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A Turbulence Dissipation Model for Particle Laden Flow

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A dissipation transport equation for the carrier phase turbulence in particle-laden flow is derived from fundamental principles. The equation is obtained by volume averaging, which inherently includes the effects of the particle surfaces. Three additional terms appear that reveal the effect of the particles; these terms are evaluated using Stokes drag law. Two of the terms reduce to zero and only one term remains which is identified as the production of dissipation due to the particles. The dissipation equation is then applied to cases where particles generate homogeneous turbulence, and experimental data are used to evaluate the empirical coefficients. The ratio of the coefficient of the production of dissipation (due to the presence of particles) to the coefficient of the dissipation of dissipation is found to correlate well with the relative Reynolds number. *AIChE J*, 55: 1416–1425, 2009
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Introduction

A recent literature review in the area of modeling particle-laden turbulent flows reveals that there is a continuing need to develop a robust model for the effect of the disperse phase on the turbulence of the conveying phase. A short review of the modeling status was published by Curtis et al.[1] who pointed out that large industries such as the chemical, pharmaceutical, agricultural and mining industries can benefit from an understanding of particle-laden flows. Such a benefit is claimed to affect cost savings and increased productivity. Curtis et al. also identified several areas of investigation that could benefit industry; these include turbulent-gas flow interactions, particle clustering, particle shape, friction effects, and particle size distribution. Other applications where the effect of particles on fluid- phase turbulence becomes important are fluidized beds, chemical reactors, drug delivery systems, pollution control, and the food processing industries, to mention a few.

In particle-laden turbulent flow, the mechanisms responsible for turbulence modulation (e.g., the effect of particles on carrier phase turbulence) are not well understood.[2] As particles are introduced, the statistics of the continuous phase turbulence are altered. Depending on the particle characteristics such as size, density, mass loading and relative velocity difference, the level of turbulent kinetic energy (TKE) and dissipation changes relative to the corresponding un-laden flow. Gore and Crowe[3] showed that there is a relationship between turbulent modulation and the ratio of the particle size to a characteristic length of the most energetic eddy in the flow. The primary reason for this modulation is attributed to the two-way-coupled kinetic energy and dissipation between the continuous and dispersed phases.

The modulation of turbulence due to the presence of particles is attributed to the altered dissipation within the continuous phase caused by the work done at the surfaces of the particles.[4] The modulation of turbulence in particle laden flows has been demonstrated by extensive experimentation over the last two decades.[5–15] However, a turbulence model that adequately predicts these modulations over a wide range of data is still lacking.

The most robust and widely used model for turbulence in single phase flows has been the two equation k-ε model. The equation for turbulence kinetic energy is:

$$\frac{Dk_t}{Dt} = (P_k)_t - \varepsilon_t + \frac{\partial}{\partial x_i}\left[\left(\nu + \frac{(\nu_T)_t}{\sigma_k}\right)\frac{\partial k_t}{\partial x_i}\right] \tag{1}$$

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where k_t is the time averaged turbulence kinetic energy $(k_t = \overline{\overline{u_i' u_i'}}/2)$, $(P_k)_t$ is the production of turbulent energy due to mean (i.e., time averaged) velocity gradients, v is the molecular kinematic viscosity, $(v_T)_t$ is the turbulent kinematic viscosity (based on time averaging), ε_t is the time averaged dissipation and σ_k is the effective Schmidt number for turbulent diffusion. The turbulence kinetic energy is affected by diffusion, the generation due to mean flow velocity gradients and time averaged dissipation. The equation for dissipation is:

$$\frac{D\varepsilon_t}{Dt} = C_{\varepsilon 1}\frac{(P_\varepsilon)_t \varepsilon_t}{k_t} - C_{\varepsilon 2}\frac{\varepsilon_t^2}{k_t} + \frac{\partial}{\partial x_i}\left[\left(v + \frac{(v_T)_t}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon_t}{\partial x_i}\right] \tag{2}$$

where $(P_\varepsilon)_t$ is the production of dissipation due to mean (i.e., time averaged) velocity gradients, $C_{\varepsilon 1}$, $C_{\varepsilon 2}$ are empirical constants and σ_ε is the turbulent Schmidt number for diffusion of dissipation. As with the turbulence kinetic energy, the dissipation is affected by diffusion of dissipation, production of dissipation and dissipation of dissipation.

The development of the equation for dissipation is not as straight forward as for the turbulence kinetic energy equation. Bernard and Wallace[16] identify the time averaged dissipation as:

$$\varepsilon_t \equiv v\left(\overline{\overline{\frac{\partial u_i'}{\partial x_j}\frac{\partial u_i'}{\partial x_j}}}\right) \tag{3}$$

They show the development of an equation for dissipation by taking the gradient of the momentum equation, multiplying it by twice the kinematic viscosity and by the gradient of the fluctuating velocity and then time averaging the entire equation, which is mathematically represented by:

$$\overline{\overline{\left(\frac{\partial}{\partial x_j}[NS_i]\right) * 2v\left(\frac{\partial u_i'}{\partial x_j}\right)}}$$

The instantaneous velocity, pressure, and shear stress are decomposed into the sum of a time averaged and a fluctuating property to obtain:

$$\begin{aligned}\frac{D\varepsilon_t}{Dt} = & -2v\frac{\partial \overline{\overline{u_k}}}{\partial x_j}\overline{\overline{\left[\frac{\partial u_i'}{\partial x_j}\frac{\partial u_i'}{\partial x_k}\right]}} - (\varepsilon_{ik})_t\frac{\partial \overline{\overline{u_i}}}{\partial x_k} - 2v\overline{\overline{\left[u_k'\frac{\partial u_i'}{\partial x_j}\right]}}\frac{\partial^2 \overline{\overline{u_i}}}{\partial x_j \partial x_k} \\ & -2v\overline{\overline{\left[\frac{\partial u_i'}{\partial x_j}\frac{\partial \delta u_k'}{\partial x_j}\frac{\partial u_i'}{\partial x_k}\right]}} - v\frac{\partial}{\partial x_k}\overline{\overline{\left[u_k'\frac{\partial u_i'}{\partial x_j}\frac{\partial u_i'}{\partial x_j}\right]}} - 2\frac{v}{\rho}\frac{\partial}{\partial x_i}\overline{\overline{\left[\frac{\partial u_i'}{\partial x_j}\frac{\partial P'}{\partial x_j}\right]}} \\ & + v\frac{\partial^2 \varepsilon_t}{\partial x_k^2} - 2v^2\overline{\overline{\left(\frac{\partial^2 u_i'}{\partial x_j \partial x_k}\right)^2}}\end{aligned} \tag{4}$$

where $(\varepsilon_{ik})_t$ is the dissipation tensor. A discussion of each of the terms is presented in Bernard and Wallace.[16] They make arguments for grouping terms together and modeling of other terms to yield the equation for dissipation, namely Eq. 2.

A typical approach to obtain two-equation models for turbulence energy and dissipation in dispersed phase flows is to begin by adding a source term to the single-phase momentum equation to account for the surface effects, namely,

$$\frac{\partial}{\partial t}(\rho u_i) + \frac{\partial}{\partial x_j}(\rho u_j u_i) = -\frac{\partial P}{\partial x_i} + \mu\frac{\partial}{\partial x_j}\left(\frac{\partial u_i}{\partial x_j}\right) + \frac{\alpha_d \rho_d}{\tau_p}f(v_i - u_i) \tag{5}$$

where u_i is the instantaneous carrier phase velocity, v_i is the instantaneous dispersed phase velocity, α_d and ρ_d are the volume fraction and material density of the dispersed phase, respectively, τ_p is the particle response time, and f is the drag factor. The derivations then proceed using the same Reynolds averaging procedures employed for single phase incompressible flows. This additional term is the drag force per unit volume on the continuous phase. The concept of adding a point force to represent the effect of a cloud of particles is open to argument.

A test to assess the viability of a turbulence model for disperse phase flows is to apply the model to the simplest possible flow configuration. This idealized case would be a uniform, steady flow through a cloud of particles fixed in position over a large region of space with no walls (shown in Figure 1). The volume averaged properties of the flow would be homogeneous and steady. In this case the production of turbulence by the particles would be equal to the dissipation because there would be no diffusion or production due to mean velocity gradients. Applying the turbulence models derived from Eq. 5 to this configuration typically yields a zero or negative value for the dissipation.[17]

The momentum equation shown in Eq. 5 cannot be derived from fundamental principles; even for very dilute flows, the momentum equations cannot be reduced to Eq. 5. The two valid approaches for developing the momentum equations for dispersed phase flow are volume averaging[18] and ensemble averaging.[19] It has been shown[18] that the additional source term needed in the momentum equations to account for the surface effects of particles arises from volume averaging the momentum equations (i.e., a volume larger than the continuum limit is necessary). The average velocities in the volume averaged or ensemble averaged equations do not represent the local (point wise) instantaneous velocity of a given flow and thereby are not amenable to the Reynolds averaging procedures used in single phase flows. In other words, the temporal fluctuations of the averaged velocities do not reflect the flow turbulence.

Crowe and Gillandt[20] developed an equation for carrier phase turbulent kinetic energy by volume averaging the mechanical energy equation. When applied to the basic test case, the equation reduces to the expected result; the rate of

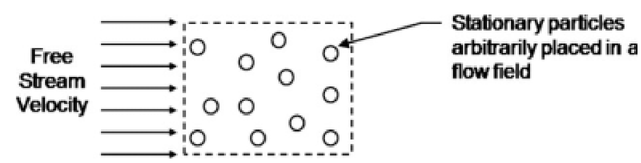

Figure 1. An idealized case used to test two-way coupled turbulence models in particle-laden flows.

For such a case, it is assumed that there are no wall effects and the particles are stationary; therefore the generation must balance with the dissipation.

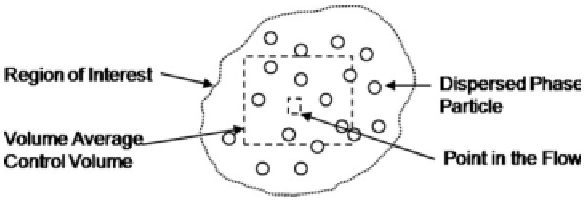

Figure 2. A qualitative comparison of the volume averaging approach and temporal averaging approach to modeling effects of particles.

Unless the particle surfaces are treated as boundary conditions, a temporal averaging approach does not include the effect of neighboring particles at a point in the flow.

turbulence generation by the particles is equal to the rate of dissipation. The turbulence kinetic energy equation developed by Crowe and Gillandt[20] has been used by Lain et al.[21] in bubble flow. The comparison showed that the numerical results using Crowe and Gillandt's[20] turbulence kinetic energy equation slightly under-predicted the average velocity but was an improvement over conventional models. Zhang and Reese[22,23] also performed an extensive study on Crowe and Gillandt[20] turbulent kinetic energy equation and found good comparison with the data of Tsuji et al.[8]

The purpose of this article is to present an approach to obtain a volume averaged dissipation equation for particle laden turbulent flows. By deriving the dissipation equation from first principles, it is possible to establish the effects of the dispersed phase on the turbulence dissipation of the carrier phase.

Review of Volume Averaging Concepts

In single phase flows, the turbulence equations are developed by Reynolds averaging the Navier- Stokes equations which describe the instantaneous properties at a point. Turbulence is described as the temporal velocity fluctuation from its time averaged value at a point in the flow. In dispersed phase flows it is not possible to describe the flow properties at a point without the inclusion of the effect of the neighboring particles (illustrated in Figure 2).

Volume averaging and ensemble averaging provide a scheme to include the effects of the dispersed phase without the necessity of including the details of the surface interaction. Crowe et al.[18] and Slattery[24] provide a detailed description of the volume average concept. The local volume average of a property B is defined as:

$$\overline{B} = \frac{1}{V}\int_{V_c} B dV \tag{6}$$

where V is the volume of the mixture and V_c is the volume of the continuous phase in the mixture volume. The phase volume average of the property B is defined as:

$$\langle B \rangle = \frac{1}{V_c}\int_{V_c} B dV \tag{7}$$

The phase average property is related to the local volume averaged property by:

$$\overline{B} = \alpha_c \langle B \rangle \tag{8}$$

where α_c is the volume fraction of the continuous phase, often referred to as the "void fraction." The averaging volume must be large enough to maintain a stationary average yet small compared to system dimensions to enable the use of differential operators.

Aside from temporal averaging, another way of defining turbulence is by the velocity deviation from the volume averaged velocity at a point in time, such as:

$$u_i = \langle u_i \rangle + \delta u_i \tag{9}$$

where u_i is the instantaneous velocity, $\langle u_i \rangle$ is the phase volume averaged velocity as illustrated in Figure 3. The turbulence energy is then defined as

$$k = \frac{1}{V_c}\int_{V_c} \frac{\delta u_i \delta u_i}{2} dV = \left\langle \frac{\delta u_i \delta u_i}{2} \right\rangle \tag{10}$$

The advantage of using volume averaging is that the effects of the surfaces are easily distinguished from the effects of the fluid. One of the identities introduced is the volume average of the gradient of a property, which brings out the effects of each particle surface within the domain. This identity is expressed as:

$$\overline{\frac{\partial}{\partial x_j}(B_i)} = \frac{\partial \overline{B_i}}{\partial x_j} - \frac{1}{V}\int_{S_d} B_i n_j dS \tag{11}$$

where the integration is performed over the particle surfaces, S_d, inside the control volume.

A very important advantage in applying the volume average concept to dispersed phase flows is that jump boundary conditions and moving meshes are not needed as would be in a point wise flow analysis (such as the RANS equations). In the presence of millions of particles, this approach would become computationally expensive. Although DNS practices use point wise forces to represent particles, the same

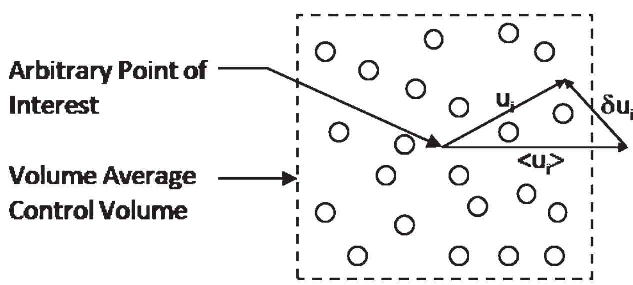

Figure 3. An illustration of the volume deviation velocity used to define turbulence in a volume averaged setting.

The instantaneous velocity, at any point in the flow, is the sum of the volume average and the volume deviation velocities at a point in time.

problem is imposed. In addition, the effect of the particle volume is ignored. An interim solution to this complex problem is volume averaging.

The difficulty with the application of volume averaging to multi-phase turbulence equations is in the comparison to experiments. Typical experiments are set up for point wise measurements, however with newly developed instrumentation such as Particle Image Velocimetry (PIV) there is potential to perform volume averaged measurements.

Derivation of the Dissipation Equation

To close the volume averaged turbulence equation set, an equation for the time rate of change of dissipation is needed. The following derivation is analogous to the derivation of the time average dissipation equation, provided by Bernard and Wallace.[16] The definition of volume average dissipation introduced by Crowe and Gillant[20] is:

$$\varepsilon = \nu\left\langle \frac{\partial \delta u_i}{\partial x_j}\frac{\partial \delta u_i}{\partial x_j}\right\rangle \tag{12}$$

To begin, a spatial gradient of the Navier-Stokes equation is taken. This is multiplied by the volume deviation velocity gradient ($\partial \delta u_i/\partial x_j$) and twice the kinematic viscosity. Finally, the result is volume averaged, which is represented mathematically by:

$$\overline{\left(\frac{\partial}{\partial x_j}[NS_i]\right) * 2\nu\left(\frac{\partial \delta u_i}{\partial x_j}\right)}$$

which can also be expressed as:

$$\overline{2\nu\frac{\partial \delta u_i}{\partial x_j}\frac{\partial}{\partial x_j}\left(\rho\frac{\partial u_i}{\partial t}\right)} + \overline{2\nu\frac{\partial \delta u_i}{\partial x_j}\frac{\partial}{\partial x_j}\left(\rho u_k\frac{\partial u_i}{\partial x_k}\right)} = -\overline{2\nu\frac{\partial \delta u_i}{\partial x_j}\frac{\partial}{\partial x_j}\frac{\partial P}{\partial x_i}} + \overline{2\nu\frac{\partial \delta u_i}{\partial x_j}\frac{\partial^2 \tau_{ik}}{\partial x_j \partial x_k}} \tag{13}$$

Decomposing the instantaneous terms in Eq. 13 into volume average and deviation terms and assuming the flow is incompressible, the above equation can be rewritten as:

$$\begin{aligned}
&\overline{2\nu\frac{\partial \delta u_i}{\partial x_j}\frac{\partial}{\partial x_j}\left(\frac{\partial\langle u_i\rangle}{\partial t}\right)} + \overline{2\nu\frac{\partial \delta u_i}{\partial x_j}\frac{\partial}{\partial x_j}\left(\frac{\partial \delta u_i}{\partial t}\right)} \\
&\quad + \overline{2\nu\frac{\partial \delta u_i}{\partial x_j}\frac{\partial\langle u_k\rangle}{\partial x_j}\frac{\partial\langle u_i\rangle}{\partial x_k}} + \overline{2\nu\langle u_k\rangle\frac{\partial \delta u_i}{\partial x_j}\frac{\partial^2\langle u_i\rangle}{\partial x_j\partial x_k}} \\
&\quad + \overline{2\nu\frac{\partial \delta u_i}{\partial x_j}\frac{\partial\langle u_k\rangle}{\partial x_j}\frac{\partial \delta u_i}{\partial x_k}} + \overline{2\nu\langle u_k\rangle\frac{\partial \delta u_i}{\partial x_j}\frac{\partial^2 \delta u_i}{\partial x_j\partial x_k}} \\
&\quad + \overline{2\nu\frac{\partial \delta u_i}{\partial x_j}\frac{\partial \delta u_k}{\partial x_j}\frac{\partial\langle u_i\rangle}{\partial x_k}} + \overline{2\nu\delta u_k\frac{\partial \delta u_i}{\partial x_j}\frac{\partial^2\langle u_i\rangle}{\partial x_j\partial x_k}} \\
&\quad + \overline{2\nu\frac{\partial \delta u_i}{\partial x_j}\frac{\partial \delta u_k}{\partial x_j}\frac{\partial \delta u_i}{\partial x_k}} + \overline{2\nu\delta u_k\frac{\partial \delta u_i}{\partial x_j}\frac{\partial^2 \delta u_i}{\partial x_j\partial x_k}} \\
&= -\overline{2\frac{\nu}{\rho}\frac{\partial \delta u_i}{\partial x_j}\frac{\partial}{\partial x_j}\frac{\partial\langle P\rangle + \delta P}{\partial x_i}} \\
&\quad + \overline{2\frac{\nu}{\rho}\frac{\partial \delta u_i}{\partial x_j}\frac{\partial^2}{\partial x_j\partial x_k}\left(\langle\tau_{ik}\rangle + \delta\tau_{ik}\right)}
\end{aligned} \tag{14}$$

Applying the volume averaging concept to each term in Eq. 14 shows (details are shown in Appendix):

$$\begin{aligned}
&\nu\frac{\partial}{\partial t}\left[\alpha_c\left\langle\frac{\partial \delta u_i}{\partial x_j}\frac{\partial \delta u_i}{\partial x_j}\right\rangle\right] + \nu\frac{\partial}{\partial x_k}\left[\alpha_c\langle u_k\rangle\left\langle\frac{\partial \delta u_i}{\partial x_j}\frac{\partial \delta u_i}{\partial x_j}\right\rangle\right] \\
&= -2\nu\alpha_c\frac{\partial\langle u_k\rangle}{\partial x_j}\left\langle\frac{\partial \delta u_i}{\partial x_j}\frac{\partial \delta u_i}{\partial x_k}\right\rangle - 2\nu\alpha_c\frac{\partial\langle u_i\rangle}{\partial x_k}\left\langle\frac{\partial \delta u_i}{\partial x_j}\frac{\partial \delta u_k}{\partial x_j}\right\rangle \\
&\quad - 2\nu\alpha_c\frac{\partial^2\langle u_i\rangle}{\partial x_j\partial x_k}\left\langle\delta u_k\frac{\partial \delta u_i}{\partial x_j}\right\rangle - 2\nu\alpha_c\left\langle\frac{\partial \delta u_i}{\partial x_j}\frac{\partial \delta u_k}{\partial x_j}\frac{\partial \delta u_i}{\partial x_k}\right\rangle \\
&\quad - \nu\frac{\partial}{\partial x_k}\left[\alpha_c\delta u_k\left\langle\frac{\partial \delta u_i}{\partial x_j}\frac{\partial \delta u_i}{\partial x_j}\right\rangle\right] - 2\frac{\nu}{\rho}\frac{\partial}{\partial x_i}\left[\alpha_c\left\langle\frac{\partial \delta u_i}{\partial x_j}\frac{\partial \delta P}{\partial x_j}\right\rangle\right] \\
&\quad + \nu^2\frac{\partial^2}{\partial x_k^2}\left[\alpha_c\left\langle\frac{\partial \delta u_i}{\partial x_j}\frac{\partial \delta u_i}{\partial x_j}\right\rangle\right] - 2\nu^2\alpha_c\left\langle\left(\frac{\partial^2 \delta u_i}{\partial x_j\partial x_k}\right)^2\right\rangle \\
&\quad - \frac{\nu^2}{V}\frac{\partial}{\partial x_k}\int_{S_d}\frac{\partial \delta u_i}{\partial x_j}\frac{\partial \delta u_i}{\partial x_j}n_k dS + \frac{2\nu}{\rho V}\int_{S_d}\frac{\partial \delta u_i}{\partial x_j}\frac{\partial \delta P}{\partial x_j}n_i dS \\
&\quad - \frac{2\nu^2}{V}\int_{S_d}\frac{\partial \delta u_i}{\partial x_j}\frac{\partial}{\partial x_j}\frac{\partial \delta u_i}{\partial x_k}n_k dS
\end{aligned} \tag{15}$$

By identifying the volume averaged dissipation tensor as $\varepsilon_{ik} = 2\nu\left\langle\frac{\partial \delta u_i}{\partial x_j}\frac{\partial \delta u_k}{\partial x_j}\right\rangle$ and noting that the volume averaged continuity equation for incompressible flow is $\frac{\partial \alpha_c}{\partial t} + \frac{\partial(\alpha_c\langle u_k\rangle)}{\partial x_k} = 0$, Eq. 15 can be rewritten in the form:

$$\begin{aligned}
\alpha_c\frac{D\varepsilon}{Dt} &= -2\nu\alpha_c\frac{\partial\langle u_k\rangle}{\partial x_j}\left\langle\frac{\partial \delta u_i}{\partial x_j}\frac{\partial \delta u_i}{\partial x_k}\right\rangle - \alpha_c\varepsilon_{ik}\frac{\partial\langle u_i\rangle}{\partial x_k} \\
&\quad - 2\nu\alpha_c\left\langle\delta u_k\frac{\partial \delta u_i}{\partial x_j}\right\rangle\frac{\partial^2\langle u_i\rangle}{\partial x_j\partial x_k} - 2\nu\alpha_c\left\langle\frac{\partial \delta u_i}{\partial x_j}\frac{\partial \delta u_k}{\partial x_j}\frac{\partial \delta u_i}{\partial x_k}\right\rangle \\
&\quad - \nu\frac{\partial}{\partial x_k}\left[\alpha_c\left\langle\delta u_k\frac{\partial \delta u_i}{\partial x_j}\frac{\partial \delta u_i}{\partial x_j}\right\rangle\right] - 2\frac{\nu}{\rho}\frac{\partial}{\partial x_i}\left[\alpha_c\left\langle\frac{\partial \delta u_i}{\partial x_j}\frac{\partial \delta P}{\partial x_j}\right\rangle\right] \\
&\quad + \nu\frac{\partial^2}{\partial x_k^2}[\alpha_c\varepsilon] - 2\nu^2\alpha_c\left\langle\left(\frac{\partial^2 \delta u_i}{\partial x_j\partial x_k}\right)^2\right\rangle \\
&\quad - \frac{\nu^2}{V}\frac{\partial}{\partial x_k}\int_{S_d}\frac{\partial \delta u_i}{\partial x_j}\frac{\partial \delta u_i}{\partial x_j}n_k dS + \frac{2\nu}{\rho V}\int_{S_d}\frac{\partial \delta u_i}{\partial x_j}\frac{\partial \delta P}{\partial x_j}n_i dS \\
&\quad - \frac{\nu^2}{V}\int_{S_d}\frac{\partial}{\partial x_k}\left[\frac{\partial \delta u_i}{\partial x_j}\frac{\partial \delta u_i}{\partial x_j}\right]n_k dS
\end{aligned} \tag{16}$$

The above equation is the general dissipation equation for two-way coupled particle laden flows. The assumptions associated with Eq. 16 are incompressible flow and no mass transfer between the dispersed and continuous phase. If the void fraction is unity and no dispersed phase surfaces are present, Eq. 16 reduces to the single-phase flow dissipation equation:

$$\frac{D\varepsilon}{Dt} = \overbrace{-2\nu\frac{\partial\langle u_i\rangle}{\partial x_k}\left\langle\frac{\partial\delta u_j}{\partial x_k}\frac{\partial\delta u_j}{\partial x_i}\right\rangle - \varepsilon_{ik}\frac{\partial\langle u_i\rangle}{\partial x_k} - 2\nu\left\langle\delta u_k\frac{\partial\delta u_i}{\partial x_j}\right\rangle\frac{\partial^2\langle u_i\rangle}{\partial x_j\partial x_k} - 2\nu\left\langle\frac{\partial\delta u_i}{\partial x_j}\frac{\partial\delta u_k}{\partial x_j}\frac{\partial\delta u_i}{\partial x_k}\right\rangle}^{\text{Production of Dissipation}}$$

$$\underbrace{-\nu\frac{\partial}{\partial x_k}\left[\left\langle\delta u_k\frac{\partial\delta u_i}{\partial x_j}\frac{\partial\delta u_i}{\partial x_j}\right\rangle\right]}_{\substack{\text{Transport}\\ \text{of}\\ \text{Dissipation}}}\underbrace{-2\frac{\nu}{\rho}\frac{\partial}{\partial x_i}\left[\left\langle\frac{\partial\delta u_i}{\partial x_j}\frac{\partial\delta P}{\partial x_j}\right\rangle\right]}_{\substack{\text{Pressure}\\ \text{Diffusion of}\\ \text{Dissipation}}}\underbrace{+\nu\frac{\partial^2}{\partial x_k^2}[\varepsilon]}_{\substack{\text{Viscous}\\ \text{Diffusion of}\\ \text{Dissipation}}}\underbrace{-2\nu^2\left\langle\left(\frac{\partial^2\delta u_i}{\partial x_j\partial x_k}\right)^2\right\rangle}_{\substack{\text{Dissipation}\\ \text{of}\\ \text{Dissipation}}} \tag{17}$$

which is the volume average equivalent to time average dissipation equation presented by Bernard and Wallace.[16] It is now apparent that the effects of the surfaces of the dispersed phase are associated with the following terms:

$$-\frac{\nu^2}{V}\frac{\partial}{\partial x_k}\int_{S_d}\frac{\partial\delta u_i}{\partial x_j}\frac{\partial\delta u_i}{\partial x_j}n_k dS + \frac{2\nu}{\rho V}\int_{S_d}\frac{\partial\delta u_i}{\partial x_j}\frac{\partial\delta P}{\partial x_j}n_i dS - \frac{\nu^2}{V}\int_{S_d}\frac{\partial}{\partial x_k}\left[\frac{\partial\delta u_i}{\partial x_j}\frac{\partial\delta u_i}{\partial x_j}\right]n_k dS \tag{18}$$

The integrals in Eq. 18 represent the dissipation effects caused by surfaces of the particles. Within these integrals are spatial gradients of volume deviation properties. To evaluate the integrals, the coupled gradients must be captured. A first attempt to evaluate these terms is based on the assumption that the particles are much smaller than the equivalent Kolmogorov length scale and that the drag force is given by Stokes drag law; in addition, the effects of particle rotation are neglected. The particle-fluid relative velocity (U_i) can be expressed as the instantaneous velocity between the particle and the surrounding fluid as:

$$U_i = u_i - v_i \tag{19}$$

which is the same as taking a particle at rest in a flow field with velocity U_i. Substituting Eq. 9 into Eq. 19 shows a relationship between the local velocity deviation and the relative velocity in the form:

$$\delta u_i = U_i + v_i - \langle u_i\rangle \tag{20}$$

The spatial velocity gradients shown in Eq. 18 are evaluated at the surfaces of the particles. Taking a spatial gradient of the above equation shows:

$$\left.\frac{\partial\delta u_i}{\partial x_j}\right|_{S_d} = \left.\frac{\partial U_i}{\partial x_j}\right|_{S_d} + \left.\frac{\partial v_i}{\partial x_j}\right|_{S_d} - \left.\frac{\partial\langle u_i\rangle}{\partial x_j}\right|_{S_d} \tag{21}$$

where S_d represents the surface of the dispersed phase particles. If the particle is rigid and rotational effects are neglected, then the velocity of the particle is constant across the particle. Therefore, the last two terms on the right hand side of the above equation are zero when evaluated along the particle surface, reducing Eq. 21 to:

$$\left.\frac{\partial\delta u_i}{\partial x_j}\right|_{S_d} = \left.\frac{\partial U_i}{\partial x_j}\right|_{S_d} \tag{22}$$

The assumption that the particles are much smaller than the equivalent Kolmogorov length scale (i.e., Stokes flow) allows the relative velocity gradients to be solved for analytically. The reason for this assumption is to obtain a simplified form of each term in Eq. 18 and then through empiricism extend these forms to flows containing particles that are larger than the equivalent Kolmogorov length scale. By transforming the gradients and unit normal vectors between Cartesian and spherical polar coordinate systems, the terms in Eq. 18 can be directly solved for. Evaluation of the diffusion term shows that the integral over the surface of each particle is zero:

$$-\frac{\nu^2}{V}\frac{\partial}{\partial x_k}\int_{S_d}\frac{\partial\delta u_i}{\partial x_j}\frac{\partial\delta u_i}{\partial x_j}n_k dS = 0 \tag{23}$$

which is primarily because of symmetry. However, this may not be the case if the particle is larger than the equivalent Kolmogorov length scale. Evaluation of the second term in Eq. 18 shows:

$$\frac{2\nu}{\rho V}\int_{S_d}\frac{\partial\delta u_i}{\partial x_j}\frac{\partial\delta P}{\partial x_j}n_i dS = \frac{2\nu}{\rho V}\int_{S_d}\frac{\partial u_r}{\partial r}\frac{\partial\delta P}{\partial r}dS + \frac{2\nu}{\rho V}\int_{S_d}\left[\frac{1}{r}\frac{\partial u_r}{\partial\theta} - \frac{u_\theta}{r}\right]\frac{1}{r}\frac{\partial\delta P}{\partial\theta}dS \tag{24}$$

where u_r and u_θ are the radial and tangential components of the free stream velocity and δP is the deviation of the volume pressure, or Stokes pressure. Evaluation of right hand side (RHS) of Eq. 24 at the surface of the particle results in:

$$\frac{2v}{\rho V}\int_{S_d}\frac{\partial u_r}{\partial r}\frac{\partial\delta P}{\partial r}dS+\frac{2v}{\rho V}\int_{S_d}\left[\frac{1}{r}\frac{\partial u_r}{\partial\theta}-\frac{u_\theta}{r}\right]\frac{1}{r}\frac{\partial\delta P}{\partial\theta}dS=0 \quad (25)$$

The reason is that the contributing gradients do not couple with the unit vector normal to the particle surface.[17] Again, this may not be the case for particles larger than the equivalent Kolmogorov length scale. Evaluation of the last term in Eq. 18 results in:

$$-\frac{v^2}{V}\int_{S_d}\frac{\partial}{\partial x_k}\left[\frac{\partial\delta u_i}{\partial x_j}\frac{\partial\delta u_i}{\partial x_j}\right]n_k dS = \frac{v^2}{V}\sum_n\frac{27U_n^2}{D_n}\int_0^{2\pi}\int_0^{\pi}\sin^3(\theta)d\theta d\varphi \quad (26)$$

where the summation represents the effects of the all the particles within the control volume and n represent each particle within the control volume. Evaluating the integral yields:

$$-\frac{v^2}{V}\int_{S_d}\frac{\partial}{\partial x_k}\left[\frac{\partial\delta u_i}{\partial x_j}\frac{\partial\delta u_i}{\partial x_j}\right]n_k dS = 72\pi\frac{v^2}{V}\sum_n\frac{|u_i-v_i|_n^2}{D_n} \quad (27)$$

This term is identified as a production of dissipation term. The deviation from Stokes drag may be approximated by multiplying by the drag factor, f, which is the ratio of the drag coefficient for a sphere to Stokes drag. Also the coefficient 72π is replaced by an empirical coefficient so Eq. 27 is represented by:

$$-\frac{v^2}{V}\int_{S_d}\frac{\partial}{\partial x_k}\left[\frac{\partial\delta u_i}{\partial x_j}\frac{\partial\delta u_i}{\partial x_j}\right]n_k dS = C_{\varepsilon3}\frac{v^2}{V}\sum_n f_n\frac{|u_i-v_i|_n^2}{D_n} \quad (28)$$

where $C_{\varepsilon3}$ is an empirical coefficient and would be determined by experiments.

The Modified Dissipation Model

The time averaged dissipation model, Eq. 2, is obtain by grouping terms in the time averaged dissipation transport equation Eq. 4.[16] These terms fall into categories representing the production of dissipation, diffusion of dissipation and dissipation of dissipation. This same approach is taken to convert the volume averaged dissipation transport equation to a model of the form:

$$\alpha_c\frac{D\varepsilon}{Dt}=C'_{\varepsilon1}\alpha_c\frac{P_\varepsilon\varepsilon}{k}-C'_{\varepsilon2}\alpha_c\frac{\varepsilon^2}{k}+C_{\varepsilon3}\frac{v^2}{V}\sum_n f_n\frac{|u_i-v_i|_n^2}{D_n}+\frac{\partial}{\partial x_i}\left[\alpha_c\left(v+\frac{v_T}{\sigma_\varepsilon}\right)\frac{\partial\varepsilon}{\partial x_i}\right] \quad (29)$$

where α_c is the void fraction, P_ε is the production of dissipation, $C'_{\varepsilon1}$ and $C'_{\varepsilon2}$ are the combined single and dispersed phase production and dissipation of dissipation coefficients respectively, $C_{\varepsilon3}$ is the production of dissipation coefficient due to particles, V is the mixture volume, v is the molecular kinematic viscosity, f is the drag factor, u_i and v_i are the instantaneous fluid and particle velocities respectively, D is the particle diameter, n is the particle number within the mixture volume, v_T is the turbulent kinematic viscosity (based on volume averaging), σ_ε is the effective Schmidt number for turbulent diffusion, and k and ε are the volume averaged turbulent kinetic energy and dissipation defined in Eqs. 10 and 12, respectively. Applying Eq. 29 to the idealized case of particle laden turbulent flows (shown in Figure 1) where the particles are stationary and the flow is steady and homogeneous with no wall effects reduces to:

$$C_{\varepsilon3}\frac{v^2}{V}\sum_n f_n\frac{|u_i|_n^2}{D_n}=C'_{\varepsilon2}\alpha_c\frac{\varepsilon^2}{k} \quad (30)$$

which shows that the production of dissipation due to the presence of particles must balance the dissipation of dissipation in the fluid. According to the experimental data reviewed by of Gore and Crowe,[3] the turbulence intensity for particle-laden flows is increased (relative to a single phase flow) for $D/L > 0.1$ and decreased for $D/L < 0.1$, where D is the particle diameter and L is the characteristic length of the most energetic eddy. Although the magnitude of the ratio of particle diameter to the fluid length scale at which this transition occurs is arguable,[4] clearly there is a transition. Non-dimensionalizing Eq. 30 by a mean velocity (U) and a characteristic length scale in the flow (L), a measure of the most energetic eddy, shows:

$$C_{\varepsilon3}\frac{1}{\mathrm{Re}_L^2}\frac{1}{\tilde{V}}\sum_n f_n\frac{|\tilde{u}_i|_n^2}{(D/L)_n}=C'_{\varepsilon2}\alpha_c\frac{\tilde{\varepsilon}^2}{\tilde{k}}$$

where the tilde is the non-dimensional form and Re_L is the Reynolds number based on the mean velocity and a characteristic length scale of the flow. The ratio D/L appears as a fundamental parameter. For this idealized case, it can be shown that the turbulence intensity ($\sigma = \sqrt{2\tilde{k}}$) is a function of D/L which corresponds to the findings of Gore and Crowe.[3]

Results and Discussion

The idealized case (depicted in Figure 1) cannot be achieved experimentally. However, if the turbulence intensity based on the particle velocity is much less than unity, dropping particles through an initially quiescent fluid approaches the conditions of the idealized case. There are several data sets available for these experimental conditions. The volume averaged turbulent kinetic energy equation developed by Crowe and Gillandt[20] reduces to:

$$\varepsilon=\frac{\beta_V}{\alpha_c\rho_c}\left|\langle v_i\rangle\right|^2=\frac{\alpha_d\rho_d}{\alpha_c\rho_c}\frac{f}{\tau_p}\left|\langle v_i\rangle\right|^2 \quad (31)$$

for $\sqrt{k}/|\langle v_i\rangle| \ll 1$. Simplifying Eq. 29 for the case of falling particles through an initially quiescent fluid results in

$$C_{\varepsilon 3}\frac{v^2}{V}\sum_n f_n \frac{|(v_i)_n|^2}{D_n} = C'_{\varepsilon 2}\alpha_c \frac{\varepsilon^2}{k} \tag{32}$$

The left hand side of the above equation represents the time rate of change of production of dissipation due to the relative velocity gradients at the particle surface while the right hand side is the dissipation of dissipation in the viscous fluid. For particles with uniform diameter, Eq. 32 further reduces to:

$$C'_{\varepsilon 2}\alpha_c \frac{\varepsilon^2}{k} = C_{\varepsilon 3} n v^2 f \frac{|\langle v_i\rangle|^2}{D} \tag{33}$$

where n is the particle number density.

There is not sufficient information at this point to evaluate both empirical coefficients. The value of $C'_{\varepsilon 2}$ for the standard single phase k-ε model is 1.92; however, adjustments have been made to fit other experimental data such as turbulence behind a grid and in the case of multi-phase flows may depend on particle loading. For this reason, the ratio of coefficients will be evaluated. By substituting in the dissipation from the turbulent kinetic energy, Eq. 31 into Eq. 33, a relationship between the coefficients associated with the production of dissipation due to presence of particles and the total dissipation of dissipation within the fluid results in:

$$\frac{C_{\varepsilon 3}}{C'_{\varepsilon 2}} = \frac{\alpha_c\left[\frac{\alpha_d\rho_d}{\alpha_c\rho_c}\frac{f}{\tau_p}|\langle v_i\rangle|^2\right]^2}{knv^2 f\frac{|\langle v_i\rangle|^2}{D}} \approx \left(\frac{\pi 18^2}{6}\right)\frac{\alpha_d}{\alpha_c}\frac{f|\langle v_i\rangle|^2}{k} \tag{34}$$

Crowe and Wang[25] compiled data from several different authors[5–15] and showed a correlation between the relative Reynolds number and the ratio of Taylor length scale to particle diameter. Most of the data fit remarkably well, however a few sets of data seemed to deviate. Crowe and Wang[25] report that the data of Kenning and Crowe[15] may be low compared with the rest of the data due to analyzing the turbulence after the particle cloud passed. There does not seem to be enough data with varying volume fractions that confirm this conclusion. The volume fractions associated with Kenning and Crowe's data are on the order of 10^{-2} when compared with Mizukami et al.[11] (10^{-6}) and Parthasarathy and Faeth[12] (10^{-4}). The volume fractions of Lance and Bataille[14] were on the order of Kenning and Crowe's data. According to Elgobashi,[26] volume fractions less than 10^{-6} have negligible effect on the turbulence of the carrier phase (one-way coupling), volume fractions in the range of 10^{-6}–10^{-3} alter the turbulence of the carrier phase (two-way coupling), and volume fractions greater than 10^{-3} further alter the turbulence of the carrier phase due to particle-particle collisions (four way coupling). The data of Kenning and Crowe[15] and Lance and Bataille[14] showed volume fractions on the order of 10^{-2}; which appears to fall into a four- way coupling category. The correlation presented by Crowe and Wang[25] for two-way coupling is:

$$\frac{\lambda}{D} \propto \left[\frac{\alpha_c}{\alpha_d}\frac{1}{18f}\frac{k_c}{\Delta u^2}\right]^{1/2} \tag{35}$$

where, λ is the Taylor length scale, D is the particle diameter, α_c and α_d are the volume fractions of the continuous and dispersed phase respectively, k_c is the turbulent kinetic energy of the carrier phase, f is the drag factor, and Δu is the velocity difference between the phases. Comparing Eqs. 34 and 35, it is clearly noted that the ratio of the particle production of dissipation coefficient to the total dissipation of dissipation coefficient is proportional to the ratio of the particle diameter to the Taylor length scale of the fluid.

$$\frac{C_{\varepsilon 3}}{C'_{\varepsilon 2}} \propto \left[\frac{D}{\lambda}\right]^2 \tag{36}$$

To evaluate the ratio of coefficients, data from experiments of particles falling in an initially quiescent fluid with volume fractions between 10^{-6} and 10^{-2} were considered along with useful data at the centerline of pipe flow experiments. The data of Parthasarathy and Faeth[12] Mizukami et al.,[11] Hosokawa et al.,[5] Chen and Faeth[13] and Varaksin et al.[9] provided information to determine the coefficient over a wide range of relative Reynolds numbers (1–1000), volume fractions (10^{-6}–10^{-2}), particle density (900–3600 kg/m^3) and carrier phase density (1–1000 kg/m^3). Mizukami et al.[11] and Parthasarathy and Faeth[12] performed experiments to determine the production of dissipation due to particles falling in an initially quiescent fluid, representative of Eq. 32. The particle distribution was mono-dispersed and particle velocities were nearly uniform. The data of Hosokawa et al.,[5] Chen and Faeth[13] and Varaksin et al.[9] involved particles in pipe flow, of which only the data at the centerline of the pipe were reduced. The data were reduced for the ratio of coefficients as a function of relative Reynolds number, defined as:

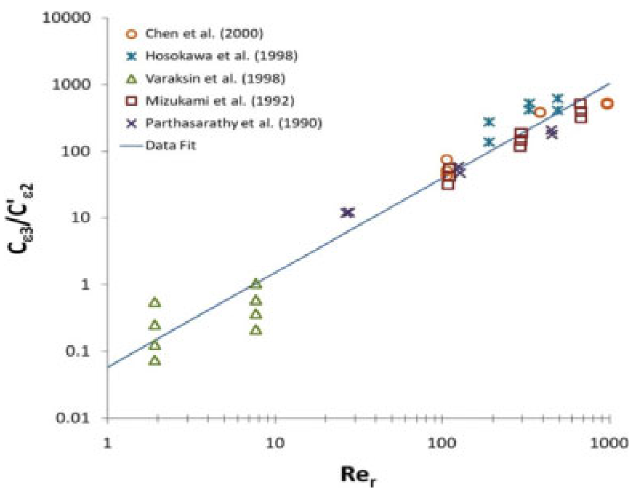

Figure 4. Variation of the ratio of the production of dissipation coefficient ($C_{\varepsilon 3}$) to the dissipation of dissipation coefficient ($C'_{\varepsilon 2}$) over a wide range of relative Reynolds numbers for various types of particle laden flows.

For low relative Reynolds numbers, the data of Varaksin et al.[9] shows that the ratio of coefficients depends on the particle mass loading—for increased loading, the ratio of coefficients is increased. [Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com.]

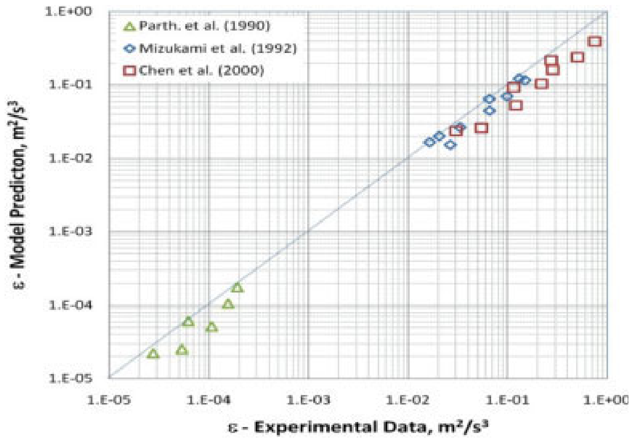

Figure 5. Comparison of the dissipation predicted by the model, Eq. 40, to the experimental data of Refs. 11–13.

[Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com.]

$$\mathrm{Re_r} = \frac{|u_i - v_i|D}{\nu_\mathrm{c}} \tag{37}$$

are shown in Figure 4. The data do appear to correlate well for relative Reynolds numbers ranging from 30 to 1000. The data of Mizukami et al.[11] and Parthasarathy and Faeth[12] involve high particle Reynolds numbers over a wide range of particle mass loadings; at high particle Reynolds numbers, the correlation does not appear to be affected by the particle mass loading. However, at low relative Reynolds numbers, the data of Varaksin et al.[9] shows that the particle mass loading (0.12–0.39) must contribute to the coefficient. The fact that there is a correlation lends credence to the model, yet more data is needed at low relative Reynolds numbers to understand the effect of particle mass loading.

Fitting the data shown in Figure 4 using the least squares method with the equation:

$$\frac{C_{\varepsilon 3}}{C'_{\varepsilon 2}} \approx C_{\varepsilon p}(\mathrm{Re_r})^m \tag{38}$$

shows $C_{\varepsilon p} = 0.0587$, $m = 1.4161$. Substituting Eq. 38 into Eq. 29 results in a general dissipation transport equation for incompressible flow with no mass transfer between the phases and negligible effect of particle rotation of the form:

$$\alpha_\mathrm{c}\frac{D\varepsilon}{Dt} = C'_{\varepsilon 1}\alpha_\mathrm{c}\frac{P_\varepsilon \varepsilon}{k} + C'_{\varepsilon 2}\left[C_{\varepsilon p}\alpha_\mathrm{d}\frac{\nu^2}{V_\mathrm{d}}\sum_n (\mathrm{Re}_r^m)_n f_n \frac{|u_i - v_i|_n^2}{D_n} - \alpha_\mathrm{c}\frac{\varepsilon^2}{k}\right] + \frac{\partial}{\partial x_i}\left[\alpha_\mathrm{c}\left(\nu + \frac{\nu_T}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial x_i}\right] \tag{39}$$

where V_d is the total volume of the dispersed phase particles within the mixture volume. For steady flow with no diffusion or mean carrier phase velocity gradients, Eq. 39 reduces to:

$$\varepsilon = \left[0.0587\ \mathrm{Re_r^{1.4161}}\frac{\alpha_\mathrm{d}}{\alpha_\mathrm{c}}\frac{k\nu^2}{V_\mathrm{d}}\sum_n f_n \frac{|u_i - (v_i)_n|^2}{D_n}\right]^{1/2} \tag{40}$$

A comparison of the model prediction of the dissipation found from experimental data is shown in Figure 5. The data deviates slightly from the prediction using Eq. 40. The data of Hosokawa et al.[5] and Varaksin et al.[9] did not include results for the dissipation.

Although only a limited amount of data is shown here to validate the model, the model does show promise to account for the effects of particles in the dissipation equation. To further test the model, a comparison to DNS simulations and flow data under different conditions are needed.

Conclusions

Volume averaging proves to be a powerful tool for analyzing multi-phase flows. This article presents the derivation of a volume averaged turbulence dissipation transport equation that accounts for the turbulence dissipation caused by particle surfaces within the flow. The governing equation shows an additional production of dissipation term that is related to the instantaneous relative velocity gradients at the particle surface. The governing equation is valid for incompressible flows with no mass transfer between the dispersed and continuous phases. By setting the volume fraction of the continuous phase to unity and the relative velocity between the phases to zero, the volume average dissipation equation reduces to the single phase equivalent of the time averaged dissipation equation.

The dissipation transport equation was applied to experimental data involving generation of homogeneous turbulence by particles. The ratio of the new production of dissipation coefficient (due to the presence of particles) and the dissipation of dissipation coefficient was found to be related to the particle diameter and the Taylor length scale. At high relative Reynolds numbers, the ratio of the coefficients was found to correlate well with the relative Reynolds number of the particles, and at low relative Reynolds numbers, the ratios of coefficients appears to be a function of loading in addition to the particle Reynolds number.

Notation

f = drag factor.
n = number of droplets per unit volume, [1/m^3].
N = number of droplets.
P = pressure, [N/m^2].
$\mathrm{Re_r}$ = relative Reynolds number between dispersed and continuous phase.
U_i = stokes relative velocity, [m/s].
u_i = instantaneous velocity of the continuous phase medium, [m/s].
v_i = instantaneous velocity of the dispersed phase medium, [m/s].
$\overline{\overline{u_i}}$ = time averaged velocity of the single phase medium, [m/s].
u'_i = time fluctuation velocity of the single phase medium, [m/s].
$\langle u_i \rangle$ = volume average velocity of the continuous phase medium, [m/s].
δu_i = volume deviation velocity of the continuous phase medium, [m/s].
$\langle v_i \rangle$ = volume average velocity of the dispersed phase medium, [m/s].
δv_i = volume deviation velocity of the dispersed phase medium, [m/s].
V = volume of evaluated section, [m^3].

Greek symbols

α = volume fraction associated with a phase.
ε = dissipation, [m^2/s^3].
μ = dynamic viscosity, [N s/m^2].
ν = kinematic viscosity, [m^2/s].
ρ = density, [kg/m^3].
σ = turbulence intensity.
τ = viscous shear stress, [N/m^2].
$\tau_{()}$ = response time in relation to subscript, [s].

Averaging definitions

$\langle B \rangle = \frac{1}{V_c}\int_{V_c} BdV$ = phase volume average of property B.
$\overline{B} = \frac{1}{V}\int_{V_c} BdV$ = local volume average of property B.
$\overline{\overline{B}} = \frac{1}{T}\int_{T} Bdt$ = local time average of property B.

Subscripts

c = related to the continuous phase.
d = related to the dispersed phase.
p = related to the particle.
t = related to the time averaging.

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Appendix: Details on the Volume Average of Each Term in Eq. 14

Volume average of the first term:

$$\overline{2\nu\frac{\partial\delta u_i}{\partial x_j}\frac{\partial}{\partial x_j}\left(\frac{\partial\langle u_i\rangle}{\partial t}\right)} = 2\nu\frac{\partial}{\partial x_j}\left(\frac{\partial\langle u_i\rangle}{\partial t}\right)\overline{\frac{\partial\delta u_i}{\partial x_j}} = 0 \tag{A1}$$

Volume average of the second term, assuming no mass transfer between the phases ($\dot{r} = 0$):

$$\overline{\nu\frac{\partial}{\partial t}\left[\frac{\partial\delta u_i}{\partial x_j}\frac{\partial\delta u_i}{\partial x_j}\right]} = \nu\frac{\partial}{\partial t}\overline{\left[\frac{\partial\delta u_i}{\partial x_j}\frac{\partial\delta u_i}{\partial x_j}\right]} + \frac{\nu}{V}\int_{S_d}\frac{\partial\delta u_i}{\partial x_j}\frac{\partial\delta u_i}{\partial x_j}(v_k n_k + \dot{r})dS = \nu\frac{\partial}{\partial t}\left[\alpha_c\left\langle\frac{\partial\delta u_i}{\partial x_j}\frac{\partial\delta u_i}{\partial x_j}\right\rangle\right] + \frac{\nu}{V}\int_{S_d}\frac{\partial\delta u_i}{\partial x_j}\frac{\partial\delta u_i}{\partial x_j}(v_k n_k)dS \tag{A2}$$

Volume average of the third term:

$$\overline{2\nu\frac{\partial\delta u_i}{\partial x_j}\frac{\partial\langle u_k\rangle}{\partial x_j}\frac{\partial\langle u_i\rangle}{\partial x_k}} = 2\nu\frac{\partial\langle u_k\rangle}{\partial x_j}\frac{\partial\langle u_i\rangle}{\partial x_k}\overline{\frac{\partial\delta u_i}{\partial x_j}} = 0 \tag{A3}$$

Volume average of the fourth term:

$$\overline{2\nu\langle u_k\rangle\frac{\partial\delta u_i}{\partial x_j}\frac{\partial^2\langle u_i\rangle}{\partial x_j\partial x_k}} = 2\nu\langle u_k\rangle\frac{\partial^2\langle u_i\rangle}{\partial x_j\partial x_k}\overline{\frac{\partial\delta u_i}{\partial x_j}} = 0 \tag{A4}$$

Volume average of the fifth term:

$$\overline{2\nu\frac{\partial\delta u_i}{\partial x_j}\frac{\partial\langle u_k\rangle}{\partial x_j}\frac{\partial\delta u_i}{\partial x_k}} = 2\nu\alpha_c\frac{\partial\langle u_k\rangle}{\partial x_j}\left\langle\frac{\partial\delta u_i}{\partial x_j}\frac{\partial\delta u_i}{\partial x_k}\right\rangle \tag{A5}$$

Volume average of the sixth term:

$$\overline{2\nu\langle u_k\rangle\frac{\partial\delta u_i}{\partial x_j}\frac{\partial^2\delta u_i}{\partial x_j\partial x_k}} = \nu\langle u_k\rangle\overline{\frac{\partial}{\partial x_k}\left[\frac{\partial\delta u_i}{\partial x_j}\frac{\partial\delta u_i}{\partial x_j}\right]} = \nu\langle u_k\rangle\frac{\partial}{\partial x_k}\left[\alpha_c\left\langle\frac{\partial\delta u_i}{\partial x_j}\frac{\partial\delta u_i}{\partial x_j}\right\rangle\right] - \frac{\nu}{V}\langle u_k\rangle\int_{S_d}\frac{\partial\delta u_i}{\partial x_j}\frac{\partial\delta u_i}{\partial x_j}n_k dS \tag{A6}$$

Volume average of the seventh term:

$$\overline{2\nu\frac{\partial\delta u_i}{\partial x_j}\frac{\partial\delta u_k}{\partial x_j}\frac{\partial\langle u_i\rangle}{\partial x_k}} = 2\nu\alpha_c\frac{\partial\langle u_i\rangle}{\partial x_k}\left\langle\frac{\partial\delta u_i}{\partial x_j}\frac{\partial\delta u_k}{\partial x_j}\right\rangle \tag{A7}$$

Volume average of the eighth term:

$$\overline{2\nu\delta u_k\frac{\partial\delta u_i}{\partial x_j}\frac{\partial^2\langle u_i\rangle}{\partial x_j\partial x_k}} = 2\nu\alpha_c\frac{\partial^2\langle u_i\rangle}{\partial x_j\partial x_k}\left\langle\delta u_k\frac{\partial\delta u_i}{\partial x_j}\right\rangle \tag{A8}$$

Volume average of the ninth term:

$$\overline{2\nu\frac{\partial\delta u_i}{\partial x_j}\frac{\partial\delta u_k}{\partial x_j}\frac{\partial\delta u_i}{\partial x_k}} = 2\nu\alpha_c\left\langle\frac{\partial\delta u_i}{\partial x_j}\frac{\partial\delta u_k}{\partial x_j}\frac{\partial\delta u_i}{\partial x_k}\right\rangle \tag{A9}$$

Volume average of the tenth term (for details see Schwarzkopf[17]):

$$\begin{aligned}2\nu\overline{\delta u_k\frac{\partial\delta u_i}{\partial x_j}\frac{\partial^2\delta u_i}{\partial x_j\partial x_k}} &= \nu\frac{\partial}{\partial x_k}\left[\alpha_c\langle u_k\rangle\left\langle\frac{\partial\delta u_i}{\partial x_j}\frac{\partial\delta u_i}{\partial x_j}\right\rangle\right] \\ &+ \nu\frac{\partial}{\partial x_k}\left[\alpha_c\left\langle\delta u_k\frac{\partial\delta u_i}{\partial x_j}\frac{\partial\delta u_i}{\partial x_j}\right\rangle\right] - \frac{\nu}{V}\int_{S_d}v_k\frac{\partial\delta u_i}{\partial x_j}\frac{\partial\delta u_i}{\partial x_j}n_k dS \\ &- \nu\langle u_k\rangle\frac{\partial}{\partial x_k}\left[\alpha_c\left\langle\frac{\partial\delta u_i}{\partial x_j}\frac{\partial\delta u_i}{\partial x_j}\right\rangle\right] + \frac{\nu}{V}\langle u_k\rangle\int_{S_d}\frac{\partial\delta u_i}{\partial x_j}\frac{\partial\delta u_i}{\partial x_j}n_k dS\end{aligned} \tag{A10}$$

Volume averaging the pressure term:

$$\begin{aligned}\overline{-2\frac{\nu}{\rho}\frac{\partial\delta u_i}{\partial x_j}\frac{\partial}{\partial x_j}\frac{\partial\langle P\rangle+\delta P}{\partial x_i}} &= -2\frac{\nu}{\rho}\frac{\partial}{\partial x_i}\left[\alpha_c\left\langle\frac{\partial\delta u_i}{\partial x_j}\frac{\partial\delta P}{\partial x_j}\right\rangle\right] \\ &+ \frac{2\nu}{\rho V}\int_{S_d}\frac{\partial\delta u_i}{\partial x_j}\frac{\partial\delta P}{\partial x_j}n_i dS + 2\frac{\nu}{\rho}\alpha_c\left\langle\frac{\partial^2\delta u_i}{\partial x_i\partial x_j}\frac{\partial\delta P}{\partial x_j}\right\rangle\end{aligned} \tag{A11}$$

An order of magnitude analysis shows that the last term on the RHS of Eq. A11 is shown to be negligible compared with the first term on the RHS (for details see Schwarzkopf[17]), thus the above equation is reduced to:

$$\begin{aligned}\overline{-2\frac{\nu}{\rho}\frac{\partial\delta u_i}{\partial x_j}\frac{\partial}{\partial x_j}\frac{\partial\langle P\rangle+\delta P}{\partial x_i}} &= -2\frac{\nu}{\rho}\frac{\partial}{\partial x_i}\left[\alpha_c\left\langle\frac{\partial\delta u_i}{\partial x_j}\frac{\partial\delta P}{\partial x_j}\right\rangle\right] \\ &+ \frac{2\nu}{\rho V}\int_{S_d}\frac{\partial\delta u_i}{\partial x_j}\frac{\partial}{\partial x_j}\delta P n_i dS\end{aligned} \tag{A12}$$

The shear term is simplified for incompressible flow:

$$\begin{aligned}\overline{2\frac{\nu}{\rho}\frac{\partial\delta u_i}{\partial x_j}\frac{\partial^2\tau_{ik}}{\partial x_j\partial x_k}} &= \overline{2\nu^2\frac{\partial\delta u_i}{\partial x_j}\frac{\partial}{\partial x_j}\frac{\partial}{\partial x_k}\left\langle\frac{\partial u_i}{\partial x_k}+\frac{\partial u_k}{\partial x_i}\right\rangle} \\ &= \overline{2\nu^2\frac{\partial\delta u_i}{\partial x_j}\frac{\partial}{\partial x_j}\frac{\partial^2 u_i}{\partial x_k^2}}\end{aligned} \tag{A13}$$

Volume averaging the shear term (details are shown in Schwarzkopf[17]):

$$\begin{aligned}\overline{2\nu^2\frac{\partial\delta u_i}{\partial x_j}\frac{\partial^2}{\partial x_j\partial x_k}\left(\frac{\partial\langle u_i\rangle}{\partial x_k}+\frac{\partial\delta u_i}{\partial x_k}\right)} &= \nu^2\frac{\partial^2}{\partial x_k^2}\left[\alpha_c\left\langle\frac{\partial\delta u_i}{\partial x_j}\frac{\partial\delta u_i}{\partial x_j}\right\rangle\right] \\ &- \frac{\nu^2}{V}\frac{\partial}{\partial x_k}\int_{S_d}\frac{\partial\delta u_i}{\partial x_j}\frac{\partial\delta u_i}{\partial x_j}n_k dS - 2\nu^2\alpha_c\left\langle\left(\frac{\partial^2\delta u_i}{\partial x_j\partial x_k}\right)^2\right\rangle \\ &- \frac{2\nu^2}{V}\int_{S_d}\frac{\partial\delta u_i}{\partial x_j}\frac{\partial}{\partial x_j}\frac{\partial\delta u_i}{\partial x_k}n_k dS\end{aligned} \tag{A14}$$

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