Effect of changes in salt content and food thickness on electromagnetic heating of rice, mashed potatoes and peas in 915 MHz single mode microwave cavity

Deepali Jain\textsuperscript{a}, Juming Tang\textsuperscript{a,⁎}, Patrick D. Pedrow\textsuperscript{b}, Zhongwei Tang\textsuperscript{a}, Shyam Sablani\textsuperscript{a}, Yoon-Ki Hong\textsuperscript{a}

\textsuperscript{a} Biological Systems Engineering Department, Washington State University, Pullman, WA 99164, USA
\textsuperscript{b} School of Electrical Engineering and Computer Science, Washington State University, Pullman, WA 99164, USA

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ABSTRACT

A mathematical model for predicting electromagnetic power dissipation within a rectangular dielectric slab heated by equal intensity 915 MHz plane waves from top and bottom was developed. A dimensionless parameter (J-T number) which is a combination of the loss factor (\(\varepsilon_r''\)), dielectric constant (\(\varepsilon_r'\)) and food thickness (\(L\)) was proposed. This unique number provided direct insight into the relationship between food dielectric properties, thickness, product temperature, and thermal lethality. For the validation tests, mashed potatoes, peas and rice samples with 0–2% salt content were processed in a pilot scale microwave assisted thermal sterilization (MATS) system. In each food, the combination of dielectric properties and thickness which gave J-T number of 1.8–2.2 at 100–121°C, provided the highest lethalities. MATS is a novel commercial technology being adapted in the food industry. A qualitative assessment of the combined effect of food properties on lethality using this model will be helpful in process development for MATS systems.

1. Introduction

Interaction of electromagnetic waves with a material is quantified by its electrical and magnetic properties known as dielectric permittivity (\(\varepsilon^*\)) and magnetic susceptibility (\(\mu^*\)) (Balanis, 2005). For non-magnetic materials such as foods, dielectric permittivity is the primary parameter which affects the electromagnetic energy absorption, reflection, transmission, and dissipation within the food (Chandrasekaran, Ramanathan, & Basak, 2013; Fakhouri & Ramaswamy, 1993). Dielectric permittivity is defined as a complex number with real and imaginary parts (Tang, 2015):

\[
\varepsilon^* = \varepsilon'_r - j\varepsilon''_r
\]

where \(\varepsilon'_r\) is dielectric permittivity of vacuum (8.85 × 10^{-12} \text{ F/m}), \(\varepsilon'_r\) & \(\varepsilon''_r\) are relative dielectric constant and loss factor, often simply referred to as dielectric constant and loss factor, respectively. The storage of electric energy is represented by the dielectric constant whereas thermal conversion of electrical energy is represented by the loss factor in a dielectric material. In microwave heating, dipole rotation and electrical conduction are the dominant loss mechanisms, therefore the overall loss factor is expressed as the combination of dipolar and conduction effects (Peng, Tang, Jiao, Bohnet, & Barrett, 2013):

\[
\varepsilon'' = \varepsilon''_d + \varepsilon''_i
\]

where \(\varepsilon''_d\) is the relative dipole loss, and \(\varepsilon''_i\) is the relative ionic loss. Ionic contribution to the loss factor is described by the following equation:

\[
\varepsilon''_i = \frac{\sigma}{2\pi f\varepsilon_0}
\]

where \(\sigma\) is electrical conductivity of the dielectric material and \(f\) is the frequency of electromagnetic waves.

Dielectric properties of a food material are not only functions of frequency and temperature but also depend on food composition. A small change to a food product formulation may have a significant effect on dielectric constant as well as loss factor which in turn affect the heating behavior of the food. For example, a change in the amount of salt from 0.8 to 1.8% in mashed potatoes increases the loss factor from 38.1 to 95.2 at 121°C and 915 MHz (Guan, Cheng, Wang, & Tang, 2004). Similar effects of salt addition on loss factor has been reported for many food products such as carrots (Peng, Tang, Barrett, Sablani, & Powers, 2014), potato slices (Wang, Zhang, Mujumdar, & Jiang, 2011),...
An understanding of the relationship between dielectric properties of foods and their microwave heating performance may help in developing process schedules based on product formation and package geometry. Maxwell’s equations coupled with heat transfer equations were used to predict the microwave heating in different scenarios for 2450 MHz multi-mode (Datta, 1999; Luan, Wang, Tang, & Jain, 2017) and 915 MHz single-mode cavities (Chen, Tang, & Liu, 2008; Jain, Tang, Liu, & Pedrow, 2018). Usually, real scenarios are very complex considering variations in food shape, thickness, rotation or translational movement of food in a microwave cavity, and temperature dependence of product dielectric properties, thermal conductivity, and specific heat. Analytical solutions to these heating problems are often difficult or impossible to obtain. Numerical techniques, e.g., finite element, finite difference, and finite difference time domain methods are used to calculate temperature profiles (Dibben, 2001). The modeling procedures using these methods are very time consuming and requires specialized software and high-performance computers. However, for practical purposes, a simple model which can predict the qualitative temperature profiles based on dielectric and thermal properties may be instrumental. Ayappa and Davis (1991); Zhang and Datta (2001); Nelson and Datta (2001); Barringer, Davis, Gordon, Ayappa, and Davis (1995); Remmen Henk, Ponne, Nijhuis, Bartels, and Kerkhof (1996) solved simplified Maxwell’s equations and the approximation Lambert’s law for thick samples at 2450 MHz. Hossan and Datta (2012) studied the temperature dependence of dielectric properties and analytically calculated the temperature profiles within the food samples for 40, 433, 915 and 2450 MHz. All of these models were developed for multi-mode cavity systems where heating patterns change with the dielectric properties and frequency. The primary focus of their studies was to analyze the change in heating patterns along the thickness, and the location of cold and hot spots for different sizes, shapes, dielectric properties and locations of the food products within the cavity. A single-mode 915 MHz microwave assisted thermal sterilization (MATS) system was developed at Washington State University (Tang, 2015). In the single-mode cavities, pre-packaged food is heated simultaneously by electromagnetic waves and hot water. In each cavity, microwaves enter from two horn shaped applicators, 50% from the top horn and 50% from the bottom horn, through the circulating water to rectangular food packages. The movement of food packages in microwave heating cavity makes physical process complex involving the propagation of microwaves from top and bottom horns, conversion of electrical energy into thermal energy and heat transfer, and changing positions of the food packages. 3-D Maxwell’s equations coupled with heat transfer equations were solved for MATS using Quick-wave software to gain insight into electric field distribution and heating patterns of the food packages (Luan, Tang, Pedrow, Liu & Tang, 2015; Resurreccion et al., 2013). While the heating pattern results obtained by computer simulations were in agreement with the experiments performed using the chemical marker and computer vision assistant technique, our previous computer simulations using workstation (HP-Z800, a dual processor of X5680 (3.33GHz), 96GB DDR3 memory) were very time-consuming. For example, Resurreccion et al. (2013) reported 42 hr per simulation run. In developing a MATS process schedule for a specific product, temperature measurements at the cold spots (in central layer) are performed to estimate lethality to target pathogens. Process conditions depend on the food formulations and package geometry, in particular, food thickness. It is desirable to develop a general understanding of how food properties, especially dielectric properties and food thickness, influence the heating rates at the cold spot in food packages during MATS processes. Such information would serve as general guidance for the efficient development of process schedules. Therefore, in this study, simplified Maxwell’s equations for plane waves were solved analytically to develop general criteria as to how food properties and thickness influence the heating rates at the cold spot inside a food package. Specific objectives of this research were (1) to develop a simplified model for analyzing the effects of dielectric properties and thickness of foods on 915 MHz microwave energy penetration and (2) to validate the model by experimental measurement of temperature in mashed potatoes, peas and rice for a broad range of loss factor values achieved by changing salt content. The analytical model developed in this study is implemented with relatively simple and less time-consuming computer software.

2. Mathematical models

In this study, a simplified Maxwell’s equation was solved to obtain electric field distribution in a rectangular food incident by 915 MHz microwaves. The power dissipation was computed from the electric field distribution which was then used as a source term in the heat transfer equation to obtain temperature profiles.

2.1. Governing equations and mathematical model

The following assumptions were applied to 3-D Maxwell’s equations to obtain a simplified model for one-dimensional propagation in z direction in a 915 MHz single-mode cavity with water immersion:

(i) Electromagnetic waves were assumed to be a plane wave traveling from water to food incident normally in phase on top and bottom sides of a rectangular food (Fig. 1). Using the coordinate system of Fig. 1, the electromagnetic waves in MATS system have non-zero electric fields in y direction and travel in z direction. The magnitude of incident electric field (Ei) was taken as 10^4 V/m on each side. 10^5 V/m is in the range of electric field intensities calculated for MATS system using computer simulations (Luan, Tang, Pedrow, Liu, & Tang, 2016).

(ii) Food products were solid, homogeneous in composition and structure, they obeyed linear material constitutive laws and had relative magnetic permeability μr = 1.

(iii) Only the microwave power term was considered as the heat source while calculating temperature profiles at the central layer of the food. This assumption is reasonable for the short time microwave heating in MATS (3–5 min), as the microwave heating is the dominating heating mechanism at the central layer of the food.

(iv) The heat diffusion rate is much slower than the heat generation from microwaves, and the thermal conductivity values were

![Fig. 1. Schematic of a food slab immersed in water heated by 915 MHz plane waves. Electromagnetic waves are incident in phase from top and bottom of the slab of height L. Dielectric properties, i.e., dielectric constant (εr) and loss factor (δ) of water and food are denoted by subscripts w and f, respectively.](image-url)
considered negligible. The heating rates were assumed to be a function of volumetric heat capacity and power dissipation.

Using the above assumptions, simplified time independent Maxwell’s equation for a uniform plane wave propagating in z direction can be written as (Sadiku, 2014):

\[
d\frac{\partial E}{\partial t} + \gamma E = 0
\]

where \( E \) is electric field intensity and \( \gamma \) represents the propagation constant expressed as

\[
\gamma = \alpha + j\beta
\]

where \( \alpha \) and \( \beta \) are attenuation constant and phase constant, respectively. They are related to dielectric properties and frequency as:

\[
\alpha = 2\pi f \sqrt{\epsilon_r} \left( \frac{\epsilon_r - \epsilon_0}{\epsilon_r + 2\epsilon_0} \right) \left( 1 + \frac{\epsilon_r - \epsilon_0}{\epsilon_r + 2\epsilon_0} \right)^{-1}
\]

\[
\beta = 2\pi f \sqrt{\mu_r} \left( \frac{\mu_r - \mu_0}{\mu_r + 2\mu_0} \right) \left( 1 + \frac{\mu_r - \mu_0}{\mu_r + 2\mu_0} \right)^{-1}
\]

where \( f \) is frequency (Hz), and \( c \) is the speed of light in vacuum \((3 \times 10^8 \text{ m/s})\). The boundary conditions for a uniform plane wave traveling from water into the food are given by (Sadiku, 2014):

\[
\hat{n} \times (E_{\text{water}} - E_{\text{food}}) = 0
\]

\[
\hat{n} \times (H_{\text{water}} - H_{\text{food}}) = 0
\]

where \( H \) is magnetic field intensity, and subscript \( w \) and \( f \) represent the water and food, respectively. The solution of the equation for the electric field inside the food a distance \( z \) from the interface is given by:

\[
E = \frac{\eta_0}{\eta_w + \eta_f} \left( \eta_w \epsilon_r + \epsilon_0 \left( \frac{\epsilon_r - \epsilon_0}{\epsilon_r + 2\epsilon_0} \right) \right) \left( 1 + \frac{\epsilon_r - \epsilon_0}{\epsilon_r + 2\epsilon_0} \right)
\]

where \( \eta_0 \) is the incident electric field intensity, \( L \) is food thickness, \( T_{\text{trans}} \) and \( R_{\text{refl}} \) are the transmission and reflection coefficient respectively in lossy medium given as (Ayappa & Davis, 1991):

\[
T_{\text{trans}} = \frac{2\eta_f}{\eta_w + \eta_f}
\]

\[
R_{\text{refl}} = \frac{\eta_0 - \eta_w}{\eta_w + \eta_f}
\]

where \( \eta_w \) and \( \eta_f \) are the complex intrinsic impedance of water and food, respectively.

2.2. Electromagnetic power dissipation and heat transfer

The power flux for a propagating electromagnetic wave is represented by the time average Poynting vector \( S \) over one period given by (Sadiku, 2014):

\[
\bar{S} = \frac{1}{2} (E \times H^*)
\]

Power dissipation in a medium is given by Poynting theorem expressed in point form as (Ayappa & Davis, 1991):

\[
V \cdot S = -\frac{1}{2} \omega \epsilon_0 \epsilon_r |E|^2 + \frac{\mu_0}{2} |H|^2 + \frac{\omega \epsilon_r}{2} E \cdot E
\]

where \( \omega = 2\pi f \) is frequency in radians, \( \mu_0 \) is permeability of vacuum.

The theorem states that the net power flow across a surface enclosing a volume is sum of the power dissipated in the medium (real part) and the stored electric and magnetic fields (imaginary part). Thus, the dissipated microwave power per unit volume is given by:

\[
P(\omega) = Re(-V \cdot S)
\]

\[
P(\omega) = 2\pi \int_0^\infty f \epsilon_0 \epsilon_r |E|^2
\]

hence with a knowledge of medium properties and electric field intensity the local power dissipation is calculated from Eq. (16).

A general form of energy transfer equation for a solid is given by:

\[
V \cdot k \nabla T + P(\omega) = \rho C_v \frac{\partial T}{\partial t}
\]

where \( T \) is temperature (°C), \( k \) (W/(m K)) and \( C_v \) are thermal conductivity, density and specific heat respectively. \( P(\omega) \) is heat source term given by Eq. (16) for the current heating case.

The solution of Eq. (17) provides transient temperature profiles in foods. As per assumption (iv), the conduction term is dropped in the calculations to give heat transfer equation as:

\[
P(\omega) = \rho C_v \frac{dT}{dt}
\]

If \( T_0 \) is the initial temperature (at \( t = 0 \)), temperature (°C) after \( t \) minutes of heating is given as:

\[
T = \frac{P(\omega) \times t + T_0}{\rho C_v}
\]

The microwave power term in the above equation was considered independent of temperature. The purpose of this model was not to predict exact temperatures at the cold spot locations during MATS processing. Rather the temperature profiles obtained from this simplified model were used to approximate the general behaviour of change in lethality with change in food formulations in relation to dielectric properties and food thickness. The solutions to Eqs. (16) and (19) were obtained using MATLAB2016.

3. Experimental validation

3.1. Sample preparation

For the validation of the mathematical model, three types of samples were chosen as model foods to represent different food products, i.e., cereal-rice, vegetable-potatoes and legume-peas. Rice (Auksoonsri, Tang, Lin, & Songsermpong, 2018), mashed potatoes (Pandit, Tang, Mikhaylenko, & Liu, 2006) and peas (Bornhorst, Tang, Sablani, Barbara-Canos, & Liu, 2017) were chosen based on the previous studies for heating pattern determination using computer vision assistant technique (Pandit, Tang, Liu, & Mikhaylenko, 2007). Sodium chloride salt (food grade, Morton, Chicago, IL) at different levels was used to change the values of loss factor. Mashed potato samples with 0, 0.1, 0.2, 0.5, 1 and 2% salt were prepared. Salt was added to the water followed by mixing 20% of dried potato flakes (Oregon Potato Company, Pasco, WA) in the salt solution. Pea samples were prepared by using dried white peas (Swad, Skokie, IL). The peas were soaked overnight at room temperature and were boiled for 60 min with a pea to water ratio of 1:1.5. Peas were ground and filled inside the trays. 0, 0.1, 0.2, 0.5, 1 and 2% salt content was used for pea samples. In preparing rice samples, medium grain rice (Nishiki, Los Angeles, CA) was used. Rice grains were pre-cooked with rice to water ratio of 1:1.2 at 95°C for 40 min. 0, 0.2, 0.5, 1, 1.5 and 2% salt was added to the water before cooking. 0.5% \( \nu \)-ribose was used as a precursor of chemical marker M2 in all the samples for heating pattern determination using computer vision assistant method (Pandit et al., 2006). MATS cavities were filled with hot water with a dielectric constant of 58. Thus to minimize the reflection of microwaves from the food-water interface, water content of the samples was adjusted to obtain the dielectric constant value in the range of 50–60 (Pathak, Liu, & Tang, 2003; Tang, Liu, Pathak, & Eugene, 2006).
3.2. Food properties measurement

Dielectric properties of all samples were measured using an HP 8752C Network Analyzer and 85070B open-end coaxial dielectric probe (Agilent Technologies, Santa Clara, CA) in the microwave frequency range: 300–3000 MHz. The probe was calibrated before measuring each sample with a short circuit (a gold-plated shorting block), an open circuit (air), and pure water at 25°C. After calibration, a sample was filled into a cylindrical test cell. The temperature of the cell was controlled by circulating an oil from an oil bath to the sample holder. Detailed design of the system was given by Wang, Wig, Tang, and Hallberg (2003). The measurements were carried out at temperatures of 60, 70, 80, 90, 100 and 110°C.

Specific heat of the samples was measured using a differential scanning calorimeter (DSC, Q1000, TA Instruments, New Castle, DE) as described in Sablani, Bruno, Kasapis, and Symaladevi (2009). 15–20 mg samples were sealed in aluminum pans and were equilibrated at room temperature. The samples were heated from room temperature to 50°C at 5°C/min and equilibrated for 10 min. The samples were scanned from 50 to 120°C at a rate of 5°C/min. Heat capacity value was divided by the weight of the sample to obtain the specific heat values for the samples. Specific heat values were taken at 60°C which was the initial temperature of the product at the entrance of microwave cavity. All the experiments were performed in duplicates.

3.3. Microwave assisted thermal sterilization (MATS) processing

Mashed potato, rice, and pea samples each having 6 varying levels of salt, altogether 18 recipes were prepared. 300 g of each sample was filled in 10 oz. rectangular tray (95 mm (x) × 140 mm (y) × 25 mm (z)) and vacuum sealed with 65 mbar, 200°C and 12 s dwell time with a Multivac T-200 sealer (Multivac Inc., Kansas City, MO). The thickness of the food inside the tray (L) was measured for all three types of foods.

A rigorous and comprehensive experimentation process was used to obtain statistically meaningful, and reliable results for MATS. First, experiments were conducted to select the appropriate amount of α-ribose (M2 precursor) in the samples to obtain color changes in the samples which could be detected by the computer vision assistant technique. Preliminary tests were conducted to develop the process schedule to achieve desired lethality for shelf stable products. After preliminary tests, heating patterns and cold spot locations in each recipe were first determined. Then temperature profiles were measured at the cold spot locations to calculate the lethality values in consequent test runs.

The pilot scale MATS system was able to process 48 food packages every run. Firstly, for the heating pattern determination, three separate runs were conducted to process the three food types. In each test, 10 dummy food packages were placed at the two ends of the conveyor belt, and 7–9 packages of each salt level were placed in between. A total of 126 food packages (seven for each recipe) were cut horizontally in the middle layer and images were taken using a camera set-up as described previously in Pandit et al. (2007). The images were analyzed by computer vision assistant technique to obtain the heating patterns and cold spot locations. The computer vision assistant method is based on the color change of the samples after processing due to chemical marker M2 generation in the browning reaction between α-ribose and amino acids (Pandit et al., 2006). Software converts the whole image into red, green and blue colors, depending upon the intensity of the thermal treatment. Areas which receives more thermal treatment are converted to red and least are turned to blue. Medium heat treatment areas are converted to green (Pandit et al., 2007). Samples with different salt levels were analyzed together at the same color scale by the software to compare the heating patterns for each kind of food.

For temperature measurements in the pre-determined cold spot locations, six runs (48 packages per run) were conducted on our MATS system using same process schedule. In one run, samples with three different salt levels for the same food were processed. For example, in one run, mashed potato samples with 0, 0.1 and 0.2% salts were treated, and samples with 0.5, 1 and 2% salt levels were treated in another run. For one recipe, temperature measurement was performed in triplicates. Prior to each run, mobile metallic Ellab temperature sensors (Ellab Inc., Hillerød, Denmark) were placed at the cold spot locations in the trays, as described in Luan, Tang, Pedrow, Liu, and Tang (2015). The food trays with temperature sensors were separated by the trays filled with same food without sensors to avoid any interference in the temperature measurements. Ellab sensors recorded the temperature at every 2 s. After completing the processing, the temperature data from the sensor was downloaded in the computer and the cumulative lethality ($F_t$) value was calculated using the following equation (Holdsworth, 1997):

$$F_t = \int_0^t (10^{(T_0 - T) / 10}) dt$$

(20)

where $T$ is the temperature (°C) at the cold spot and $t$ is the time in minutes.

The MATS system used in this study had four sections, i.e., pre-heating, microwave heating, holding and cooling. Each section had a circulating water heated to a pre-set temperature controlled by a plate heat exchanger. The food samples were placed on a mesh conveyor belt and loaded to the preheating section. After preheating, the samples were moved through the microwave heating section then the holding section and finally to the cooling section. System description is given in detail by Resurreccion et al. (2013). The water temperature in preheating, heating and cooling sections were set at 61, 124, 124 and 23°C, respectively. The power setting for the four microwave heating cavities in the MATS system were 6, 5, 3.0 and 3.2 kW based on previous tests to reach the desired thermal lethality. Food was pre-heated for 35 min in the system, moved through the microwave heating section over 4.5 min. The food trays then moved through the holding section over 4.7 min and cooled down in the cooling section for 5 min and unloaded.

4. Results and discussion

4.1. Analytical results

4.1.1. Electromagnetic power dissipation along the food thickness

This section presents the analysis of the electromagnetic power dissipation along the microwave propagation direction (z) as a function of food thickness (L) and dielectric properties. The equal intensity (1 V/mm) in phase 915 MHz microwaves was applied to top, and bottom surfaces of rectangular food placed in water at 124°C. The relative dielectric constant and relative loss factor of water at this temperature and 915 MHz are 56 and 3, respectively (Tang, 2015).

A typical thickness for single meal trays or pouches used in the MATS system is in the range of 16–25 mm. Thus the effect of food thickness on the power dissipation per unit volume P(x) were calculated for food thickness (L) = 15, 18, 22, 24, 26 and 30 mm using Eqs. (10) and (16). The calculations were made for dielectric constant ($\varepsilon_r$) values of 40, 50, 60 and 70 and loss factor ($\tan \delta$) values of 5, 30, 100 and 150. These dielectric properties were chosen based on the values obtained at processing temperatures for various food products with different formulations (Wang et al., 2003).

Fig. 2 shows the power dissipation profiles in foods with different thickness as influenced by dielectric constants at loss factor ($\tan \delta$) = 30. The loss factor value was chosen based on the experience for optimum penetration of 915 MHz microwaves in MATS (Wang et al., 2003). As the food thickness increases, the power penetration to the central layer of the food decreases from $1.7 \times 10^6$ W/m² for $L = 15$ mm to 0.95 × 10^6 W/m² for $L = 30$ mm when $\varepsilon_r = 40$. For the foods with the thickness 15–24 mm (Fig. 2 a-d), a single broad peak is observed around the central layer. However, as the food thickness increases above
24 mm (Fig. 2e & f), multiple resonances, i.e., multiple peaks and dips were observed which may result in alternate hot and cold spots along the food thickness. For food with thickness 30 mm (Fig. 2f), power dissipation at the surface is almost same as that of the central layer, whereas, for all other thicknesses, power dissipation is more at the central layer compared to at the surface (Fig. 2a-e).

For a given thickness, food dielectric constant doesn’t affect the overall number of resonances of the power distribution. In the packages with comparatively shallow depths ($L = 15, 18$ and $22$ mm) overall power dissipation decreases as the dielectric constant value of the food increases (Fig. 2a-c). For $L = 24$ and $26$ mm (Fig. 2d & e), power dissipation follows a similar trend in central region ($0.2 < z/L < 0.8$). Near the surface a maximum power dissipation is obtained when food had $\varepsilon_r = 70$. For $L = 30$ mm (Fig. 2f), power dissipation at the surface and the central region (region of maxima) increases with increasing dielectric constant of the food, but it decreases in the areas close to minima.

Fig. 3 shows power dissipation distribution along the microwave propagating direction ($z$) for foods with loss factors 5, 30, 100 and 150 in packages of various thickness and dielectric constant ($\varepsilon_r$) = 50. For foods with loss factor values of 5 and 30, the sum of the forward moving and backward moving electromagnetic waves within the samples leads to maximum power dissipation at the central layer for all thicknesses ($L = 15$–$30$ mm, Fig. 3a-f). For loss factors 100 and 150, most of the microwave power is absorbed in the surface for samples with thicknesses $L = 18$ mm to $30$ mm (Fig. 3b-f). For $15$ mm thick sample, surface layer absorbs more power than central layer only at $\varepsilon_r = 150$ (Fig. 3a). For $L = 15$ mm and $18$ mm (Fig. 3a & b), when loss factor is 150, the power declines rapidly into the foods reaching a minimum at the central layer. But the dissipated power in the center was still higher in the foods with loss factor = 150 compared to foods with loss factor = 5 where most of the microwave power is absorbed around the central area.

Our results showed that food with thickness between 15 and 24 mm and a range of food dielectric properties ($\varepsilon_r$ = 40-70, $\varepsilon_r$ = 30-70) is suited for MATS processing; where circulating water heats the food surface, and microwaves provide the maximum dissipation at the center of the food packages, resulting in the relatively uniform heating profiles in $z$ direction. Higher loss factor values such as $\varepsilon_r > 100$ (equivalent to 2% added salt in mashed potatoes at above 80 °C (Guan et al., 2004)) would lead to most of the power absorption at the surface, which in turn would result in severe non-uniform heating and therefore not recommended for MATS processing.

4.1.2. Electromagnetic power dissipation at the central layer

This section analyzes the effect of a change in dielectric properties of food products on the power absorption profile at the central layer ($L/2$) which is a cold spot location in food packages during MATS processing (Luan, Tang, Pedrow, et al., 2015; Resurreccion et al., 2013; Tang, 2015). Fig. 4 shows the volumetric power dissipation at $z = L/2$ as a function of loss factor for $L = 15, 20$ and $25$ mm thick food and dielectric constants ($\varepsilon_r$) = 40, 50, 60 and 70.

The microwave power dissipation in the central layer first increased with the increase in loss factor of the food. After reaching a maximum value, the power dissipation decreases. This trend is same for all values of dielectric constants and food thickness. Eq. (16) shows that microwave power absorption is directly proportional to the loss factor and square of electric field intensity. As discussed in the previous section, the electric field intensity at the central layer of food depends on the penetration ability of the microwaves. The penetration depth is inversely proportional to food loss factor, and less microwaves penetrate to the central layer inside food with higher values of loss factor. Therefore, after reaching a maximum power dissipation, further increase in the loss factor of food would contribute to more dissipation but it would be limited to the surface of the food and the central layer would be heated less.

The effect of the dielectric constant is complex, however, but not as drastic as loss factor. For low loss factor up to 30, microwave power dissipation decreases as the dielectric constant increases. This

Fig. 2. Influence of food thickness ($L$) on power distribution along the wave propagation direction ($z$) in a food with a loss factor ($\varepsilon_r$) = 30 and dielectric constants ($\varepsilon_r$) of 40 (−−), 50 (—), 60 (…) and 70 (−•−).
relationship is similar to that observed in low loss dielectric materials such as water where the material is relatively transparent to microwaves (Tang, 2015). In those cases, low values of dielectric constant provide higher penetration depths (Komarov & Tang, 2004; Tang, 2015). However, for high loss factor foods, power dissipation values increase with the dielectric constant.

The hot water filled cavity and symmetrical design of the MATS system provides cold spot location at the central layer inside food packages (Chen et al., 2008; Jain et al., 2017; Luan et al., 2015; Resurreccion et al., 2013; Tang, 2015). Therefore, it is desirable to obtain higher microwave power dissipation at the central layer compared to surface layers to increase the heating efficiency and uniformity of food packages processed in MATS. Upon analyzing microwave power absorption at the central layer of the food it was observed that the value of loss factor which provides the maximum power absorption at the central layer varies with the product thickness as well as the dielectric constant of the foods (Fig. 4). For a given product thickness, the value of loss factor at which the maximum power dissipation ($P_{\text{max}}$) occurs increases with increasing dielectric constant. Whereas for a given dielectric constant, the value of loss factor decreases with increasing thickness to maximum power dissipation ($P_{\text{max}}$). For example in product with thickness ($L$) of 15 mm, and dielectric constant ($\varepsilon_r$) = 40, $P_{\text{max}}$ is obtained when loss factor ($\varepsilon_r''$) is 49. For same thickness ($L = 15$ mm), if dielectric constant is 70, the loss factor value to obtain maximum power dissipation changed to 59. For $L = 20$ mm and $\varepsilon_r'' = 40$, loss factor should be 34, and at $\varepsilon_r'' = 70$, the loss factor should be 43 to obtain maximum dissipation. Similarly, for $L = 25$ mm, at $\varepsilon_r'' = 40$, loss factor should be 26 and at $\varepsilon_r'' = 70$, the loss factor should be 34 for maximum power dissipation. Based on the above observations, it appears to be a complicated process to estimate the optimum set of food properties and thickness to achieve the maximum dissipation at the central layer in MATS. Thus, to estimate the overall influence of these factors (thickness, dielectric constant and loss factor) on the microwave power absorption, we proposed a dimensionless number as:

$$
\frac{2\pi\beta\varepsilon_r''}{c\sqrt{\varepsilon_r'}} = \frac{2\pi\beta\varepsilon_r''}{c\sqrt{\varepsilon_r'\varepsilon_r'' + \varepsilon_r''^2}} = \frac{2\pi\beta\varepsilon_r'}{\varepsilon_r'' \lambda_{\text{air}}}
$$

where $c$ is the speed of microwave propagation in the air ($3 \times 10^8$ m/s) and $\lambda_{\text{air}}$ is the wavelength of microwaves in free space. In this number, the loss factor ($\varepsilon_r''$) incorporates the microwave power absorption, dielectric permittivity term ($\varepsilon_r'$) incorporates the transmission and reflection of microwaves, and thickness ($L$) of the food includes the penetration effect. Fig. 5 shows the plot of power absorption in the central layer vs. this number. The power dissipation in the central layer of the food package reaches a maximum, corresponding to the highest heating rates when the dimensionless number is in between 1.8 and 2.2 for all thicknesses and dielectric constants. This value provides maximum heating rates as a function of dielectric properties and thickness, in the central layer of a rectangular dielectric slab incident by in phase and equal intensity 915 MHz microwaves from top and bottom. We refer this unique combination of parameters as Jain-Tang (J-T) number.

4.1.3. Temperature profiles

The absorbed electromagnetic power obtained from simplified Maxwell’s Eq. (16) was used as a heat source in the energy Eq. (19) for calculating temperature distribution within the food. Specific heat $C_p$ and density $\rho$ of a substance are important factors for calculating the heating rates and temperature achieved. A material with a lower value of volumetric specific heat ($\rho \times C_p$) requires less amount of energy for a unit change in temperature. Thus, even if power dissipation within two types of food is equal, one food might get heated faster than the other, depending upon the volumetric specific heat values.

Fig. 6 shows the temperature calculated for product center ($z = L/2$) in 18 mm thick foods with different volumetric specific heat values when the food is exposed to 915 MHz microwaves for 3 min. Electric field intensity of 1 V/mm was applied to on top and bottom of the food package. The temperature profiles are similar to the power dissipation patterns; however temperature values depends upon the volumetric
Fig. 4. Influence of dielectric properties on power dissipation profiles at the central layer of 15, 20 and 25 mm thick food.

Fig. 5. Microwave power dissipation at the central layer of a package vs J-T number for foods with dielectric constant ($\varepsilon_r'$) = 40 (−−), 50 (−−−), 60 (...) and 70 (—) and package thickness $L = 15, 20$ and 25 mm.

Fig. 6. Temperature as a function of J-T number and volumetric specific heat at the central layer of 18 mm thick food heated by 915 MHz microwaves for 3 min when incident by 1 V/mm electric field on both faces of food with dielectric constant ($\varepsilon_r'$) = 40 (−−), 50 (−−−), 60 (...) and 70 (—).
specific heat. For the same value of the J-T number and dielectric constant, there is a difference of 10–30°C when volumetric specific heat changes from 4.5 to 2.5 MJ/(m³·°C). Lethality is a cumulative effect of time and temperature. Thus even small temperature differences lead to more pronounced changes in $F_0$ value. An advantage of this temperature approximation is that it will aid in the assessment of the effect of a change in product formulation on the heating performance of food packages within a MATS system.

4.2. Food properties and analytical temperature distribution in mashed potatoes, peas and rice

For MATS processing, 300 g of mashed potatoes ($\rho = 1.1 \times 10^3$ kg/m³), peas ($\rho = 1.3 \times 10^3$ kg/m³) and rice ($\rho = 0.96 \times 10^3$ kg/m³) were vacuum sealed at 65 millibar. The rectangular slabs obtained after the vacuum provided a thickness of $L = 23$ mm for mashed potatoes, 18 mm for peas and 25 mm for rice in a 140 mm long and 95 mm wide tray. Tables 1 and 2 show the dielectric constant, loss factor and J-T number at 915 MHz in the temperature range of 60–121°C. Table 3 lists the specific heat values for peas, mashed potatoes, and rice, respectively for different salt levels. Loss factor of food increased with increasing salt content for all three foods. The increase in loss factor of the samples with increasing salt content was due to the increase in ionic contribution of the loss factor due to dissolved ions of sodium chloride. The loss factor also increased with the temperature, except for rice at 0% salt level for which loss factor does not change with temperature. However, salt does not influence the dielectric constant significantly. These results are in agreement with the previous finding in different foods (Guan et al., 2004; Jain et al., 2017; Peng et al., 2014; Wang et al., 2011; Zhang et al., 2007). The salt content also does not change the specific heat values of the samples.

The one-dimensional plane wave model developed in this study was used to compare the end temperature of mashed potatoes, peas and rice after microwave heating in MATS, using the experimentally measured physical and dielectric properties of the food samples as inputs. The food is exposed to 1 V/mm 915 MHz microwaves for 4.5 min from the top and bottom faces. The volumetric specific heat of the food sample was calculated by multiplying its density to the average specific heat of all salt levels. It was 4.77 MJ/(m³·°C) for peas, 3.83 MJ/(m³·°C) for mashed potatoes and 3 MJ/(m³·°C) for rice. Fig. 7 shows analytical calculations for temperature achieved in mashed potatoes, peas and rice vs. J-T number (0–6) corresponding to a wide range of loss factors ($\epsilon'' = 0$–300). Even though rice had the largest thickness (25 mm) compared to mashed potatoes (23 mm) and peas (18 mm), the temperature achieved by supplying the same amount of power was more for rice for the same value of J-T number. The higher temperature

<table>
<thead>
<tr>
<th>Salt content (%)</th>
<th>Temperature (°C)</th>
<th>$\epsilon'$</th>
<th>$\epsilon''$</th>
<th>$2\delta\epsilon''/\sqrt{\epsilon'}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>60</td>
<td>60.6 ± 0.2</td>
<td>191 ± 0.1</td>
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</tr>
<tr>
<td>70</td>
<td>58.7 ± 0.0</td>
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<tr>
<td>80</td>
<td>57.9 ± 0.1</td>
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<tr>
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</tr>
<tr>
<td>110</td>
<td>55.4 ± 0.4</td>
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<td>1.5 ± 0.1</td>
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<tr>
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<td>293.2 ± 2.0</td>
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</tr>
<tr>
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<td>3.9 ± 0.0</td>
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</tr>
<tr>
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<td>1012.0 ± 0.2</td>
<td>4.2 ± 0.0</td>
<td></td>
</tr>
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<td>863.5 ± 5.5</td>
<td>3.7 ± 0.2</td>
</tr>
<tr>
<td>70</td>
<td>61.5 ± 0.6</td>
<td>1010.0 ± 1.4</td>
<td>4.1 ± 0.0</td>
<td></td>
</tr>
<tr>
<td>80</td>
<td>60.7 ± 0.7</td>
<td>1136.6 ± 6.1</td>
<td>4.4 ± 0.2</td>
<td></td>
</tr>
<tr>
<td>90</td>
<td>59.9 ± 0.8</td>
<td>1249.6 ± 6.0</td>
<td>4.7 ± 0.1</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>59.3 ± 1.1</td>
<td>1384.5 ± 5.4</td>
<td>5.0 ± 0.1</td>
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</tr>
<tr>
<td>110</td>
<td>58.8 ± 1.3</td>
<td>1534.5 ± 5.1</td>
<td>5.3 ± 0.1</td>
<td></td>
</tr>
<tr>
<td>121</td>
<td>58.5 ± 1.4</td>
<td>1676.4 ± 4.6</td>
<td>5.5 ± 0.1</td>
<td></td>
</tr>
</tbody>
</table>
values in rice samples are due to the lowest value of volumetric specific heat.

Peas have the highest volumetric specific heat, yet for the J-T number < 2.2 (in the low loss factor range), heating in peas is more than in mashed potatoes. That is because of the smaller thickness (18 mm for peas vs. 23 mm for mashed potatoes) and lower values of dielectric constants (52 for peas vs. 56 for mashed potatoes) which lead to more microwaves reaching the central layer in the pea packages. For J-T numbers > 2.2 (higher loss factors), the heating is more in mashed potatoes compared to peas, this reversal is observed previously in Fig. 4 where power dissipation decreases with decrease in dielectric constant for high loss factor range. Thus, less power dissipation in the central layer of peas combined with high volumetric specific heat results in less heating of peas compared to mashed potatoes for large values of J-T numbers.

4.3. Experimental results

4.3.1. Heating patterns

Fig. 8 shows heating patterns in the central layers of mashed potato, pea and rice samples obtained using computer vision assistant technique after MATS processing. Since the background color of food matrices and reaction rates for chemical marker formation are different among the three types of food samples, the color change of samples cannot be used to compare the heat treatment intensity between the various food types. However, for each type of food, the sample colors with different salt levels were analyzed on the same scale. Among the mashed potato samples, 0.1% salt samples had the highest color change. Among the pea samples, the sample with 0.5% salt indicated the most intense heat treatment. For the rice samples, the sample with 1% salt had the most severe heat treatment. The heating patterns of the samples with different salt contents were similar with two symmetrical hot areas. Similar heating patterns were obtained in the past for processing with MATS using whey protein gels of the various dielectric properties (Luan et al., 2015; Tang, 2015) and have been confirmed by computer modeling (Resurreccion et al., 2015). However, the rice sample at 0% salt was heated more at the edges, and there was less heating at the center. Rice at this salt level had a low loss factor value of 7. The sample was almost transparent to the microwaves; thus there was less microwave power absorption, and the sample was heated mainly by circulating hot water.

4.3.2. Lethality measurement

Fig. 9 shows the lethality values at the cold spots of food samples after MATS processing vs salt content. These data confirm the heating pattern results obtained using chemical marker technique. 0.1% salt

<table>
<thead>
<tr>
<th>Sample</th>
<th>Salt level (%)</th>
<th>$C_p$ (kJ/(kg·°C))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mashed potatoes</td>
<td>0.0</td>
<td>3.46 ± 0.00</td>
</tr>
<tr>
<td></td>
<td>0.1</td>
<td>3.56 ± 0.08</td>
</tr>
<tr>
<td></td>
<td>0.2</td>
<td>3.30 ± 0.02</td>
</tr>
<tr>
<td></td>
<td>0.5</td>
<td>3.46 ± 0.17</td>
</tr>
<tr>
<td></td>
<td>1.0</td>
<td>3.52 ± 0.01</td>
</tr>
<tr>
<td></td>
<td>2.0</td>
<td>3.55 ± 0.07</td>
</tr>
<tr>
<td>Peas</td>
<td>0.0</td>
<td>3.50 ± 0.06</td>
</tr>
<tr>
<td></td>
<td>0.1</td>
<td>3.46 ± 0.03</td>
</tr>
<tr>
<td></td>
<td>0.2</td>
<td>3.65 ± 0.04</td>
</tr>
<tr>
<td></td>
<td>0.5</td>
<td>3.75 ± 0.02</td>
</tr>
<tr>
<td></td>
<td>1.0</td>
<td>3.61 ± 0.09</td>
</tr>
<tr>
<td></td>
<td>2.0</td>
<td>3.67 ± 0.02</td>
</tr>
<tr>
<td>Rice</td>
<td>0.0</td>
<td>3.04 ± 0.01</td>
</tr>
<tr>
<td></td>
<td>0.2</td>
<td>3.26 ± 0.00</td>
</tr>
<tr>
<td></td>
<td>0.5</td>
<td>3.17 ± 0.00</td>
</tr>
<tr>
<td></td>
<td>1.0</td>
<td>3.22 ± 0.02</td>
</tr>
<tr>
<td></td>
<td>1.5</td>
<td>3.25 ± 0.05</td>
</tr>
<tr>
<td></td>
<td>2.0</td>
<td>3.23 ± 0.07</td>
</tr>
</tbody>
</table>

Fig. 7. Temperature at the central layer of mashed potatoes (…), peas (—) and rice (— —) calculated analytically using plane wave model for 4.5 min of microwave heating by 915 MHz ($E_0 = 1$V/mm on each face).
level among mashed potato samples, 0.5% salt among pea and 1% salt among rice samples provided the highest lethality among each product.

Dielectric properties of a food sample are dependent on temperature; therefore dielectric constants and loss factors of food samples were measured from the initial temperature of food in MATS processing (60°C) to sterilization temperature (121°C). Corresponding J-T numbers are given in Tables 1, 2 and 3 for mashed potatoes (L = 23 mm), peas (L = 18 mm) and rice (L = 25 mm), respectively. The samples which had J-T numbers between 1.8 and 2.2 at temperature 100–121°C (0.1% for mashed potatoes, 0.5% for peas and 1% for rice) were heated the most. Foods with a lower amount of salt levels had J-T number values < 1.6 at all temperatures (60–121°C). These food samples accumulated less lethality. Foods with a higher amount of salt (> ≥0.5% for mashed potatoes, > 1.0% for peas and > 1.5% for rice) had a J-T number value of ≥2.0 at 60–121°C. These samples were also heated less. To explain this observation, the pea sample with 1% salt are taken as an example. This sample had J-T number = 2.0–2.2 at 60–70°C. This implies that peas with 1% salt absorbed the maximum power at 60–70°C just at the start of the microwave heating, which raised the temperature further. However, at higher temperatures (80–121°C), J-T number = 2.3–2.9, microwave power absorption in the same product

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**Fig. 8.** Experimental heating patterns in the central layer of (a) mashed potatoes, (b) peas, and (c) rice samples as determined by chemical marker M2. Numbers on the top represent the salt levels from 0 to 2%. Red color represents more heated areas, blue and green represent lowest and medium heat treatments, respectively. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

**Fig. 9.** Lethality ($F_0$) in minutes measured experimentally as a function of salt for mashed potatoes (▲), peas (•) and rice (■).

**Fig. 10.** Lethality ($F_0$) in minutes measured experimentally as a function of loss factor for mashed potatoes (▲), peas (•) and rice (■).
Lower power absorption at the central layer for low loss factor foods.

Dielectric Loss Factor ($\varepsilon_r$) • Less penetration
• More dissipation on surface
• More central heating by increasing up-to 50-70; further increase will cause less heating at central layer
• Lower power dissipation in the central layer and higher surface heating
• Lower heating rates

Volumetric specific heat ($\rho C_p$) • Lower heating rates

was reduced, leading to lower heating rates. Lethality starts to add up when the cold spot temperatures reach > 100 °C. Therefore, the pea sample with 1% salt accumulated lesser lethality compared to samples which absorbed the maximum power in the temperature range 100–121°C or samples which had J-T number 1.8–2.2 at 100–121°C. Thus, a value of dielectric loss factor and J-T number calculated by taking an average of its values at 100, 110 and 121°C was used to compare the heating behavior of food samples.

Fig. 10 summarizes experimental results on the influence of loss factor on lethality at the cold spots in mashed potato, peas, and rice samples. The graph indicates that for peas ($L = 18$ mm), the maximum lethality was achieved when loss factor was 51. Whereas for mashed potatoes ($L = 23$ mm), the highest lethality was obtained at loss factor of 36, and for rice ($L = 25$ mm) loss factor was 30. These experimental results agree with the analytical results (Fig. 4) which showed that with an increase in the thickness, loss factor values decreases to obtain the maximum power dissipation. Fig. 11 shows experimental results of lethality values as a function of the J-T number. The results indicate that maximum heating is achieved at a J-T number between 1.9 and 2.0 for the three different foods. These experimental data matches closely with the calculations (Figs. 4, 6 and 7), which showed that irrespective of the food type, J-T number for the maximum absorption is constant. Rice samples accumulated the more lethality compared to peas and mashed potatoes for the same value of the J-T number because of the low volumetric specific heat of rice (Fig. 7). The shift in heating trends among mashed potatoes and peas was also observed in the experimental results similar to analytical results (Figs. 7 and 11). The close agreement between experimental data and analytical calculations confirms that the model developed in this research works well for predicting microwave power dissipation at the central layer of food packages processed in MATS.

Table 5 summarizes the effects of dielectric properties, thickness and volumetric specific heat of the food on the heating efficiency of food products. Based on the results of this work, the table provides recommendations on values of these parameters to obtain highest power dissipation at the central layer of a rectangular food package during MATS processing.

### Conclusion

An analytical mathematical model was developed for two side electromagnetic incidence on top and bottom of a finite thickness rectangular dielectric material. Influence of change in salt content and thickness on the heating rate of rice, peas and mashed potatoes were studied using the model. Dielectric constant of 49-55 and loss factor in the range of 28-56 provided highest power absorption at the middle layer of 15–25 mm thick food slab when 915 MHz electromagnetic waves were incident. We proposed and validated a dimensionless parameter called J-T number to assess relative heating rates of different food formulations and food packages processed under the same conditions in microwave assisted thermal sterilization (MATS). J-T number between 1.8 and 2.2 are the best for microwave heating. Validation experiments on peas, mashed potatoes, and rice were conducted in the pilot scale MATS system. The process developers can use this dimensionless number to select the process parameters for new formulations in food packages. The model can be used to develop process for multi-compartment trays in which two or more types of foods are processed simultaneously in the same MATS system; the J-T number will be useful to asses which food would heat more than the other and select appropriate formulations or shielding to achieve similar lethality in different compartments. The calculations and experiments were conducted for microwave assisted sterilization conditions. More studies are needed to evaluate the number for the microwave assisted pasteurization system (MAPS).

### Acknowledgments

Authors would like to thank USDA NIFA grant 2016-68003-24840 for the support of this study.