not hold since 0.65> 0.5 therefore

$$for\ i_{nw} + \underline{c}_{new} < \ i_{new} < i_{nw} + \tilde{c}_{new}$$

$$i_{new} > \frac{n_h}{n_m + n_h}(\tilde{c}_{new}) - \frac{n_m}{n_m + n_h}(\tilde{r}_{ne}(1-t) - \tilde{c}_{new}) + i_{nw}$$

which is,

$$i_{new} > \frac{1}{2}1.1 - \frac{1}{2}(1.5(0.7) - 1.1) + 0 = 0.575$$

Therefore choose $i_{new} = 0.59$. Now given $i_{new} = 0.59$ to determine i_{ew} per proposition 3.2.13

$$for\ i_{ew} \leq i_{nw} + \underline{c}_{eew}$$

$$i_{ew} \geq \left(\tilde{c}_{new} - \underline{c}_{new}\right) + \left(\overline{r}_{ne}(1-t) - \underline{c}_{new}\right) - \left(\overline{r}_e(1-t) - \underline{c}_{eew}\right) - \frac{n_m}{n_h}[(\tilde{r}_{ne}(1-t) - \tilde{c}_{new}) + (i_{new} - i_{nw})] + i_{new}$$

which is,

$$i_{ew} \geq (1.1 - 0.5) + (2(0.7) - 0.5) - (2.6(0.7) - 1) - (1.5(0.7) - 1.1) - (1.04 - 0) + 1.04 = 0.73$$

and

$$i_{ew} \geq \left(\overline{r}_{ne}(1-t) - \underline{c}_{new}\right) - \left(\overline{r}_{e}(1-t) - \underline{c}_{eew}\right) + i_{new}$$

which is,

$$i_{ew} \geq (2(0.7) - 0.5) - (2.6(0.7) - 1) + 0.59 = 0.67$$

and

$$i_{ew} \geq \frac{n_l}{n_h}\left[\left(\underline{r}_{ne}(1-t) - \overline{c}_{new}\right) + (i_{new} - i_{nw})\right] - \frac{n_m}{n_h}[\overline{c}_{new} - \tilde{c}_{new}] + \left(\overline{r}_{ne}(1-t) - \underline{c}_{new}\right)$$
$$- \left(\overline{r}_{e}(1-t) - \underline{c}_{eew}\right) - (\overline{c}_{new} - \tilde{c}_{new}) + i_{new}]$$

which is,

$$i_{ew} \geq (1(0.7) - 1.25) + (0.59 - 0) - (1.25 - 1.1) - (2(0.7) - 0.5) - (2.6(0.7) - 1) - (1.25 - 1.1) + 0.59$$
$$= -1.39$$

Therefore, choose $i_{ew} = 0.73$ which is less than $i_{nw} + \underline{c}_{eew} = 1$ therefore the condition on the transfers holds.

Given the tax rates $t = 0.30$, $T = 0.345$, and transfers $i_{ew} = 0.73$, $i_{new} = 0.59$, and $i_{nw} = 0$ the firm chooses the minimum wages that sustain equilibrium 1

$$w_{ne} = \frac{\tilde{c}_{new} + (i_{nw} - i_{new})}{1-t} \text{ and } w_e = \frac{\underline{c}_{eew} - \underline{c}_{new} + (i_{new} - i_{ew})}{1-t} + w_{ne}$$

which are,

$$w_{ne} = \frac{1.1 + (0 - 0.59)}{0.7} \cong 0.729 \text{ and } w_e = \frac{1 - 0.5 + (0.59 - 0.73)}{0.7} + 0.729 \cong 1.245.$$

The individual types self-select into the appropriate education and work combinations as long the following incentive compatibility constraints hold.

$$IC^{hm}: i_{ew} + (1-t)w_e - \underline{c}_{eew} \geq i_{new} + (1-t)w_{ne} - \underline{c}_{new}$$

$$IC^{hl}: i_{ew} + (1-t)w_e - \underline{c}_{eew} \geq i_{nw}$$

$$IC^{mh}: i_{new} + (1-t)w_{ne} - \tilde{c}_{new} \geq i_{ew} + (1-t)w_e - \tilde{c}_{eew}$$

$$IC^{ml}: i_{new} + (1-t)w_{ne} - \tilde{c}_{new} \geq i_{nw}$$

$$IC^{lh}: i_{nw} \geq i_{ew} + (1-t)w_e - \overline{c}_{eew}$$

$$IC^{lm}: i_{nw} \geq i_{new} + (1-t)w_{ne} - \overline{c}_{new}.$$

which are,

$$IC^{hm}: 0.73 + (0.7)1.245 - 1 \geq 0.59 + (0.7)0.729 - 0.5 \; or \; 0.6015 \geq 0.44721$$

$$IC^{hl}: 0.73 + (0.7)1.245 - 1 \geq 0 \; or \; 0.6015 \geq 0$$

$$IC^{mh}: 0.59 + (0.7)0.729 - 1.1 \geq 0.73 + (0.7)1.245 - 1.8 \; or \; 0.0003 \geq -0.1985$$

$$IC^{ml}: 0.59 + (0.7)0.729 - 1.1 \geq 0 \; or \; 0.003 \geq 0$$

$$IC^{lh}: 0 \geq 0.73 + (0.7)1.245 - 2.05 \; or \; 0 \geq -0.4485$$

$$IC^{lm}: 0 \geq 0.59 + (0.7)0.729 - 1.25 \; or \; 0 \geq -0.1497.$$

Therefore, the high-type chooses to get and education and work at the educated wage, the medium type chooses to not get an education and work at the uneducated wage, and the low-type chooses to not work. Individuals enjoy utility

$$i_{ew} + (1-t)w_e - \underline{c}_{eew} + i_{new} + (1-t)w_{ne} - \tilde{c}_{new} + i_{nw}$$

which is,

$$0.73 + (0.7)1.245 - 1 + 0.59 + (0.7)0.729 - 1.1 + 0 = 0.6015 + 0.0003 = 0.6018.$$

The government budget constraint is

$$BC: t(n_h w_e + n_m w_{ne}) + T\big(n_h(\overline{r}_e - w_e) + n_m(\tilde{r}_{ne} - w_{ne})\big) \geq n_h i_{ew} + n_m i_{new} + n_l i_{nw}$$

which is,

$$BC: 0.3(1.245 + 0.729) + 0.345\big((2.6 - 1.245) + (1.5 - 0.729)\big) \geq 0.73 + 0.59 + 0 \; or \; 1.32567 \geq 1.32$$

Firm profits are

$$\pi^1 = (1-T)[n_h(\overline{r}_e - w_e) + n_m(\tilde{r}_{ne} - w_{ne})]$$

which is

$$\pi^1 = (0.655)[(2.6 - 1.245) + (1.5 - 0.729)] = 1.39253.$$

All other possible profits given the tax rates $t = 0.30$, $T = 0.345$, and transfers $i_{ew} = 0.73$, $i_{new} = 0.59$, and $i_{nw} = 0$ given the firm chooses the minimum wages that sustains each equilibrium are

$$\pi^2 = (0.655)[2.6 - 0.515] = 1.365675$$

$$\pi^3 = (0.655)[(2.6 - 1.529) + (2 - 1.529)] = 1.01$$

$$\pi^4 = (0.655)[(2.6 - 1.886) + (2 - 1.886) + (1.5 - 1.886)] = 0.28951$$

$$\pi^5 = (0.655)[(2.6 - 1.743) + (2 - 1.743) + (1 - 0.943)] = 0.767005$$

$$\pi^6 = (0.655)[(2.6 - 1.458) + (1.5 - 0.943) + (1 - 0.943)] = 1.15018$$

$$\pi^7 = (0.655)[(2 - 0.943) + (1.5 - 0.943) + (1 - 0.943)] = 1.094505$$

$$\pi^8 = (0.655)[(2 - 0.729) + (1.5 - 0.729)] = 1.33751$$

$$\pi^9 = (0.655)[(2 - 0)] = 1.31$$

$$\pi^{10} = 0$$

Therefore $\pi^1 > \pi^k \; for \; k = 2, 3, \ldots, 10$.

Thus, the tax rates $t = 0.30$, $T = 0.345$, and transfers $i_{ew} = 0.73$, $i_{new} = 0.59$, and $i_{nw} = 0$ solves the government's maximization program.

Proof of Proposition 4.3:

Choose tax rates $t = 0.30$, $T = 0.6$, and transfer $i_{nw} = 0$ to determine i_{new} per proposition 3.2.13

$$for \; i_{new} \leq i_{nw} + \underline{c}_{new}$$

$$i_{new} < \frac{n_m + n_h}{n_l}(\overline{c}_{new} - \tilde{c}_{new}) - \left(\underline{r}_{ne}(1-t) - \overline{c}_{new}\right) + i_{nw}$$

which is,

$$i_{new} > \frac{2}{1}(1.25 - 1.1) - (1(0.7) - 1.25) + 0 = 0.85$$

Therefore choose $i_{new} = 0.84$. Now given $i_{new} = 0.84$ to determine i_{ew} per proposition 3.2.13

$$for \; i_{new} \leq i_{nw} + \tilde{c}_{new}$$

$$i_{ew} < (\tilde{r}_{ne}(1-t) - \tilde{c}_{new}) - (\tilde{r}_e(1-t) - \tilde{c}_{eew}) - \frac{n_h}{n_m}\left[\left(\underline{c}_{eew} - \underline{c}_{new}\right) - (\tilde{c}_{eew} - \tilde{c}_{new})\right] + i_{new}$$

which is,

$$i_{ew} < (1.5(0.7) - 1.1) + (2(0.7) - 1.8) - (1 - 0.5) - (1.8 - 1.1) + 0.84 = 1.39$$

and

Therefore, choose $i_{ew} = 1.38$.

Given the tax rates $t = 0.30,\ T = 0.6$, and transfers $i_{ew} = 1.38$, $i_{new} = 0.84$, and $i_{nw} = 0$ the firm chooses the minimum wages that sustain equilibrium 1

$$w_{ne} = \frac{\tilde{c}_{new} + (i_{nw} - i_{new})}{1 - t} \text{ and } w_e = \frac{\underline{c}_{eew} - \underline{c}_{new} + (i_{new} - i_{ew})}{1 - t} + w_{ne}$$

which are,

$$w_{ne} = \frac{1.1 + (0 - 0.84)}{0.7} \cong 0.372 \text{ and } w_e = \frac{1 - 0.5 + (0.84 - 1.38)}{0.7} + 0.372 \cong 0.315.$$

The individual types self-select into the appropriate education and work combinations as long the following incentive compatibility constraints hold.

$$IC^{hm}: i_{ew} + (1 - t)w_e - \underline{c}_{eew} \geq i_{new} + (1 - t)w_{ne} - \underline{c}_{new}$$

$$IC^{hl}: i_{ew} + (1 - t)w_e - \underline{c}_{eew} \geq i_{nw}$$

$$IC^{mh}: i_{new} + (1 - t)w_{ne} - \tilde{c}_{new} \geq i_{ew} + (1 - t)w_e - \tilde{c}_{eew}$$

$$IC^{ml}: i_{new} + (1 - t)w_{ne} - \tilde{c}_{new} \geq i_{nw}$$

$$IC^{lh}: i_{nw} \geq i_{ew} + (1 - t)w_e - \overline{c}_{eew}$$

$$IC^{lm}: i_{nw} \geq i_{new} + (1 - t)w_{ne} - \overline{c}_{new}.$$

which are,

$$IC^{hm}: 1.38 + (0.7)0.315 - 1 \geq 0.84 + (0.7)0.372 - 0.5 \ or \ 0.6005 \geq 0.6004$$

$$IC^{hl}: 1.38 + (0.7)0.315 - 1 \geq 0 \ or \ 0.6005 \geq 0$$

$$IC^{mh}: 0.84 + (0.7)0.372 - 1.1 \geq 1.38 + (0.7)0.315 - 1.8 \ or \ 0.0004 \geq -0.1995$$

$$IC^{ml}: 0.84 + (0.7)0.372 - 1.1 \geq 0 \ or \ 0.004 \geq 0$$

$$IC^{lh}: 0 \geq 1.38 + (0.7)0.315 - 2.05 \ or \ 0 \geq -0.4495$$

$$IC^{lm}: 0 \geq 0.84 + (0.7)0.372 - 1.25 \ or \ 0 \geq -0.1496.$$

Therefore, the high-type chooses to get and education and work at the educated wage, the medium type chooses to not get an education and work at the uneducated wage, and the low-type chooses to not work. Individuals enjoy utility

$$i_{ew} + (1 - t)w_e - \underline{c}_{eew} + i_{new} + (1 - t)w_{ne} - \tilde{c}_{new} + i_{nw}$$

which is,

$$1.38+(0.7)0.315-1+0.85+(0.7)0.372-1.1+0=0.6005+0.0004=0.6019.$$

The government budget constraint is

$$BC\colon t(n_h w_e+n_m w_{ne})+T\big(n_h(\overline{r}_e-w_e)+n_m(\tilde{r}_{ne}-w_{ne})\big)\geq n_h i_{ew}+n_m i_{new}+n_l i_{nw}$$

which is,

$$BC\colon 0.3(0.315+0.372)+0.6\big((2.6-0.315)+(1.5-0.372)\big)\geq 1.38+0.85+0 \;\; or \; 2.2539\geq 2.23$$

Firm profits are

$$\pi^1=(1-T)[n_h(\overline{r}_e-w_e)+n_m(\tilde{r}_{ne}-w_{ne})]$$

which is

$$\pi^1=(0.4)[(2.6-0.315)+(1.5-0.372)]=1.3652.$$

All other possible profits given the tax rates $t=0.30$, $T=0.6$, and transfers $i_{ew}=1.38$, $i_{new}=0.85$, and $i_{nw}=0$ given the firm chooses the minimum wages that sustain each equilibrium are

$$\pi^2=(0.4)[2.6-(-0.043)]=1.0572$$

$$\pi^3=(0.4)[(2.6-0.6)+(2-0.6)]=1.36$$

$$\pi^4=(0.4)[(2.6-0.957)+(2-0.957)+(1.5-0.957)]=1.2916$$

$$\pi^5=(0.4)[(2.6-0.814)+(2-0.814)+(1-0.586)]=1.3544$$

$$\pi^6=(0.4)[(2.6-0.529)+(1.5-0.586)+(1-0.586)]=1.3596$$

$$\pi^7=(0.4)[(2-0.586)+(1.5-0.586)+(1-0.586)]=1.0968$$

$$\pi^8=(0.4)[(2-0.371)+(1.5-0.371)]=1.1032$$

$$\pi^9=(0.4)[(2-0.057)]=0.772$$

$$\pi^{10}=0$$

Therefore $\pi^1>\pi^k \; for \; k=2,3,\dots.,10$.

Thus, the tax rates $t=0.30$, $T=0.59$, and transfers $i_{ew}=1.38$, $i_{new}=0.84$, and $i_{nw}=0$ solves the government's maximization program.

Proof of Proposition 4.4:

Choose tax rates $t = 0.30$, $T = 0.98$, and transfer $i_{nw} = .45$ to determine i_{new} per proposition 3.2.13

$$for\ i_{new} \leq i_{nw} + \underline{c}_{new}$$

$$i_{new} > \frac{n_h}{n_m}\left(\tilde{c}_{new} - \underline{c}_{new}\right) - (\tilde{r}_{ne}(1-t) - \tilde{c}_{new}) + i_{nw}$$

which is,

$$i_{new} > (1.1 - 0.5) - (1.5(0.7) - 1.1) + 0.45 = 1.10$$

but $i_{new} \leq i_{nw} + \underline{c}_{new}$ does not hold since $1.10 > 0.45 + 0.5$ therefore

$$for\ i_{nw} + \underline{c}_{new} < i_{new} < i_{nw} + \tilde{c}_{new}$$

$$i_{new} > \frac{n_h}{n_m + n_h}(\tilde{c}_{new}) - \frac{n_m}{n_m + n_h}(\tilde{r}_{ne}(1-t) - \tilde{c}_{new}) + i_{nw}$$

which is,

$$i_{new} > \frac{1}{2}1.1 - \frac{1}{2}(1.5(0.7) - 1.1) + 0.45 = 1.025$$

Therefore choose $i_{new} = 1.04$. Now given $i_{new} = 1.04$ to determine i_{ew} per proposition 3.2.13

$$for\ i_{ew} \leq i_{nw} + \underline{c}_{eew}$$

$$i_{ew} \geq \left(\tilde{c}_{new} - \underline{c}_{new}\right) + \left(\overline{r}_{ne}(1-t) - \underline{c}_{new}\right) - \left(\overline{r}_{e}(1-t) - \underline{c}_{eew}\right) - \frac{n_m}{n_h}[(\tilde{r}_{ne}(1-t) - \tilde{c}_{new}) + (i_{new} - i_{nw})] + i_{new}$$

which is,

$$i_{ew} \geq (1.1 - 0.5) + (2(0.7) - 0.5) - (2.6(0.7) - 1) - (1.5(0.7) - 1.1) - (1.04 - 0.45) + 1.04 = 1.18$$

and

$$i_{ew} \geq \left(\overline{r}_{ne}(1-t) - \underline{c}_{new}\right) - \left(\overline{r}_{e}(1-t) - \underline{c}_{eew}\right) + i_{new}$$

which is,

$$i_{ew} \geq (2(0.7) - 0.5) - (2.6(0.7) - 1) + 1.04 = 1.12$$

and

$$i_{ew} \geq \frac{n_l}{n_h}\left[\left(\underline{r}_{ne}(1-t) - \overline{c}_{new}\right) + (i_{new} - i_{nw})\right] - \frac{n_m}{n_h}[\overline{c}_{new} - \tilde{c}_{new}] + \left(\overline{r}_{ne}(1-t) - \underline{c}_{new}\right)$$

$$- \left(\overline{r}_e(1-t) - \underline{c}_{eew}\right) - (\overline{c}_{new} - \tilde{c}_{new}) + i_{new}]$$

which is,

$$i_{ew} \geq (1(0.7) - 1.25) + (1.04 - 0.45) - (1.25 - 1.1) - (2(0.7) - 0.5) - (2.6(0.7) - 1) - (1.25 - 1.1) + 1.04$$

$$= -0.94.$$

Therefore, choose $i_{ew} = 1.18$ which is less than $i_{nw} + \underline{c}_{eew} = 1.45$ therefore the condition on the transfers holds.

Given the tax rates $t = 0.30$, $T = 0.98$, and transfers $i_{ew} = 1.18$, $i_{new} = 1.04$, and $i_{nw} = .45$ the firm chooses the minimum wages that sustains equilibrium 1

$$w_{ne} = \frac{\tilde{c}_{new} + (i_{nw} - i_{new})}{1-t} \text{ and } w_e = \frac{\underline{c}_{eew} - \underline{c}_{new} + (i_{new} - i_{ew})}{1-t} + w_{ne}$$

which are,

$$w_{ne} = \frac{1.1 + (0.45 - 1.04)}{0.7} \cong 0.729 \text{ and } w_e = \frac{1 - 0.5 + (1.04 - 1.18)}{0.7} + 0.729 \cong 1.245.$$

The individual types self-select into the appropriate education and work combinations as long the following incentive compatibility constraints hold.

$$IC^{hm}: i_{ew} + (1-t)w_e - \underline{c}_{eew} \geq i_{new} + (1-t)w_{ne} - \underline{c}_{new}$$

$$IC^{hl}: i_{ew} + (1-t)w_e - \underline{c}_{eew} \geq i_{nw}$$

$$IC^{mh}: i_{new} + (1-t)w_{ne} - \tilde{c}_{new} \geq i_{ew} + (1-t)w_e - \tilde{c}_{eew}$$

$$IC^{ml}: i_{new} + (1-t)w_{ne} - \tilde{c}_{new} \geq i_{nw}$$

$$IC^{lh}: i_{nw} \geq i_{ew} + (1-t)w_e - \overline{c}_{eew}$$

$$IC^{lm}: i_{nw} \geq i_{new} + (1-t)w_{ne} - \overline{c}_{new}.$$

which are,

$$IC^{hm}: 1.18 + (0.7)1.245 - 1 \geq 1.04 + (0.7)0.729 - 0.5 \text{ } or \text{ } 1.0515 \geq 0.89721$$

$$IC^{hl}: 1.18 + (0.7)1.245 - 1 \geq 0.45 \text{ } or \text{ } 1.0515 \geq 0.45$$

$$IC^{mh}: 1.04 + (0.7)0.729 - 1.1 \geq 1.18 + (0.7)1.245 - 1.8 \text{ } or \text{ } 0.4503 \geq 0.2515$$

$$IC^{ml}: 1.04 + (0.7)0.729 - 1.1 \geq 0.45 \; or \; 0.4503 \geq 0.45$$

$$IC^{lh}: 0.45 \geq 1.18 + (0.7)1.245 - 2.05 \; or \; 0.45 \geq 0.0015$$

$$IC^{lm}: 0.45 \geq 1.04 + (0.7)0.729 - 1.25 \; or \; 0.45 \geq 0.3003.$$

Therefore, the high-type chooses to get and education and work at the educated wage, the medium type chooses to not get an education and work at the uneducated wage, and the low-type chooses to not work. Individuals enjoy utility

$$i_{ew} + (1-t)w_e - \underline{c}_{eew} + i_{new} + (1-t)w_{ne} - \tilde{c}_{new} + i_{nw}$$

which is,

$$1.18 + (0.7)1.245 - 1 + 1.04 + (0.7)0.729 - 1.1 + 0.45 = 1.9518.$$

The utility enjoyed by individuals is greater than any possible total surplus in any other equilibrium. The government budget constraint is

$$BC: t(n_h w_e + n_m w_{ne}) + T\big(n_h(\overline{r}_e - w_e) + n_m(\tilde{r}_{ne} - w_{ne})\big) \geq n_h i_{ew} + n_m i_{new} + n_l i_{nw}$$

which is,

$$BC: 0.3(1.245 + 0.729) + 0.98\big((2.6 - 1.245) + (1.5 - 0.729)\big) \geq 1.18 + 1.04 + 0.45 \; or \; 2.67568 \geq 2.67$$

Firm profits are

$$\pi^1 = (1-T)[n_h(\overline{r}_e - w_e) + n_m(\tilde{r}_{ne} - w_{ne})]$$

which is

$$\pi^1 = (0.02)[(2.6 - 1.245) + (1.5 - 0.729)] = 0.04252.$$

All other possible profits given the tax rates $t = 0.30$, $T = 0.98$, and transfers $i_{ew} = 1.18$, $i_{new} = 1.04$, and $i_{nw} = 0.45$ given the firm chooses the minimum wages that sustains each equilibrium are

$$\pi^2 = (0.02)[2.6 - 0.515] = 0.0417$$

$$\pi^3 = (0.02)[(2.6 - 1.529) + (2 - 1.529)] = 0.03084$$

$$\pi^4 = (0.02)[(2.6 - 1.886) + (2 - 1.886) + (1.5 - 1.886)] = 0.00884$$

$$\pi^5 = (0.02)[(2.6 - 1.743) + (2 - 1.743) + (1 - 0.943)] = 0.02343$$

$$\pi^6 = (0.02)[(2.6 - 1.458) + (1.5 - 0.943) + (1 - 0.943)] = 0.03512$$

$$\pi^7 = (0.02)[(2 - 0.943) + (1.5 - 0.943) + (1 - 0.943)] = 0.03342$$

$$\pi^8 = (0.02)[(2 - 0.729) + (1.5 - 0.729)] = 0.04084$$

$$\pi^9 = (0.02)[(2 - 0)] = 0.04$$

$$\pi^{10} = 0$$

Therefore $\pi^1 > \pi^k \; for \; k = 2, 3, \ldots., 10$.

Thus, the tax rates $t = 0.30$, $T = 0.98$, and transfers $i_{ew} = 1.18$, $i_{new} = 1.04$, and $i_{nw} = .45$ solves the government's maximization program.