MgtOp 215 **Chapter 9** **Dr. Ahn**

A (statistical) **hypothesis** is a claim or statement (about the population parameter). Examples include

Dr. Ahn is a wonderful person.
This is a fair coin, that is, π=0.5.

Given a hypothesis there exists a competing hypothesis, whether stated explicitly or not. In the above examples competing hypotheses are

Dr. Ahn is not a wonderful person.
This is not a fair coin.

Of a hypothesis and its competing hypothesis, one of them will be called the **null hypothesis**, denoted by H_0, and the other one will be called the **alternative hypothesis**, denoted by H_1. It is very important to identify the null and the alternative hypothesis.

The null hypothesis represents the status quo and will be given the benefit of the doubt. Therefore, it is like a defendant in a court trial. The alternative hypothesis is an assertion that requires substantial and convincing evidence and often called a researcher's hypothesis. In the following four examples, state the null and the alternative hypothesis.

Example 1. When an athlete is tested for performance-enhancing drugs like steroids, we test two hypothese: “No illegal steroids use” vs. “Illegal steroids use.”

Example 2. Researchers in A Corp. claims that their new product can increase A Corp.'s market share.

Example 3. A better business bureau is investigating a meat company that sells ground beef in 5-pound package. There have been complaints that the company is short-weighting its customers.

Example 4. During the election season, the incumbent is running agaist the challenger.

MSL Homework: 9.12

The null and the alternative hypothesis take one of the following forms.

$$H_0: \theta = \theta_0 \qquad H_0: \theta \geq \theta_0 \qquad H_0: \theta \leq \theta_0$$
$$H_1: \theta \neq \theta_0 \qquad H_1: \theta < \theta_0 \qquad H_1: \theta > \theta_0$$

The hypothesis $\theta = \theta_0$ is called a **simple hypothesis** and the others are called **composite hypotheses**. The first set of the null and alternative hypothesis is called a **two-sided test** because the values of the parameter claimed under the alternative hypothesis are on both sides of θ_0, and the rejection region is usually in both tails. The last two sets of the null and alternative hypothesis are called **one sided tests** because the values of the parameter claimed under the alternative hypothesis are on only one side of θ_0, and the rejection region is usually in one tail whose side is determined by the alternative hypothesis.

The conclusion drawn from a hypothesis test is either

rejecting the null hypothesis (hence accepting the alternative hypothesis)

or

not rejecting the null hypothesis.

When the conclusion is not rejecting the null hypothesis, avoid using "accepting the null hypothesis." The null hypothesis is rejected if the evidence is beyond the reasonable doubt, and thus rejection of the null hypothesis is a strong conclusion, while non-rejection of the null hypothesis is a weak conclusion.

We can summarize the decision of a hypothesis test in the following table.

Decision	True State H_0	H_1
Do not reject H_0	correct	**Type II error**
Reject H_0	**Type I error**	correct

Our main concern in the hypothesis test is committing one of the errors
The probability of rejecting H_0 when H_0 is true is called the **Type I error** probability and denoted by α. Probability of not rejecting H_0 when is H_1 is true is called the **Type II error** probability and denoted by β.

The ideal decision rule should be such that both α and β are all as small as possible. However, in reality we cannot make both smaller at the same time: if we try to make α smaller, then β gets larger and vice versa. For a fixed value of α, in order to make β smaller we must increase the sample size *n*. Therefore for a hypothesis we traditionally control α. by deciding how much one is willing to tolerate committing the Type I error depending on the seriousness of committing the Type I error.

The **level of significance** of a hypothesis tests is the maximum Type I error probability allowed, and is usually denoted by α also,

Example 5. When evaluating a loan applicant, a financial officer is faced with the problem of granting loans to people who are good risks and denying loans to people who appears to be poor risks. In fact the financial officer is testing the null hypothesis

H_0

against the alternative hypothesis

H_1

a) When does the officer commit the Type I error?
b) When does the officer commit the Type II error?

Should the significance level α be smaller or larger in the following instances?

1) Lending money is tight, interest rates are high, and loan applicants are numerous.
2) Lending money in plentiful, interest rates are moderate, and there is intense competition for loan applicants.

MSL Homework: 9.9, 9.11, 9.13.
HC Homework: 9.10 and the following problem.

Problem: A drug company claims that its newly developed drug is more effective than the existing drugs for treating a certain disease.

a. State the null and the alternative hypothesis.
b. Can these hypotheses be considered as statistical hypotheses?
c. When is the Type I error committed? Also, when is the Type II error committed?
d. For which case should the level of significance be smaller?
   1. The disease is the AIDS.
   2. The disease is a common cold.

The **power** of a test is the probability of (correctly) rejecting the null hypothesis when the alternatively hypothesis is true, and it s 1-β.

A **test statistic** is a random variable whose value is used to determine whether we reject the null hypothesis.

The **decision rule** specifies the set of values of the test statistic for which H_0 is rejected in favor of H_1 and the set of values for which H_0 is not rejected.

The **rejection region** of a test, also called the **critical region** consists of all values of the test statistics for which H_0 is rejected.

A **critical value** of the test statistic is the value in the critical region that separates the rejection region from the non-rejection region. (It is also called a **rejection point** in the textbook.)

Whenever the test statistic falls in the rejection region, we say that the test statistic is (**statistically**) **significant**.

Procedure for testing hypothesis

1. State H_0 and H_1.
2. Select α.
3. Determine the test statistic.
4. Determine the critical region.
5. Take a sample and compute the value of the test statistic.
6. Draw a conclusion.

The **observed significance level**, often called the ***p*-value** of the test statistic is the probability of obtaining a value of the test statistic as extreme or more extreme (into the direction of the rejection region) than the value of the test statistic that has been observed when H_0 is true. It represents the smallest level of significance at which H_0 can be rejected.

The null hypothesis is (not) rejected if and only if the p-value $\leq \alpha$ (p-value $> \alpha$). Most of computer software for statistics provide the p-value, in which case we do not need to look up tables for the critical values instead just compare the p-value and α.

Testing hypothesis about the population mean μ

$H_0: \mu = \mu_0$ $H_1: \mu \neq \mu_0$	$H_0: \mu \geq \mu_0$ $H_1: \mu < \mu_0$	$H_0: \mu \leq \mu_0$ $H_1: \mu > \mu_0$

Test statistic:

	σ_x Known	σ_x Unknown
Normal Population	$Z = \frac{\bar{X}-\mu_0}{\sigma/\sqrt{n}}$	$T = \frac{\bar{X}-\mu_0}{s/\sqrt{n}}$
Non-normal Population	$Z = \frac{\bar{X}-\mu_0}{\sigma/\sqrt{n}}$	$Z = \frac{\bar{X}-\mu_0}{s/\sqrt{n}}$

Reject H_0, if

$\lvert z_{stat}\rvert \geq z_{\alpha/2}$ $\lvert t_{stat}\rvert \geq t_{\alpha/2,n-1}$	$z_{stat} \leq -z_\alpha$ $t_{stat} \leq -t_{\alpha,n-1}$	$z_{stat} \geq z_\alpha$ $t_{stat} \geq t_{\alpha,n-1}$

p-value

$P(\lvert Z\rvert \geq \lvert z_{stat}\rvert)$ $P(\lvert T\rvert \geq \lvert t_{stat}\rvert)$	$P(Z \leq z_{stat})$ $P(T \leq t_{stat})$	$P(Z \geq z_{stat})$ $P(T \geq t_{stat})$

Example 6. A standard manufacturing process has produced millions of TV tubes, with a mean life 1200 hours and a standard deviation 300 hours. Assume the life time is approximately normally distributed. A new process is recommended by the engineering department as a better process. A sample of 100 tubes from the new process shows an average lifetime of 1230 hours. Test the claim by the engineering department at α=0.01. Obtain the observed significance level.

PHStat2>One-Sample Tests > Z Test for the Mean, sigma known…
Then: enter 1200 for the Null Hypothesis, 0.01 for the Level of Significance, and 300 for the Population Standard Deviation, and 100 for the Sample Size and 1230 for the Sample Mean, then click Upper-Tail Test.. Then you will get the following

Z Test of Hypothesis for the Mean

Data	
Null Hypothesis μ=	**1200**
Level of Significance	**0.01**
Population Standard Deviation	**300**
Sample Size	**100**
Sample Mean	**1230**

Intermediate Calculations	
Standard Error of the Mean	30
Z Test Statistic	1

Upper-Tail Test	
Upper Critical Value	2.326341928
p-Value	0.15865526
Do not reject the null hypothesis	

Example 7. It is claimed that the mean salary of accounting graduates from MBA degree programs is $64,000. A random sample of size 100 is taken and it is found that the sample mean is $63,775 and the sample standard deviation is $1000. Test the claim at α=0.05. Find the *p*-value.

PHStat2>One-Sample Tests > Z Test for the Mean, sigma known…
(Although in this case we use the sample standard deviation, *s*, in place of the population standard deviation, σ, we "pretend" that we know the standard deviation.
The is because we need to compute the *z*-statistic.)
Then, enter 64000 for the Null Hypothesis, 0.05 for the Level of Significance, and 63775 for the Population Standard Deviation, and 100 for the Sample Size and 1230 for the Sample Mean, then click Two-Tailed Test.. Then you will get the following

Z Test of Hypothesis for the Mean

Data	
Null Hypothesis μ=	64000
Level of Significance	0.05
Population Standard Deviation	1000
Sample Size	100
Sample Mean	63775

Intermediate Calculations	
Standard Error of the Mean	100
Z Test Statistic	-2.25

Two-Tailed Test	
Lower Critical Value	-1.959961082
Upper Critical Value	1.959961082
p-Value	0.024448867
Reject the null hypothesis	

The relationship between the two sided test at α and the $(1-\alpha)$ 100% confidence interval: The $H_0: \theta = \theta_0$ (vs. $H_1: \theta \neq \theta_0$) is rejected if and only if θ_0 is not in the confidence interval; and the $H_0: \theta = \theta_0$ is not rejected if and only if θ_0 is in the confidence interval:

MSL Homework: 9.1, 9.3, 9.6, 9.7, 9.8
HC Homework: 9.16 (Use Excel/PHstat2).

Example 8. Based on past experience of raising Pure Rock chickens, their average weight at age five month had been 1.35 pounds and the weights are normally distributed. In an effort to increase their weight during that period, a special additive was mixed with the chicken feed. The subsequent weights of a random sample of five-month-old chickens were (in pounds):

1.41, 1.37, 1.33, 1.35, 1.30, 1.39 1.36, 1.38, 1.40, 1.39.

Test at α=0.05 if the special additive is effective. Is the p-value larger than 0.05?

First enter the data.

PHStat2>One-Sample Tests > t Test for the Mean, sigma unknown…

Then, enter 1.35 for the Null Hypothesis, 0.05 for the Level of Significance, and click Sample Statistics Unknown and enter the Data Cell Range, then click Upper-Tail Test.. Then you will get the following

t Test for Hypothesis of the Mean

Data	
Null Hypothesis μ=	**1.35**
Level of Significance	**0.05**
Sample Size	**10**
Sample Mean	**1.368**
Sample Standard Deviation	**0.033928028**

Intermediate Calculations	
Standard Error of the Mean	0.010728985
Degrees of Freedom	9
t Test Statistic	1.677698368

Upper-Tail Test	
Upper Critical Value	**1.833113856**
***p*-Value**	**0.06385876**
Do not reject the null hypothesis	

MSL Homework: 9.18, 9.19, 9.20, 9.21, 9.22, 9.23, 9.24, 9.36, 9.37, 9.38, 9.39, 9.40, 9.42, 9.44, 9.45, 9.47, 9.50

HC Homework: 9.32 (Use Excel/PHStat2), 9.51 (Use Excel/PHStat2)

In order to get p-values using MS Excel for the t-statistics f_x>Statistical>T.DIST. For example, if you need $P(T > 1.678)$ with 9 d.f, then using $P(T > 1.678) = 1 - P(T \leq 1.678) =$ $1 -$T.DIST(1.678,9,1);

if you need $P(T < -2.022)$ with 13 d.f, then T.DIST(-2.022,13,1);

if you need $P(|T| > |-1.352|)$ with 15 d.f, then using $P(|T| > |-1.352|) = 1 - P(|T| \leq 1.352) = 1 - \{P(T < 1.352) - P(T < -1.352)\} = 1 -${T.DIST(1.352,15,1) $-$ T.DIST(-1.352,15,1)};

Testing hypothesis about the population proportion π

$H_0: \pi = \pi_0$ $H_1: \pi \neq \pi_0$	$H_0: \pi \geq \pi_0$ $H_1: \pi < \pi_0$	$H_0: \pi \leq \pi_0$ $H_1: \pi > \pi_0$

Test statistic: $Z = \dfrac{p-\pi_0}{\sqrt{\frac{\pi_0(1-\pi_0)}{n}}}$, when $n \geq 25$, $n\pi_0 \geq 5$, and $n(1-\pi_0) \geq 5$

Reject H_0, if

$\lvert z_{stat}\rvert \geq z_{\alpha/2}$	$z_{stat} \leq -z_\alpha$	$z_{stat} \geq z_\alpha$

p-value

$P(\lvert Z\rvert \geq \lvert z_{stat}\rvert)$	$P(Z \leq z_{stat})$	$P(Z \geq z_{stat})$

Example 9. The marketing survey department of a large food producer believes that consumers will be indifferent to two proposed packaging designs for a new cereal. Test this claim at α =0.05 when in a random sample of 225 potential consumers of the cereal 130 preferred the package design A. Calculate the *p*-value.

PHStat2>One-Sample Tests > Z Test for the Proportion…
Then, enter 0.5 for the Null Hypothesis, 0.05 for the Level of Significance, and enter 130 for the Number of Successes and 225 for the Sample Size, then click Two-Tailed Test. Then you will get the following

Z Test of Hypothesis for the Proportion

Data	
Null Hypothesis *p*=	**0.5**
Level of Significance	**0.05**
Number of Successes	**130**
Sample Size	**225**
Intermediate Calculations	
Sample Proportion	0.577777778
Standard Error	0.033333333
Z Test Statistic	2.333333333
Two-Tailed Test	
Lower Critical Value	**-1.959961082**
Upper Critical value	**1.959961082**
***p*-Value**	**0.019630614**
Reject the null hypothesis	

MSL Homework: 9.52, 9.53, 9.55
HC Homework: 9.56 (Use Excel/PHStat)

Name: ID: