

Microwave Drying of Food and Agricultural Materials: Basics and Heat and Mass Transfer Modeling

Hao Feng · Yun Yin · Juming Tang

Received: 24 October 2011 / Accepted: 4 January 2012
© Springer Science+Business Media, LLC 2012

Abstract Microwave drying is based on a unique volumetric heating mode facilitated by electromagnetic radiation at 915 or 2,450 MHz. The responses of a lossy food product to dielectric heating result in rapid energy coupling into the moisture and lead to fast heating and drying. A significant reduction in drying time in microwave drying is often accompanied by an improvement in product quality, making it a promising food dehydration technology. The need for improvement in engineering design and process optimization for microwave drying has stimulated the development of computer simulation techniques to predict temperature and moisture history and distribution in the product to be dried. In this article, we present the basics of dielectric heating and drying, examine the heat and mass transfer models developed for the simulation of microwave drying processes, and discuss dielectric properties of selected food products as influenced by moisture, temperature, and porosity. In addition, we analyze nonuniform heating caused by the geometry and composition of the product, as well as by the nonuniform distribution of electromagnetic field in a microwave cavity, followed by a discussion on how to improve the microwave heating uniformity. We focus the discussion on heat and mass transfer models developed over

the years to simulate microwave drying, including simplified ones, those based on diffusion theory, and coupled heat and mass transfer analysis with the Philip and de Vries theory, Luikov scheme, Whitaker method, and two-region model. In the end, the determination of the heat source term in the energy equation, numerical schemes used to solve the partial differential equations, and the model validation are also discussed.

Keywords Microwave · Drying · Dehydration · Heat and mass transfer · Modeling · Dielectric heating

Introduction

The term “microwave” refers to electromagnetic radiation in the frequency range of 300 MHz–300 GHz with a wavelength of 1 m–1 mm. It is the propagation of electromagnetic energy through space by means of time-varying electric and magnetic fields [100]. What is unique to microwaves is that as they travel through a lossy medium an increase in temperature throughout the medium can be observed. This has led to many applications in the food and agricultural industries and our daily life. An example is the widespread application of home microwave ovens as a food reheating tool. The global demand for microwave ovens was estimated to be \$9.0 billion in 2006 [72]. The total sales number of microwave ovens in the United States stays at a constant level of approximately 10 million per year [91]. Current estimates hold that 95% of American homes have a microwave oven [40].

Microwave heat generation is caused by an interaction between the microwave and the medium by which part of the electromagnetic energy is dissipated volumetrically in the form of heat. The mechanisms under which the energy

H. Feng (✉)
Department of Food Science and Human Nutrition, University of Illinois at Urbana-Champaign, Urbana, IL, USA
e-mail: haofeng@illinois.edu

H. Feng · Y. Yin
Department of Agricultural and Biological Engineering,
University of Illinois at Urbana-Champaign, Urbana, IL, USA

J. Tang
Department of Biosystems Engineering, Washington State
University, Pullman, WA, USA

dissipates depend on the characteristics of the medium and the frequency of the wave. In the frequency range of our interest, the mechanisms may include free water polarization (γ dispersion), bound water polarization (δ dispersion), Maxwell–Wagner polarization (β dispersion), and ionic conductivity [103]. The quantification of such energy conversion can be realized by utilizing knowledge of electromagnetic theory together with an understanding of the dielectric properties of the medium. The volumetric heat generation in microwave heating distinguishes this technique from other surface heating methods and brings about such advantages as rapid heating and relatively high energy efficiency.

Drying is one of the most energy-intensive unit operations in the process industries. In a drying process, a large amount of energy is needed for sensible heating and phase change of water. The high energy consumption is caused by both the energy needed for water removal via a phase change, as well as the low heat transfer efficiency during the falling rate period of a (hot-air) drying process. In the falling rate period, drying becomes inefficient because the dried product surface yields a layer with high heat and mass transfer resistance, and the temperature gradient could be in the opposite direction of the moisture gradient. In addition, in the falling rate period, the moisture content is low, the water molecules thus have a higher evaporation enthalpy, and the removal of these molecules by evaporation requires higher energy input. When drying foods and agricultural products with conventional hot-air drying methods, this low heat and mass transfer efficiency coupled with a high energy demand for phase change results in prolonged drying time and hence a severe quality degradation in the final products.

The advantages of microwave drying arise from the volumetric heating and internal vapor generation. Heating from the interior of a food product leads to the buildup of an internal vapor pressure that drives the moisture out of the product. This results in a significant reduction in drying time, leading to significantly improved product quality [91]. In microwave drying of foods, a reduction in drying time of up to 25–90% [33, 34, 62, 77, 85, 99] and an increase in drying rate of 4–8 times [11, 62], when compared with convective drying, have been reported (Fig. 1). Other advantages of microwave drying include:

1. A high energy efficiency in the falling rate period can be achieved. It is partially due to the fact that the energy is directly coupled with the moisture, which eliminates the need to transfer heat from the low-moisture surface into the high-moisture interior. It is also the result of an increased driving force for moisture transfer due to the generation of elevated internal vapor pressure.

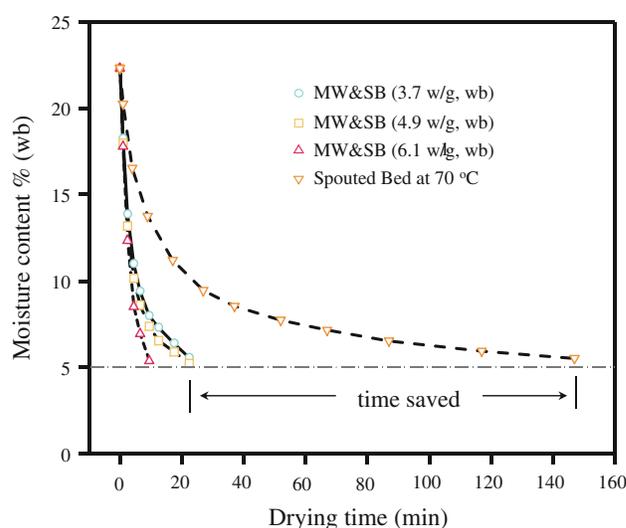


Fig. 1 Microwave and spouted bed combined drying of diced apples at different power level in comparison with spouted bed hot-air drying

2. Case hardening may be avoided or lessened because of the surface moisture accumulation and the liquid pumping phenomena. The unique surface moisture accumulation in microwave heating has been widely reported [[112], [68]].
3. An improvement in product quality can also be achieved. Better aroma retention [33, 34, 70], faster and better rehydration [28, 32, 42, 77], better color retention [32, 99, 109], and higher porosity [107] have been reported for microwave-dried food products.

The rapid and volumetric heat generation of microwaves has been utilized to improve conventional drying processes. But strictly speaking, a stand-alone microwave drying does not exist. Microwaves are used to assist or enhance another drying operation. The most widely used is a combination of microwave heating with hot-air drying [4]. If microwaves are used in the initial stage of a drying, a rapid heating and evaporation often results in a high porous-dried product. More importantly, microwaves can be applied in the final drying stage to reduce the total drying time and reduce shrinkage [37]. Microwave-assisted vacuum drying is one of the successful applications in food dehydration operations, and many research efforts have been made over the years, including those by Lin et al. [58], Gunasekaran [46], and Durance and Wang [31], among others. For particulate foods, the combination of microwave heating and fluidized bed drying, especially with spouted bed has been successfully used to dry heat-sensitive foods [32, 34, 119]. In freeze-drying, the coupling of energy directly with the wet core of a semi-dried food facilitated by microwave radiation has been intensively investigated [19, 30, 118]. Microwave has also been employed to speed up another relatively slow drying

process, osmotic dehydration, under vacuum and at atmospheric pressure [22, 78, 81].

A significantly reduced drying time, improvement in product quality, and other advantages in microwave drying seem to have paved the road for potential widespread applications of this relatively new technology. Industrial adoption of this technique, nevertheless, has been slow. It may be partially due to some unique engineering problems associated with design of microwave drying chambers [69, 90]. A major drawback is the heating nonuniformity, stemmed from the uneven microwave field distribution in the cavity caused by the superposition of the sinusoidal microwaves [121]. This is an inherent characteristic of microwaves. The control of the mass transfer rate raises another issue. In some cases, the mass transfer rate is too high, causing puffing and even disintegration of the product. Another factor negatively impacting the adoption of microwave drying is the relatively high cost and low life span of the magnetron. Methods have been developed over the years to improve microwave heating uniformity. The remaining obstacles for the application of microwave drying could be a lack of understanding of the microwave interaction with product, a scarcity in dielectric property data, and a lack of an effective means to predict the moisture and temperature history and distribution during microwave drying. The reluctance for industry to adopt new technology also plays an important role hindering the application of microwave drying. Efforts have to be made to advance our understanding of the interactions between microwaves and food products. Means to predict the temperature and moisture distributions using coupled heat and mass transfer analysis has to be developed, which will help to enhance our understanding of the underlying physics and develop better strategies for the control of microwave drying processes.

Interaction Between Microwave and Lossy Materials

The interaction of the electric field component of microwave energy with mainly polar molecules (dipolar polarization) and ions (ionic conduction) in food products is manifested in the form of heat generation. When exposed to microwave radiation, water molecular dipoles try to align in the direction of the applied electric field. As the applied field oscillates, the dipoles attempt to follow the oscillations, and the electromagnetic energy is converted to thermal energy through molecular friction and dielectric loss. In addition, charged ions oscillate back and forth under the influence of alternating electric fields. The ions bump into their neighboring molecules or atoms, resulting in agitation or motion and creating heat. The ionic conduction at relatively low microwave frequencies has a

much stronger effect than the dipolar rotation in heat generation. Therefore, a product containing ions, such as salts, will be heated up more rapidly by microwaves than those containing only water.

Dielectric Property as Influenced by Moisture, Temperature, and Other Factors

Dielectric properties of a material determine how much heat can be produced when it is exposed to microwave radiation and the way the heat is generated. The electrical parameter of a dielectric material that defines its interaction with electromagnetic fields is the complex permittivity $\varepsilon = \varepsilon' - j\varepsilon''$. It consists of a real part, ε' , called the dielectric constant, and an imaginary part, the loss factor, ε'' . The dielectric constant represents the ability of a material to store the electric field energy in the material, while the loss factor determines how much energy is dissipated in the form of heat. Microwave heat generation at any point in a material is given by:

$$P = 5.56 \times 10^{-4} \times f\varepsilon''E^2 \quad (1)$$

where P is the conversion of microwave energy into thermal energy per unit volume (W/cm^3), f is frequency (GHz), ε'' is relative (to air) loss factor (dimensionless), and E is the local electric field intensity (V/cm). The constant 5.56×10^{-4} has a unit of faraday/cm. When microwave travels through a material, its intensity decreases because part of its energy dissipates in the form of heat. The decay can be measured by penetration depth D_p . The penetration depth is defined as the distance from a product surface where the available power drops to 37% ($1/e$) of its surface value. D_p can be estimated by:

$$D_p = \frac{\lambda_0}{2\pi(2\varepsilon')^{0.5}} \left[\left(1 + \left(\frac{\varepsilon''}{\varepsilon'} \right)^2 \right)^{0.5} - 1 \right]^{-0.5} \quad (2)$$

where λ_0 is the wavelength in free space (m). From Eqs. 1 and 2, it can be seen that at a given frequency, microwave heating depends on the loss factor ε'' and the local electrical field intensity E . The penetration depth, on the other hand, is a function of both dielectric constant and loss factor. What makes microwave heating complicated is the fact that dielectric properties are dependent upon temperature and moisture.

Moisture Dependency

Moisture content is a very important factor that affects dielectric properties. For example, the dielectric constant for free water at room temperature is as high as 78, while for solid materials its value is around 2 [89]. In general,

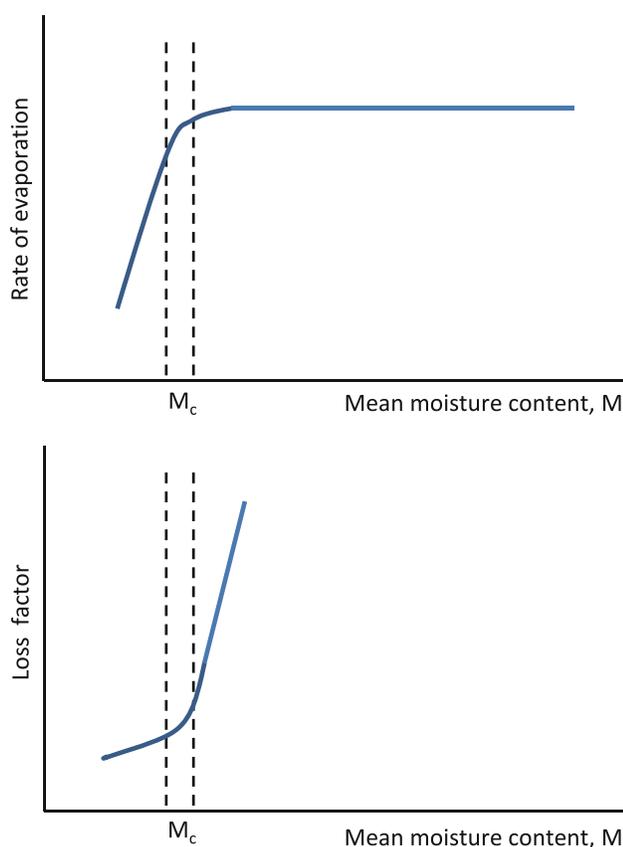


Fig. 2 Moisture content dependency of dielectric properties [66]

dielectric properties decrease with moisture decreases as illustrated in Fig. 2. The critical moisture content M_c is used to delineate the free water and the bound water. Water at moisture content below the critical moisture is called bound water, while the one above M_c is free water. The sharp decrease in dielectric properties as moisture is reduced is attributed to the reduction in the mobility of water dipoles. The bound water has a hindered ability to follow the rotation of the electromagnetic field, and hence a reduced ability to extract energy from the field. Feng et al. [39] designed a method to measure the dielectric constant (ϵ') and loss factor (ϵ'') of Red Delicious apples at a moisture range of 3.8–87.5% (wet basis), as shown in Table 1. It was found that when moisture content was relatively high (>70%), both free water dispersion and ionic conduction contributed to the dielectric behavior. At a moisture content of around 23%, the ionic conduction dominated the frequency response of the dielectric properties. At low-moisture contents of about 4%, the bound water was responsible for the dispersion.

Temperature Effect

The temperature dependency of dielectric properties is complex because more than one dispersion mechanisms can

be involved. This has been well documented in the literature. Sun et al. [101] reported that for food products, the dielectric constant decreased with a temperature increase, while the loss factor increased with temperature when salt content was high but decreased if salt content was low. In a study for the determination of dielectric properties of pea puree, Tong et al. [106] noticed that at 915 MHz, the loss factor increased with temperature, while at 2,450 MHz, it decreased with increased temperature until reaching a minimum at temperatures between 25 and 75 °C. Goeden et al. [43] showed that the dielectric constant increased when temperature increased from 20 to 65 °C then became nearly constant from 60 to 95 °C, while the loss factor increased linearly from 25 to 95 °C and decreased when no salt was present. Feng et al. [39] reported that an increase in temperature at low-moisture contents resulted in increased dielectric properties, and the dielectric responses to temperature were difficult to predict at high-moisture contents. Sipahioglu and Barringer [95] measured the dielectric properties of 15 fruits and vegetables at 5–103 °C. They found that dielectric constant of vegetables and fruits decreased with temperature and ash content, and loss factor changed quadratically with increasing temperature, that is, first decreasing and then increasing. To fully understand the temperature effect, one needs to understand the dielectric dispersion due to free water, bound water, and ionic conduction. Figure 3 illustrates both the frequency dependency and temperature effect of different dispersion mechanisms in microwave heating. From Fig. 3, we can see that, in the microwave frequencies (300 MHz–300 GHz), bound water, free water, and ionic dispersions may be involved. The percentage of bound water and free water determines the positive or negative response of dielectric properties to frequency and temperature changes. The influence of ionic conduction is always positive when temperature increases.

Porosity Effect

The effect of porosity on the dielectric properties is due to the low dielectric properties of air. Air has a relative dielectric constant of 1 and loss factor of zero. Therefore, air is considered as transparent to microwaves. High porosity materials have more entrapped air, and thus lower dielectric properties.

Heating Uniformity

It is well known that microwave can reach the moisture inside the food and facilitates a volumetric heating. However, it is also important to remember that microwave heating is not uniform. There are a number of scenarios where a nonuniform microwave heating can be encountered.

Table 1 Dielectric properties of Red Delicious apples at two temperatures (22 and 60 °C) and different moisture contents (modified from [39])

MC (wb) %	915 MHz (22 °C)		2,540 MHz (22 °C)		MC (wb) %	915 MHz (60 °C)		2,540 MHz (60 °C)	
	ϵ'	ϵ''	ϵ'	ϵ''		ϵ'	ϵ''	ϵ'	ϵ''
87.5	56.0	8.0	54.5	11.2	68.7	32.77	9.11	30.84	7.54
80.7	43.03	6.96	40.98	9.25	53.6	28.45	8.54	26.03	7.20
79.6	38.22	6.06	36.25	8.56	34.6	22.48	6.82	19.65	6.59
78.0	38.97	6.27	37.06	8.49	22.4	14.44	4.50	11.86	4.45
69.7	33.03	6.66	30.32	8.53	12.7	7.35	2.10	6.23	1.74
55.1	26.92	6.87	23.09	8.45	11.0	5.34	1.69	4.45	1.37
46.6	22.24	6.71	18.38	7.83	5.93	4.14	1.00	3.68	0.72
36.4	16.21	6.10	12.27	6.22	3.8	3.61	0.68	3.56	0.47
30.3	14.39	5.94	10.74	5.52					
23.8	5.65	2.10	4.52	1.58					
19.0	3.70	0.81	3.30	0.65					
14.1	2.80	0.33	2.63	0.29					
9.2	2.24	0.17	2.21	0.14					
3.8	1.695	0.086	1.710	0.066					

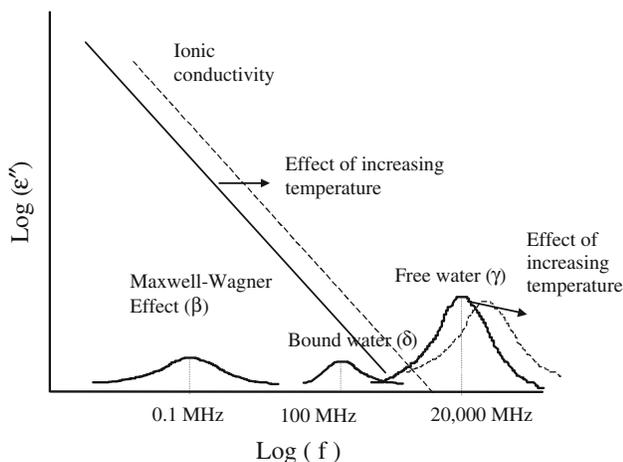


Fig. 3 Frequency and temperature effects of different dispersion mechanisms on loss factor [103]

Nonuniformity Due to Geometry and Composition

Microwaves behave like a beam of light. Reflection and transmission may occur when they reach an object. The transmitted microwaves may focus at a region of the object because of the geometry of the product. Figure 4 shows some of the possible situations where localized and uneven heating may occur. In case (a), the microwave decays when it travels into the product, and the surface region receives more microwave radiation than the interior of the material, resulting in a surface heating. In case (b), the exponential decay of microwaves from both sides of a material may form a central overheated area by superposition. In case (c), waves from two sides of a rectangular shape object may cause corner overheating. In addition, for the heating

of a spherical object in a microwave oven, the waves will be focused at the center, and a center overheating will take place. As a result, cooking of eggs in their shells in a microwave will explode. Therefore, even a microwave field could be uniform in an unloaded cavity; localized overheating due to the geometry of the food product as shown in the three cases outlined above is likely to occur. On the other hand, many food products are having more than one component. The differences in the dielectric properties among the components in a food will cause differential heating and thus temperature heterogeneity. This may bring about a food safety issue for pasteurization and sterilization applications, and attention must be paid to find the cold spot(s) and make sure enough heating and lethality will be achieved there. For drying applications, however, this differential heating could be beneficial. The wet core of a food product during drying normally has a higher loss factor compared to the same material at the dry zone, which allows more microwave energy to be drawn to the moisture and thus energy is supplied only to places that need it.

Nonuniformity Due to Electromagnetic Field

Microwaves are sinusoidal propagating electromagnetic waves. At any point in space, the magnitude of the local electromagnetic field intensity changes with both time and direction. Microwaves is reflected at a metallic surface and partially absorbed by lossy materials. In a drying cavity with metal walls, the incident microwave from a magnetron will form a complex wave pattern inside the cavity because of the superposition of the reflected waves from the walls, forming hot spots and cold spots. Figure 5 shows

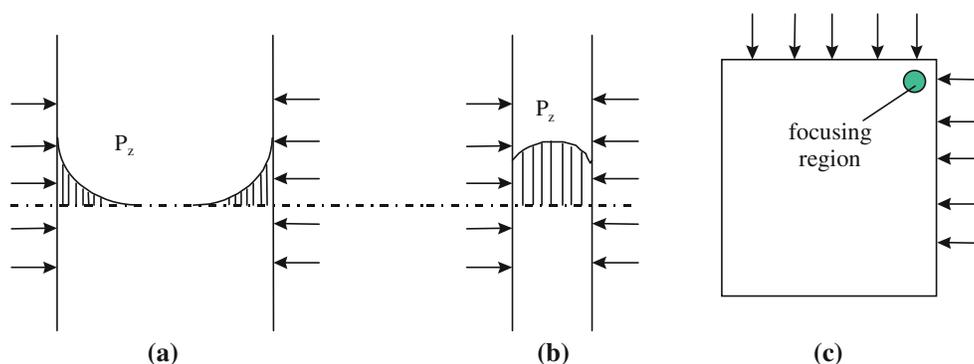
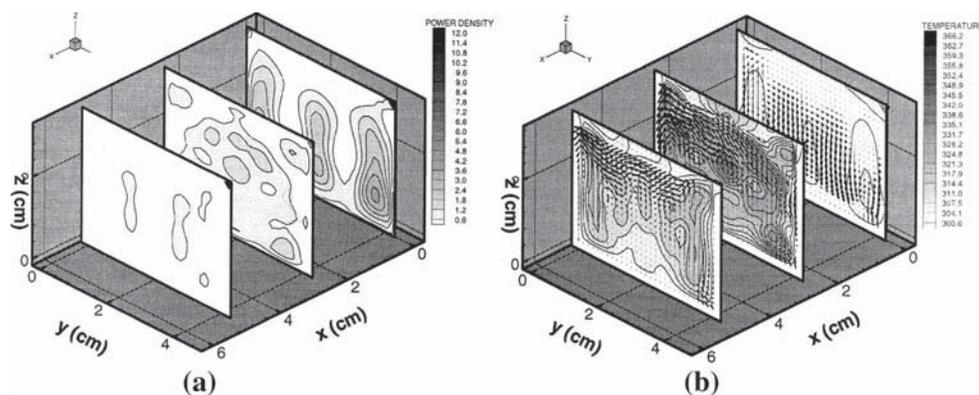


Fig. 4 Microwave power density distribution inside a material. Arrows indicate incident microwaves

Fig. 5 Microwave power **a** and temperature **b** distribution at different locations in a cavity. [122]



the microwave power and temperature distribution in a microwave cavity, simulated by the finite-difference time-domain (FDTD) scheme by solving the transient Maxwell's equations for determining the microwave power and by the finite control volume method for obtaining unsteady temperature profiles. Comparison is made at three locations in the cavity in the y - z plane, and an uneven power and temperature distribution can be clearly seen on all three planes examined [122]. The microwave energy distribution in a microwave cavity can be visualized with different methods. For instance, Dai et al. [26] tested the application of gypsum plates with cobalt chloride as an indicator to map the microwave energy distribution in a multimode microwave cavity. They observed different but all non-uniform heating patterns at different locations inside the cavity.

Means to Overcome Nonuniform Heating

Nonuniform heating in a microwave cavity will cause problems associated with quality degradation and microbial safety. Efforts have been made in recent years to address this issue. One means to improve microwave heating uniformity is to alter the distribution of the microwave radiation using novel packaging designs [120]. This method reduces the microwave exposure at regions

where the focus effect takes place or the local material has a high loss factor [41]. Another approach is to constantly change the spatial location of the products during heating. The turntable design in domestic microwave ovens is an example of this strategy. This method is especially useful for industrial application of microwave to such areas as food reheating, drying, pasteurization, and sterilization. There are three ways to provide a time-averaged, spatial homogeneous microwave heating for products. One is to agitate the products that are placed in a microwave cavity. Welt et al. [116] achieved a uniform temperature distribution in an artificial liquid food using agitation. The apparatus is suitable for solutions with viscosity less than or equal to 0.2 Pa s. The limitation of this method is that it is suitable mainly for liquid or pumpable foods. Another method is to mechanically change the spatial location of the product. Torringa et al. [107] used a shaft driven by a motor in a microwave drying test to rotate the drying chamber and improve heating uniformity. Numerous designs can be worked out following this idea. Fluidization provides pneumatic agitation of particles inside a fluidized bed. The combination of fluidization technology with microwave has been reported to achieve a significantly improved microwave heating. Research conducted at Washington State University [32] reported uniform microwave heating of evaporated diced apple, achieved

Table 2 Heat and mass transfer models for microwave drying and the classification of the models

Model	Example	Thermal physical and transport parameter(s) needed in the simulation
<i>Empirical model</i>	1. Page's model 2. exponential model	Drying constant k drying constant k and exponent
<i>Diffusion model</i> (mass transfer model)	Fick's second law	Effective diffusivity D_{eff}
<i>Heat–mass transfer models</i>		
Multiphase media model (porous media model)	Whitaker model	Thermal physical properties for water, vapor and air, porous properties
Continuum physics model	Luikov model	Phenomenological coefficients
Continuum physics model (unsaturated porous media)	Philip and DeVries model	Thermal physical and transport properties for the medium
Thermo-hydro-mechanical model	Kowalski model	Thermal physical and transport properties for the medium

with spouted bed, a special fluidization technique. The Washington State University benchtop spouted bed microwave dryer was scaled up by a factor of six in a more recent study reported by Yan et al. [119]. With the new system, the authors produced puffed potato cubes with desired expansion ratio, breaking force, and rehydration ratio. Other methods that have been proposed and tested to improve the heating uniformity include the variable frequency technique [80] and the phase control heating technique [47].

Heat and Mass Transfer Modeling in Microwave Drying

A number of excellent reviews have been published in recent years to summarize research efforts in microwave heating simulations with an emphasis on heat generation, heat transfer, and coupling of electromagnetic field with microwave heating [12, 15, 53, 87]. Relatively fewer endeavors have been made to evaluate mathematical modeling of both heat and mass transfer taking place in a microwave field for drying-related applications [87]. Microwave drying involves simultaneous transport of heat, mass, and momentum accompanied by volumetric heat generation. The internal heat generation facilitates an internal total gas pressure gradient that distinguishes microwave drying from other drying methods. It also produces a positive temperature gradient in contrast to that when heat is supplied to the product surface. Accurate estimation of internal heat generation is a challenge because of the complicated responses of dielectric properties to temperature, moisture, porosity, and composition changes. Traveling microwaves can also decay, focus, and superimpose to further complicate the calculation of the source term in heat transfer equations. Special care must be taken when analyzing the heat and mass transfer

characteristics in microwave drying. Microwave drying models can be classified into three categories, which are tabulated in Table 2. These numerical models enable scientists and engineers to study the drying behavior of a food product with regard to moisture and temperature distribution during drying, and facilitate parameter studies to optimize the drying process. However, almost all these models are applied to only a specific drying set up, a target product, and the drying conditions applied. The simulation results from an individual study cannot be directly applied to the drying analysis of a different product or even to a same product that is processed under different drying conditions. Validation tests must be conducted before applying the simulation result in process design and optimization.

The Empirical Models

Empirical models are simple to apply and often used to describe drying curves for obtaining kinetic parameters. The most commonly used empirical models are the Page equation [71] and the exponential models. Page's equation can be written as:

$$\frac{X - X_e}{X_0 - X_e} = \exp(-kt^n) \quad (3)$$

where X , X_0 , and X_e are moisture content at time t , time $t = 0$, and at equilibrium condition with surrounding air, respectively, and k and n are constants. This equation has been used by Prabhanjan et al. [76] and Tulasidas et al. [108] to analyze microwave drying of wheat and grapes and to describe the thin-layer drying kinetics of parsley leaves [99]. McMinn [64] evaluated the drying characteristics of lactose powders by the Page equation, along with 9 other empirical models. Giri and Prasad [42] presented an analysis of microwave–vacuum drying of button mushroom (*Agaricus bisporus*) and concluded that empirical

Page's model adequately described the microwave–vacuum drying data. A number of other exponential equations have also been used in microwave drying studies, such as those by Adu et al. [1], Skansi et al. [96], Drouzas et al. [29], Sutar and Prasad [102], and Dadali and Özbek [25] for drying of a variety of biological materials, especially with thin-layer drying method. An empirical model was proposed by Andrés et al. [4] to estimate the drying rate of apple cylinders pretreated by osmotic dehydration in four distinct periods in a combined hot-air–microwave drying. Regression techniques are often used to fit the drying data to selected models to correlate the drying rate with moisture content, microwave power density, microwave frequency, temperature, air flowrate, and sample geometry [7, 8, 51].

Diffusive Theory

Moisture migration in a porous medium can be driven by a concentration gradient for liquid and by a partial vapor pressure gradient for vapor. The governing equation for moisture transport is Fick's second law:

$$\frac{\partial X}{\partial t} = \text{div}(D_{\text{eff}} \text{grad } X) \quad (4)$$

where D_{eff} is effective moisture diffusivity. Equation 4 can be solved in closed form under the assumptions of constant moisture diffusivity, no shrinkage, and sufficient surface mass transfer rate, so that the moisture content can reach equilibrium with the air at any time during drying. Solutions for Eq. 4 with various geometrical and boundary conditions have been compiled by Crank [23]. The solution for a sphere is given by:

$$\frac{\bar{X} - X_e}{X_0 - X_e} = \frac{6}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \exp\left[-n^2 \frac{4\pi^2 D_{\text{eff}} t}{d^2}\right] \quad (5)$$

where \bar{X} is average moisture content, d is the diameter of the sphere, and t is time. It has been found that under certain conditions, only the first term in the infinite series is important. For instance, in the falling rate period and when the moisture ratio $(\bar{X} - X_e)/(X_0 - X_e)$ is <0.3 [36], only the first term in Eq. 5 is predominant and a simplified equation results:

$$\frac{\bar{X} - X_e}{X_0 - X_e} = \frac{6}{\pi^2} \exp\left[-\frac{4\pi^2 D_{\text{eff}} t}{d^2}\right] \quad (6)$$

Equation 6 is often used to estimate effective moisture diffusivity of different materials with a simple drying test [24]. The D_{eff} in Eq. 6 can be related to temperature by an Arrhenius-type equation [92]. Efforts have also been made to correlate D_{eff} with moisture changes [105]. The effect of microwave power on effective moisture diffusivity (D_{eff}) of

bamboo shoot was investigated by Bal et al. [6]. The diffusion model has been used to study microwave drying of agricultural and food products. Ptaszniak et al. [79] used the first term in the solution of Fick's second law to simulate the drying of broad bean. A similar study for the microwave drying of potato was conducted by Bouraoui et al. [10]. The first 15 terms in Eq. 5 were used by Shivhare et al. [94] to model microwave drying of soybean, and an agreement with experiment results was attained. Adu and Otten [2] took into consideration the variable diffusion coefficient in a simulation of microwave drying of white bean. The Fick's model was also used by Therdthai and Zhou [104] to describe the drying kinetics of mint (*Mentha cordifolia* Opiz ex Fresen) leaves for microwave–vacuum drying and hot-air drying. Goksu et al. [44] applied the Fick's law of diffusion and used the first term in the infinite series to obtain effective moisture diffusivity in a microwave-assisted fluidized bed drying of macaroni beads.

Heat and Mass Transfer Models

Simultaneous heat and mass transport takes place during a microwave drying process, with heat generated throughout a food product and moisture transfer from inside to the surface of the product. An analysis of coupled moisture and energy transport is, therefore, often used to elucidate the underlying physics. The numerical analysis with different heat and mass transfer models can also be used to assist operation design and optimization. Dependent on the need, numerous heat and mass transfer models have been proposed over the year to analysis a drying process. The representative methods for such an analysis include (1) the irreversible thermodynamics approach proposed by Luikov [61]; (2) the unsaturated porous media theory established by Philip and de Vries [74]; and the volume-averaging technique developed by Whitaker [117], as shown in Table 2. Methods originated from various simplified approaches have also been used.

Simplified Approaches

A simplified heat and mass transfer model considering both the liquid and vapor transport was developed by Lu et al. [59, 60] to analyze microwave drying of sliced and spherical food products. A careful analysis was conducted to estimate the microwave power generation inside the product. The model was validated by experiments, and a good agreement was achieved. A comparison of their model predictions for moisture content with experiments for spherical potato samples is given in Fig. 6. Grolmes and Bergman [45] proposed a microwave drying model to characterize the drying of nonhygroscopic material. They

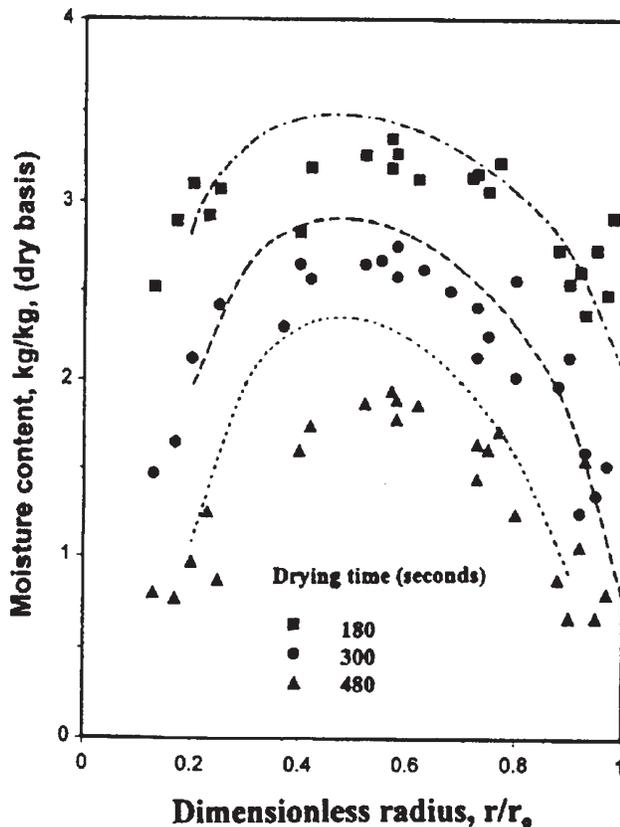


Fig. 6 Comparison between model predicted and experimentally determined moisture distributions [59]

used a macrobalance method to generate the governing differential equations. They observed three drying regimes. An initial regime occurred in which the material was heated convectively and dielectrically, followed by a transition regime, and ultimately, a final regime during which the material was dielectrically heated and convectively cooled. Melendez et al. [65] developed a heat and mass transfer model utilizing the characteristic drying curve method. The model simultaneously solved the energy and mass balance equations for air and the wet and dry regions of the solid. McMinn et al. [63] reported an one-dimensional moisture diffusion model to describe drying in an infinite slab or cylinder. A simplified solution of the model using two constants as a function of Bi number was used to predict drying curves (moisture vs. time). Feng [38] employed an air–solid two-phase flow model to simulate the drying of particulates in microwave drying with pneumatic agitation. The energy equation took into account the mass transfer coupling effect under forced convection, while the moisture equation was based on a simple diffusion model. The resulting equations were solved with an implicit finite-difference scheme, and a good agreement

with microwave drying of apple dice was achieved under the conditions tested.

Philip and de Vries Method

A two-dimensional formulation based on the Philip and de Vries theory [74] was developed by Lian et al. [57] to simulate the drying of a slab in a microwave cavity. In Philip and de Vries theory, the total potential for moisture transport consists of two components, the temperature potential and the capillary potential. Lian et al. [57] introduced moisture content and temperature to take the place of the potentials. Coupled heat and mass transfer equations were obtained. They assumed a uniform microwave distribution at the slab surface and used Lambert's law to calculate the decay of microwave into the slab. The average moisture contents obtained in the experiment agreed well with predicted ones.

Luikov Method

The Luikov method is basically a phenomenological model. It is concise and symmetric [114]. Bajza [5] employed this method to analyze heat and mass transfer during microwave drying of tanned leather. The coupled one-dimensional heat and mass transfer equations Bajza developed are:

$$\rho C_p \frac{\partial T}{\partial t} = K_T \frac{\partial^2 T}{\partial x^2} + K_M \frac{\partial^2 X}{\partial x^2} + \rho \Delta h \frac{\partial X}{\partial t} + Q \quad (7)$$

$$\frac{\partial X}{\partial t} = D_M \frac{\partial^2 X}{\partial x^2} + D_T \frac{\partial^2 T}{\partial x^2} \quad (8)$$

where ρ is density, C_p is specific heat, K_T is thermal conductivity, K_M is moisture gradient-induced heat transfer coefficient, Δh is latent heat, D_M is moisture diffusivity, and D_T is the temperature gradient-induced moisture transfer coefficient. The heat source Q can be calculated based on Eq. 1. The model was validated with experiments under different conditions, and a good agreement with the model prediction was achieved. Jun et al. [50] proposed a similar model for one-dimensional transport in spherical coordinates. Their model prediction agreed well with experimental results for drying of whole apples. It is well known that for the Luikov model, some coefficients are not directly related to physical phenomena and therefore are difficult to evaluate. The moisture gradient-induced heat transfer coefficient K_M and the temperature gradient-induced moisture transfer coefficient D_T in Bajza's model as shown in Eq. 7 did not have a clear physical meaning, and their values were not given in the article. The corresponding heat and mass transport coefficients in the study by Jun and coworkers were also not well documented.

Whitaker Method

The partial differential equations for drying problems are usually derived from an infinitely small control volume and therefore are often referred to as “point” equations. They are meant to describe the transport behavior in microscale. However, there is a need to define the scale in an analysis. A homogeneous material at macroscale may be heterogeneous at microscale. For cellular porous food products, materials are heterogeneous if we examine them at cell level. A precise description of the transport in such materials at the cell level requires the transport equations for the gas, liquid, and solid phases within and outside the cells separately. The complexity and heterogeneity of the food matrix prevent a general solution for the detailed moisture and temperature field at this level. To overcome this problem, physical phenomena in porous media are generally described by “macroscopic” equations, valid at a length scale termed the representative elementary volume (REV) [9, 75]. A method based on this idea was first proposed by Whitaker [117] and has been widely used in the heat and mass transfer analysis of drying. Whitaker gave a rigorous derivation of the drying-governing equations by means of a volume-averaging technique. The equations are representative on the REV to averaged values of microscopic variables. The advantage of this method is that the physical meaning of the model and the transport parameters is well defined, the assumptions are clear, and most importantly, the transport parameters are well defined and measurable. However, rigorous derivation following Whitaker’s method is seldom used in recent drying studies due to its complexity. Many drying models in later studies were developed utilizing Whitaker’s volume-averaging concept without going through the lengthy and complicated derivation process to obtain the partial differential equations [67].

Early studies in microwave drying using the Whitaker’s volume-averaging concept were conducted by Wei et al. [115] and Jolly and Turner [49]. The models they developed were for nonhygroscopic porous media. Wei et al. noticed an increase in both the liquid volume fraction and air density toward the sample surface. A pressure maximum built up inside the sample before it fell to nearly atmospheric pressure. Jolly and Turner reported the significant influence of sample size on predicted temperature and moisture profiles.

The importance of the gas pressure gradient in microwave drying was not fully recognized until Turner and Jolly [110] noticed that without considering the pressure effect, it was difficult to account for the phenomena of “water pumping” in microwave drying. They also realized the importance of the contribution of the gas pressure to product quality. They introduced a third transport equation, a total gas pressure equation, into the drying model. Since then, the importance of the additional driving force due to

the gaseous pressure gradient in microwave heating has been well documented in microwave drying studies. Turner and Rudolph [111] utilized this approach to simulate the drying of glass beads. Toringa et al. [107] used a vapor equation in place of the total gas pressure equation to analyze the drying of food products. Constant et al. [21] developed a coupled heat and mass transfer model to study the microwave drying of light concrete by taking into account the effects of moisture, temperature, and total pressure gradient in the model equations. They predicted and experimentally demonstrated the liquid pumping phenomenon. A study conducted by Turner et al. [112] extended their previous model to hygroscopic media. The formulation they proposed is the following:

$$a_{X1} \frac{\partial X}{\partial t} + a_{T1} \frac{\partial T}{\partial t} = \frac{\partial}{\partial z} \left[K_{X1} \frac{\partial X}{\partial z} + K_{T1} \frac{\partial T}{\partial z} + K_{P1} \frac{\partial P_g}{\partial z} + K_{gr1} \right] \quad (9)$$

$$a_{X2} \frac{\partial X}{\partial t} + a_{T2} \frac{\partial T}{\partial t} + a_{P2} \frac{\partial P_g}{\partial t} = \frac{\partial}{\partial z} \left[K_{X2} \frac{\partial X}{\partial z} + K_{T2} \frac{\partial T}{\partial z} + K_{P2} \frac{\partial P_g}{\partial z} + K_{gr2} \right] + \Phi \quad (10)$$

$$a_{X3} \frac{\partial X}{\partial t} + a_{T3} \frac{\partial T}{\partial t} + a_{P3} \frac{\partial P_g}{\partial t} = \frac{\partial}{\partial z} \left[K_{X3} \frac{\partial X}{\partial z} + K_{T3} \frac{\partial T}{\partial z} + K_{P3} \frac{\partial P_g}{\partial z} + K_{gr3} \right] \quad (11)$$

where a_{ij} and K_{ij} are capacity and kinetic coefficients, respectively. Subscript i could represent moisture, temperature, pressure, and gravity, while $j = 1, 2,$ and 3 . P_g is total gas pressure. Turner et al. also carefully examined the microwave power distribution in the drying cavity by solving the electromagnetic field equations under the plane wave assumption. Model predictions agreed with their experiments conducted with wood. Turner and Perré [113] conducted a simulation of vacuum drying of sapwood and heartwood with radiative heating using two models, one was a comprehensive heat and mass transfer model developed with the volume-averaging approach and the other was a simplified one.

A number of microwave drying simulation studies applying the REV approach have been reported although the authors did not explicitly reference the volume-average concept. Chen et al. [14] applied a coupled heat and mass transfer model to study the drying behavior of spherical particulates in microwave field by considering capillary flow of water with the Darcy’s law and a diffusive flow of vapor by the Fick’s law. In microwave heating pattern study, they considered uniform heating, sinusoidal, and rectangular heating modes. The intermittent heating was found to save drying time by 2/3, but was accompanied by an increase in energy consumption of 26–37%. An empirical model for calculating moisture diffusivity was used in the work of

Sanga et al. [88] for the simulation of convective microwave drying of a cylindrical piece of carrot. The moisture and temperature equations obtained were solved with a finite-element technique, and a good agreement was achieved when compared with the experimental results. Shrinkage was taken into consideration in their model by continuously redefining the computational domain of the problem. An effort was made by Dinčov et al. [27] to simultaneously consider the electromagnetic field distribution in a microwave cavity and the heat and mass transfer caused by microwave heating. A finite-difference time-domain analysis was conducted to compute the electromagnetic field, and a finite-volume scheme was employed to solve the two-fluid (water–vapor) transport the food product. The coupling between the two set of calculations was made by the changes in the dielectric properties of the sample. The pressure-driven flow was also considered by the Darcy’s law. A set of one-dimensional heat and mass transfer equations was proposed by Salagnac et al. [86] to examine the hot-air, infrared, and microwave drying of a cellular concrete material. Their simulation results were in good agreement with experimental measurements. The heat and mass transfer models developed by Abbasi Souraki and Mowla [97] were used to investigate the drying of garlic in an inert medium in a fluidized bed. The model only considered diffusive flow, but a relatively good agreement between the predicted and experimentally determined average moisture content of the particulates was achieved.

It is worth mentioning that most of the heat and mass transfer studies in microwave drying were for nonhygroscopic materials. Comprehensive microwave drying analysis for hygroscopic porous media related to foods and agricultural products is challenging and less reported. For hygroscopic food and agricultural materials, the bound water transport cannot be ignored, as the bound water, or nonfreezable water, can account for up to 16–28% of the total moisture [55]. To obtain a dry product with a water activity of below 0.6 to avoid microbial growth, the removal of bound water in the drying is needed. Not many drying models have considered the transport of bound water. One reason is that the flow of bound water in a porous product is complicated and cannot be simply attributed to diffusive flow [16]. When chemical potential gradient is considered as the driving force for bound water transport, the bound water flow is given by

$$n_b = D_b(1 - \zeta') \frac{\partial \mu_b}{\partial z} = -D_b(1 - \zeta') \left(\frac{\zeta \nabla P_v}{\rho_v} - \frac{S_v}{M_v} \nabla T \right) \tag{12}$$

where D_b is bound water diffusivity, ζ' is porosity (gas volume/total volume), ζ is porosity ((gas + liquid)/total volume), μ_b is chemical potential of bound water, P_v is vapor

pressure, S_v is saturation, M_v is molar mass of vapor, T is temperature, and ρ_v is vapor density. In the comprehensive heat and mass transfer models developed by Feng et al. [37], the water exists in plant tissues, and the transport of such water during drying was first analyzed. Plant tissue can be considered as a hygroscopic capillary porous medium that is divided internally into numerous repeating units [56]. The water in a plant tissue can be classified in accordance with its physicochemical state as (a) liquid water in the cell (in protoplasm and vacuoles), vascular tissues, and intercellular spaces, (b) vapor in intercellular space and vascular tissues, and (c) constitutive water bound chemically in cell walls. The liquid water that exists within a cell and in either vascular tissues or intercellular spaces is different in their resistance to migration. The water inside the cell has to penetrate the cell wall to reach either vascular or intercellular space. The cellular water in foods is thus referred to as bound water in the drying literature [16, 84], while water in vascular tissues and intercellular spaces is termed as free water. The model of Feng et al. [37] considered mass transport of both free water and bound water, and is shown in Eqs. 13, 14, and 15.

$$\frac{\partial X_l}{\partial t} = \frac{1}{r^2} \frac{\partial}{\partial r} \left[D_X r^2 \frac{\partial X_l}{\partial r} + D_T r^2 \frac{\partial T}{\partial r} + D_P r^2 \frac{\partial P_g}{\partial r} \right] \tag{13}$$

$$C_{TX} \frac{\partial X_l}{\partial t} + C_{TT} \frac{\partial T}{\partial t} = \frac{1}{r^2} \frac{\partial}{\partial r} \left[D_{TX} r^2 \frac{\partial X_l}{\partial r} + D_{TT} r^2 \frac{\partial T}{\partial r} + D_{TP} r^2 \frac{\partial P_g}{\partial r} \right] + \Phi \tag{14}$$

$$C_{PX} \frac{\partial X_l}{\partial t} + C_{PT} \frac{\partial T}{\partial t} + C_{PP} \frac{\partial P_g}{\partial t} = \frac{1}{r^2} \frac{\partial}{\partial r} \left[D_X^a r^2 \frac{\partial X_l}{\partial r} + D_T^a r^2 \frac{\partial T}{\partial r} + D_P^a r^2 \frac{\partial P_g}{\partial r} \right] \tag{15}$$

Equation 13 is the total moisture equation where the coefficient D_X , D_T , and D_P included the water transfer caused by free water, bound water, and water vapor driven by moisture, temperature, and pressure gradients, respectively. Equations 14 and 15 are temperature equation and total pressure equation. The model of Feng and coworkers was solved with a finite-difference scheme, and the model prediction of moisture and temperature agreed with experimental results for microwave and spouted bed drying (MWSB) of particulate materials. For the first time, the pressure built up inside a food product during microwave drying was measured by Feng et al. [37] where the measured and the predicted pressures were in a relatively good agreement, as shown in Fig. 7.

Two-Region Model

A mathematical model was developed by Chen and Pei [16] to analyze simultaneous heat and moisture transfer during

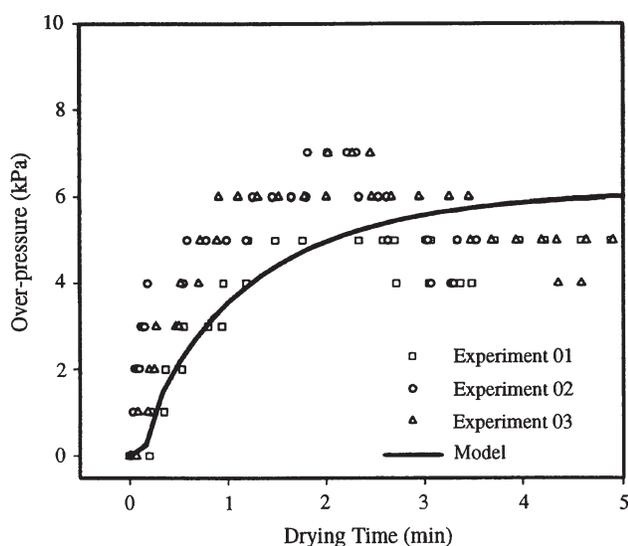


Fig. 7 Over-pressure in microwave-assisted spouted bed drying of diced apples with microwave power of 10 W/g and hot-air temperature of 70 °C [37]

drying. They assumed the existence of a receding evaporation zone separating the material into a wet zone and a dry zone. The interface between the two zones was defined by $S(t)$ which was a function of drying time. In the wet zone or wet region ($0 < x < S(t)$), the main mechanism of transfer is capillary flow of free water while in the dry zone (sorption region, $S(t) < x < L$), the moisture transport is due to movement of bound water and vapor transfer. A heat source term accounting for dielectric heating was introduced in the energy equation. Using the model developed, they investigated the drying behavior of brick, wood boobins, and corn kernels by predicting the temperature and moisture content distributions in the materials. This method was applied to the microwave drying analysis of hygroscopic and nonhygroscopic materials in a later study [17]. Moisture and temperature predictions were found in agreement with experiments for polymer pellets, glass beads, and alumina spheres. Jansen and Van der Wekken [48] published a similar analysis using the two-region method.

Continuous Thermo-Hydro-Mechanical Model

The drying model of Kowalski et al. [54] takes into account the balances of mass, linear momentum, angular momentum, energy, entropy, and the irreversible thermodynamics. The balance equations are derived starting from an Euler description, and after simplification, one energy equation and one liquid moisture equation were obtained. What is unique is that the moisture transport coefficient was assumed to take different forms in the constant rate drying and falling rate drying period. As a result, experimentally obtained critical moisture (X_{cr}) was introduced into the

moisture equation. In addition, a phase transition coefficient was used in the “free temperature” equation, which took a different form in two drying periods, and also when the moisture was below the equilibrium moisture content (X_e) of the sample. In model simulation, microwave drying experiments with samples made of kaolin were first conducted to determine convective vapor transfer coefficient and convective heat transfer coefficient. With experimentally determined parameters inserted into the model, a good agreement between the predicted temperature and moisture distribution and those from the experiments was achieved.

Important Considerations in the Simulation of Microwave Drying

Transport Mechanisms

An appropriate appreciation of the transport mechanisms involved in microwave drying is the most important aspect in heat and mass transfer analysis. A close look at the drying literature reveals that, in a sense, the history of drying modeling is a chronological record of the understanding of the transport mechanism(s). Classic studies conducted by Sherwood [93], Ceaglske and Hougen [13], and King [52] utilized single mechanism, such as liquid diffusion, vapor diffusion, and capillary flow to describe the drying of solids. To address the complexity of drying problems and to be applicable to a wide spectrum of materials, more complicated models were developed using multimechanism approaches. The most cited studies under this category were the works of Krischer, Luikov, and Whitaker. The theory of Philip and de Vries and the receding front theory were also used in coupled heat and mass transfer studies. In recent years, a consensus has been reached that a drying process is the one with the simultaneous transport of heat, free water, bound water, vapor water, and air and that a complete description of a drying problem requires the use of three coupled nonlinear partial differential equations to account for the effect of moisture, temperature, and pressure fields [[9], [67]]. Different mechanisms have been assigned to the transport of different liquid and gaseous fluxes. The inclusion or exclusion of bound water transfer determines whether the model can be used to analyze hygroscopic materials or not. A summary of the mass transport mechanisms used in microwave drying analyses is given in Table 3.

Determination of the Heat Source Term Due to Microwave Heat Generation

In microwave heating and drying, heat generation analysis is crucial for an accurate estimation of the heat source term

Table 3 Heat and mass transfer models developed for analysis of microwave drying processes

Reference	Theory	Transport mechanism			Power estimation	Numerical method	Model validation	Power measurement
		Free water	Bound water	Air				
Kowalski et al. [54]	Thermo-hydro-mechanical	Diffusive flow	NA	NA	Empirical	Finite-difference method (FDM)	Temperature distribution in samples from thermovision camera and drying curves	NA
Dinčov et al. [27]	Whitaker (implicitly)	Capillary flow	NA	Capillary flow	Electric field analysis	Finite-difference time-domain for microwave field and finite volume for heat and mass transfer	Not conducted	NA
Sanga et al. [88]	Whitaker (implicitly)	Diffusive flow	NA	NA	Lambert law	Finite-element method (FEM)	Average moisture	NA
Chen et al. [14]	Whitaker (implicitly)	Capillary flow	NA	NA	Empirical	FDM	Not conducted	NA
Feng et al. [37]	Whitaker (implicitly)	Capillary flow	Chemical potential	Capillary flow + diffusion	Empirical	FDM	Average moisture, temperature and pressure were measured	Incident and reflected
Jun et al. [50]	Luikov	NA ^a	NA	NA	NA	FEM	Average moisture and temperature	Incident
Ni et al. [68]	Whitaker (implicitly)	Capillary flow	NA	Capillary flow + diffusion	Empirical	FDM	Not conducted	NA
Turner et al. [112]	Whitaker (implicitly)	Capillary flow	Molecular diffusion	Capillary flow + diffusion	Electric field analysis	FDM	Moisture	Incident
Lian et al. [57]	Philip and de Vries theory	Darcy's law	NA	Fick's law	Lambert law	FEM; Using commercial CFD software	Average moisture	Incident
Bajza [5]	Luikov	NA	NA	NA	NA	NA	Moisture	Rated power of the oven
Constant et al. [21]	Whitaker (implicitly)	Capillary flow	NA	Capillary flow + diffusion	Empirical	FDM	Average moisture, temperature, and pressure	Incident and reflected
Torranga et al. [107]	Lumped diffusion	Fick's law	NA	NA	Uniform throughout the sample	FDM	Moisture and temperature	Incident and reflected
Adu and Otten [3]	Lumped diffusion	Fick's law	NA	NA	Uniform throughout the sample	FDM	Moisture and temperature	Incident and reflected
Constant et al. [20]	Whitaker (implicitly)	Capillary flow	NA	Capillary flow + diffusion	Electric field analysis	FDM	Control volume technique	NA

Table 3 continued

Reference	Theory	Transport mechanism			Power estimation			Numerical method	Model validation	Power measurement
		Free water	Bound water	Vapor	Air	Electric field analysis	Capillary flow + diffusion			
Turner and Jolly [110]	Whitaker (implicitly)	Capillary flow	NA	Capillary flow	Capillary flow + diffusion	Electric field analysis	FDM	Not conducted	NA	
Chen and Schmidt [17]	Two-region model	Capillary flow	NA	Capillary flow + diffusion	Capillary flow + diffusion	Uniform throughout the sample	Integral method	Moisture and temperature were measured	Incident and reflected	
Jolly and Turner [49]	Whitaker	NA	NA	NA	NA	NA	NA	NA	NA	
Chen and Pei [16]	Two-region model	Capillary flow	Darcy's law	Capillary flow + diffusion	Capillary flow + diffusion	NA	Moving FEM	Only hot-air drying tests were conducted	NA	
Wei et al. [115]	Whitaker (implicitly)	Capillary flow	NA	Capillary flow	NA	Lambert law	FDM	Not conducted	NA	

^a Not available or not applicable

in the energy equation. There are three facets when considering the heat source determination in microwave drying simulations. The first is the material used in drying tests. The heat source term in microwave drying is a function of location of the product, local temperature, local moisture content, local porosity, and local composition. The complexity also comes from the interactions among the above-mentioned parameters. For example, the decay of microwave energy determines the local electromagnetic field intensity and hence the heat generation. This decay is a function of the dielectric constant ϵ' and the loss factor ϵ'' of the product, while ϵ' and ϵ'' themselves are a complex function of temperature and moisture content. On the other hand, the temperature and moisture inside the food to be heated by microwaves are function of location. In order to predict the temperature and moisture distribution in the food product, the knowledge about the changes of ϵ' and ϵ'' as affected by temperature and moisture content is also important. If porosity change during drying is considerable, we also need to know the moisture dependency of product porosity (Table 3).

Before heat source calculations, the microwave distribution at the surface and inside the material must be known in advance. Microwave heating is not uniform in a drying chamber. There exist hot and cold spots, which are caused by the nonuniform distribution of the electromagnetic field inside the cavity. As a result, the microwave distribution at both the surface and inside the product is not uniform. An accurate calculation of the heat generation is thus linked to the distribution of the electromagnetic field. The estimation of the electromagnetic distribution can be done by solving the Maxwell equations. A complete electromagnetic analysis for heterogeneous food products with different geometry remains a challenge. Various simplified approaches have been proposed to address this problem. The most important assumption in the simplified analysis is the uniform distribution of microwave energy at product surface. Under this assumption, the electromagnetic field distribution inside the product was assumed to be (1) uniform [3, 17]; (2) exponentially decayed [57, 115]; (3) decayed following empirical relations [14, 21, 37, 68]; (4) decayed following the Lambert's relation [88]; and (5) decayed calculated by solving electric field equations [20, 27, 112].

Ability to experimentally determine the microwave power absorbed by the product is also important. This measured power absorption can be used to calculate average surface microwave power density, which is the starting point for the heat source calculation [21]. The accurate estimation of absorbed power relies on instrumentation and the experimental system used in the study. An absorbed power measuring system needs to have a directional coupler for measuring both the incident and the reflected

power. The system also needs a stub tuner to adjust the matching between the impedance of waveguide and microwave chamber with loads, a circulator to protect the magnetron, and a stand-alone magnetron to generate the power. The absorbed power is the difference between the incident and the reflected power if the loss in the waveguide and in the cavity is negligible [35]. In some microwave drying studies, however, the information about the absorbed power determination was not reported. Some research groups only reported the incident power or rated power of microwave ovens, probably due to limitations of their experimental systems.

Model Validation

Product temperature, moisture, and pressure predictions shall be validated with experimentally measured values. In a microwave field, a reliable online measurement can only be performed with fiber optical probes. Fiber optical probes for temperature and pressure measurement are available in the marketplace but at relatively high prices. The typical sampling rate is one second, which may not be fast enough to record rapid temperature and pressure changes for high-power and high-moisture microwave drying. The resolution for presently available fiber optic pressure probes is 10 Pa. For temperature measurement, off-line infrared technique can be used to map the temperature distribution of a product [54]. Due to the limitation of experimental set up, not many research groups have conducted detailed validation tests to verify the heat and mass transfer models that they developed. Moisture measurements have been widely used to examine the validity of the models. Plotting the predicted average moisture of the product versus time along with experimentally determined average moisture has been used to verify the simulation by Chen and Schmidt [17], Adu and Otten [2, 3], Constant et al. [21], and Turner et al. [112]. The weight change of the product during drying was also used to compare with the model simulation [54]. Lian et al. [57] determined the moisture profile across the width of a slab made of a food paste and achieved a relatively good agreement with computer simulated moistures. The temperatures of polymer pellets and glass and alumina beads were measured by Chen and Schmidt [17] with fluoro-optic temperature probes at both the surface and the center of a sample. They observed a reasonable agreement between the model simulation and the experimental values. Adu and Otten [2, 3] measured the temperature of Navy beans, Constant et al. [21] determined the internal temperature of a light concrete slab, and Feng et al. [37] measured the internal temperature of diced apples, all with optical fiber probes. The temperature profiles of a cylindrical kaolin sample were obtained with a thermocamera by Kowalski et al. [54] in an off-line

measurement. In microwave drying, the internal pressure generated due to volumetric heating is not negligible. Some comprehensive studies of microwave drying have incorporated a total pressure equation to address the pressure-induced transport. The measurement of pressure buildup inside the product to be dried has been done by Constant et al. [21] and Feng et al. [37] with optical fiber probes.

Numerical Analysis

The resulting microwave drying equations are coupled and highly nonlinear. Numerical techniques have to be used to solve these equations. Chen and Schmidt [18] provided a good summary of the numerical techniques used in the simulation of microwave drying. These methods include (1) the orthogonal collocation method proposed by Wei et al. [115]; (2) the control-volume finite-difference method [110]; (3) the moving finite-element method [16]; and (4) the integral method [17]. The control-volume finite-difference method was proposed by Patankar [73] and has been widely used in numerical heat transfer studies. The application of the moving finite-element method is to facilitate calculation for receding evaporation front models. The integral method was introduced to address the efficiency in numerical calculation. In this method, the temperature and moisture profiles need to be specified, often taking the form of parabolic and polynomial functions. The Crank-Nicolson finite-difference scheme was also used because of its high accuracy [68]. The finite-difference scheme and its variations have been used by Souraki and Mowla [97, 98] in their microwave drying studies. A two-dimensional finite-element model was developed to simulate the puffing of a dough ball being dehydrated under vacuum with microwave energy by Ressing et al. [83]. Rattanadecho and Klinbun [82] employed the finite control volume method based on the SIMPLE algorithm to predict the heat transfer and fluid flow in the product during drying.

Final Remarks

Microwave drying technique has gained a renewed interest in both academia and industry. The unique volumetric heat generation and the increased mass transport rate in microwave drying are proven to be advantageous over traditional drying technologies, and as a result, microwave drying is featured by an enhanced drying rate and improved product quality. Progress has been made in recent years to expand the life span of magnetrons and lower the capital investment. Techniques to overcome the heating nonuniformity have also been developed. The techniques and instrumentation for the measurement of dielectric properties have been fully developed, and more and more dielectric properties have

been measured and reported. Built upon these progresses, the heat and mass transport in microwave drying can be simulated with both simplified and comprehensive mathematical models. The models developed and validated with one food product can be used to investigate the drying of similar products, or in the design and optimization of the drying process. The use of fiber optic technique enables us to measure the temperature and pressure changes during drying, which can be used to validate the models developed. It is foreseeable that the further development of microwave drying technology, as well as more investigations into how to better simulate a microwave drying process, will stimulate the adoption of this relatively new drying method in the food industry.

References

- Adu B, Otten L, Brown RB (1994) Modeling thin layer microwave drying of soybeans. *Can Agric Eng* 36:135–141
- Adu B, Otten L (1996) Diffusion characteristics of white beans during microwave drying. *J Agric Eng Res* 64:61–69
- Adu B, Otten L (1996) Modeling microwave heating characteristics of granular hygroscopic solids. *J Microwave Power Electromagn Energy* 31:35–42
- Andrés A, Bilbao C, Fito P (2004) Drying kinetics of apple cylinders under combined hot air-microwave dehydration. *J Food Eng* 63:71–78
- Bajza Z (1997) The influence of fatliquor concentration on microwave drying kinetics. *J Soc Leather Technol Chem* 81: 227–230
- Bal LM, Kar A, Satya S, Naik SN (2010) Drying kinetics and effective moisture diffusivity of bamboo shoot slices undergoing microwave drying. *Int J Food Sci Technol* 45:2321–2328
- Beke J (1992) Microwave drying of corn. In: Mujumdar AS (ed) *Drying' 92*. Elsevier, Amsterdam
- Beke J, Mujumdar AS, Bosisio RG (1995) Drying of fresh and rewetted shelled corn in microwave fields. *Dry Technol* 13:463–475
- Bories SA (1991) Fundamental of drying of capillary-porous bodies. In: Kakac S (ed) *Convective heat and mass transfer in porous media*. Kluwer, Dordrecht
- Bouraoui M, Richaed P, Durance T (1994) Microwave and convective drying of potato slices. *J Food Process Eng* 17: 353–363
- Brygidyr AM, Rzepecka MA, McConnell MB (1977) Characterization and drying of tomato paste foam by hot air and microwave energy. *Can Inst Food Sci Technol J* 10:313–319
- Budd CJ, Hill ADC (2011) A comparison of models and methods for simulating the microwave heating of moist food-stuffs. *Int J Heat Mass Transf* 54:807–817
- Ceaglske NH, Hougen OA (1937) Drying granular solids. *Ind Eng Chem* 29:805–813
- Chen G, Wang W, Mujumdar AS (2001) Theoretical study of microwave heating patterns on batch fluidized bed drying of porous material. *Chem Eng Sci* 56:6823–6835
- Chen H, Tang J (2011) Computer simulation for microwave heating. In: Knoerzer K (ed) *Innovative food processing technologies: advances in multiphysics simulation*. Wiley, NJ
- Chen P, Pei DCT (1989) A mathematical model of drying processes. *Int J Heat Mass Transf* 32:297–310
- Chen P, Schmidt PS (1990) An integral model for drying of hygroscopic and nonhygroscopic materials with dielectric heating. *Dry Technol* 8:907–930
- Chen P, Schmidt PS (1997) Mathematical modeling of dielectrically-enhanced drying. In: Turner I (ed) *Mathematical modeling and numerical techniques in drying technology*. Marcel Dekker, New York
- Cohen JS, Yang TCS (1995) Progress in food dehydration. *Trends Food Sci Technol* 6:20–25
- Constant T, Perré P, Moyne C (1992) Microwave drying of light concrete: from transport mechanisms to explanation of energy savings. In: Mujumdar AS (ed) *Drying' 92*. Elsevier, Amsterdam
- Constant T, Moyne C, Perré P (1996) Drying with internal heat generation: theoretical aspects and application to microwave heating. *AIChE J* 42:359–367
- Contreras C, Martin ME, Martinez-Navarrete N, Chiralt A (2005) Effect of vacuum impregnation and microwave application on structural changes which occurred during air-drying of apple. *LWT-Food Sci Technol* 38:471–477
- Crank J (1975) *The mathematics of diffusion*, 2nd edn. Clarendon Press, Oxford
- Dadali G, Demirhan E, Özbek B (2007) Microwave heat treatment of spinach: drying kinetics and effective moisture diffusivity. *Dry Technol* 25:1703–1712
- Dadali G, Özbek B (2008) Microwave heat treatment of leek: drying kinetic and effective moisture diffusivity. *Int J Food Sci Tech* 43:1443–1451
- Dai J, Orsat V, Raghavan GSV (2010) Visualization of microwave energy distribution in a multimode microwave cavity using COCl₂ on gypsum plates. *J Microw Power Electromagn Energy* 44:144–152
- Dinčov DD, Parrott KA, Pericleous KA (2004) Heat and mass transfer in two-phase porous materials under intensive microwave heating. *J Food Eng* 65:403–412
- Drouzas AE, Schubert H (1996) Microwave application in vacuum drying of fruits. *J Food Eng* 28:203–209
- Drouzas AE, Tsami E, Saravacos GD (1999) Microwave/vacuum drying of model fruits gels. *J Food Eng* 39:117–122
- Duan X, Zhang M, Mujumdar AS, Wang R (2010) Trends in microwave-assisted freeze drying of foods. *Dry Technol* 28: 444–453
- Durance TD, Wang JH (2002) Energy consumption, density and rehydration rate of vacuum microwave- and hot-air convection-dehydrated tomatoes. *J Food Sci* 67:2212–2216
- Feng H, Tang J (1998) Microwave finish drying of diced apples in a spouted bed. *J Food Sci* 63:679–683
- Feng H, Tang J, Mattinson DS, Fellman JK (1999) Microwave and spouted bed drying of blueberries: the effect of drying and pretreatment methods on physical properties and retention of flavor volatiles. *J Food Process Preserv* 23:463–479
- Feng H, Tang J, Cavalieri R (1999) Combined microwave and spouted bed drying of diced apples: effect of drying conditions on drying kinetics and product temperature. Invited contribution to the special Hall Issue. *Dry Technol* 17:1981–1998
- Feng H (2000) *Microwave drying of particulate foods in a spouted bed*, PhD thesis. Washington State University, Washington
- Feng H, Tang J, Dixon-Warren SJ (2000) Determination of moisture diffusivity of red delicious apple tissues by thermogravimetric analysis. *Dry Technol* 18:1183–1199
- Feng H, Tang J, Cavalieri RP, Plumb OA (2001) Heat and mass transport in microwave drying of porous materials in a spouted bed. *AIChE J* 47:1499–1512
- Feng H (2002) Analysis of microwave assisted fluidized-bed drying of particulate product with a simplified heat and mass transfer model. *Int Commun Heat Mass Transf* 29:1021–1028

39. Feng H, Tang J, Cavalieri RP (2002) Dielectric properties of dehydrated apples as affected by moisture and temperature. *Trans ASAE* 45:129–135
40. Finkelstein EA, Strombotne KL (2010) The economics of obesity. *Am J Clin Nutr* 91:1520S–1524S
41. George RM (1993) Recent progress in product, package and process design for microwavable foods. *Trends Food Sci Technol* 4:390–394
42. Giri S, Prasad S (2007) Drying kinetics and rehydration characteristics of microwave-vacuum and convective hot-air dried mushrooms. *J Food Eng* 78:512–521
43. Goedeken DL, Tong CH, Virtanen AJ (1997) Dielectric properties of a pregelatinized bread system at 2450 MHz as a function of temperature, moisture, salt and specific volume. *J Food Sci* 62:145–149
44. Goksu EI, Sumnu G, Esin A (2005) Effect of microwave on fluidized bed drying of macaroni beads. *J Food Eng* 66:463–468
45. Grolmes JL, Bergman TL (1990) Dielectrically-assisted drying of a nonhygroscopic porous material. *Dry Technol* 8:953–975
46. Gunasekaran S (1999) Pulsed microwave-vacuum drying of food materials. *Dry Technol* 17:395–412
47. Jambhale AS, Barbadekar BV (2008) Microwave drying system with high-tech phase controller: a modified applicator. *World Acad Sci Eng Technol* 46:1–5
48. Jansen W, Van der Wekken B (1991) Modeling of dielectrically assisted drying. *J Microwave Power Electromagn Energy* 26:227–236
49. Jolly P, Turner I (1989) Microwave drying of porous media. In: *Proceedings of 4th August heat and mass transfer conference* pp 331–342
50. Jun W, Zhang J, Wang J, Xu N (1999) Modeling simultaneous heat and mass transfer for microwave drying on apple. *Dry Technol* 17:1927–1934
51. Khraisheh MAM, Cooper TJR, Magee TRA (1995) Investigation and modeling of combined microwave and air drying. *Food Bioprod Process* 73:121–126
52. King CJ (1971) *Freeze drying of foods*. Butterworth, London
53. Knoerzer K, Regier M, Schubert H (2011) Simulating and measurement transient tree-dimensional temperature distributions in microwave processing. In: Knoerzer K (ed) *Innovative food processing technologies: advances in multiphysics simulation*. Wiley, NJ
54. Kowalski SJ, Musielak G, Banaszak J (2010) Heat and mass transfer during microwave-convective drying. *AIChE J* 56:24–35
55. Kuprianoff J (1966) Fundamental and practical aspects of the freezing of foodstuffs. In: Louis René Rey (ed) *Advances in freeze-drying*. Hermann, Paris
56. Lewicki PO, Lenart A (1995) Osmotic dehydration of fruits and vegetables. In: Mujumdar AS (ed) *Handbook of industrial drying*. Marcel Dekker, Inc., New York, pp 691–713
57. Lian G, Harris CS, Evans R, Warboys M (1997) Coupled heat and moisture transfer during microwave vacuum drying. *J Microwave Power Electromagn Energy* 32:34–44
58. Lin TM, Durance TD, Scaman CH (1998) Characterization of vacuum microwave, air and freeze-dried carrot slices. *Food Res Int* 31:111–117
59. Lu L, Tang J, Liang L (1998) Moisture distribution in spherical foods in microwave drying. *Dry Technol* 16:503–524
60. Lu L, Tang J, Ran X (1999) Temperature and moisture changes during microwave drying of sliced food. *Dry Technol* 17:413–432
61. Luikov AV (1964) *Advances in heat transfer*. Pergamon Press, New York
62. Maskan M (2001) Drying, shrinkage and rehydration characteristics of kiwifruits during hot air and microwave drying. *J Food Eng* 48:177–182
63. McMinn WAM, Khraisheh MAM, Magee TRA (2003) Modeling the mass transfer during convective, microwave and combined microwave-convective drying of solid slabs and cylinders. *Food Res Int* 36:977–983
64. McMinn WAM (2006) Thin-layer modelling of the convective, microwave, microwave-convective and microwave-vacuum drying of lactose powder. *J Food Eng* 72:113–123
65. Melendez W, Flake BA, Schmidt PS (1989) A process simulation model for convective and dielectrically-enhanced single-zone tunnel dryers. In: Boehm RF (ed) *Simulation of thermal energy systems*. ASME, New York
66. Metaxas AC, Meredith RJ (1983) *Industrial microwave heating*. Peter Peregrinus, London
67. Moyne C, Perré P (1991) Processes relation to drying: part I, theoretical model. *Dry Technol* 9:1135–1152
68. Ni H, Datta AK, Torrance KW (1999) Moisture transport in intensive microwave heating of biomaterials: a multiphase porous media model. *Int J Heat Mass Transf* 42:1501–1512
69. Nijhuis HH, Torringa E, Luyten H, René F, Jones P, Funebo T, Ohlsson T (1996) Research needs and opportunities in the dry conservation of fruits and vegetables. *Dry Technol* 14:1429–1457
70. Nury FS, Salunkhe DK (1968) Effects of microwave dehydration on components of apples. USDA, ARS
71. Page GE (1949) Factors influencing the maximum rates of air drying shelled corn in thin-layer, MS thesis. Purdue University, Indiana
72. Parker PM (2005) *The 2006–2011 world outlook for microwave ovens*. ICON Group International
73. Patankar SV (1980) *Numerical heat transfer and fluid flow*. Hemisphere, Washington
74. Philip JR, de Vries DA (1957) Moisture movement in porous materials under temperature gradient. *Trans Am Geophys Union* 38:222–232
75. Plumb OA (2000) Modeling the drying process. In: Kambiz V (ed) *Handbook of porous media*, 2nd edn. CRC Press, Boca Raton
76. Prabhanjan DG, Raghavan GSV, Bosisio RG (1993) Microwave drying of wheat using surface wave applicator. *Powder Handl Process* 5:73–77
77. Prabhanjan DG, Ramaswamy HS, Raghavan GSV (1995) Microwave-assisted convective air drying of thin layer carrots. *J Food Eng* 25:283–293
78. Prothon F, Ahrne LM, Funebo T, Kidman S, Langton M, Sjöholm I (2001) Effects of combined osmotic and microwave dehydration of apple on texture, microstructure and rehydration characteristics. *LWT-Food Sci Technol* 34:95–101
79. Ptasznik W, Zygmunt S, Kudra T (1990) Simulation of RF-assisted convective drying for seed quality broad bean. *Dry Technol* 8:977–992
80. Qiu Y, Hawley MC (1998) Computer controlled variable frequency microwave processing of graphite/epoxy in a single mode cavity. In: *Proceedings of 33rd microwave power symposium* pp 42–46
81. Raghavan GSV, Silveira AM (2001) Shrinkage characteristics of strawberries osmotically dehydrated in combination with microwave drying. *Dry Technol* 19:405–414
82. Rattanadecho P, Klinbun W (2011) Theoretical analysis of microwave heating of dielectric materials filled in a rectangular waveguide with various resonator distances. *J Heat Transf* 13:031008–031017
83. Ressing H, Ressing M, Durance T (2007) Modeling the mechanisms of dough puffing during vacuum microwave drying using the finite element method. *J Food Eng* 82:498–508
84. Rotstein E (1986) *Advances in transport phenomena and thermodynamics in the drying of cellular food systems*. In: Mujumdar AS (ed) *Drying* 86. Hemisphere, New York

85. Ruíz Díaz G, Martínez-Monzó J, Chiralt PFA (2003) Modelling of dehydration-rehydration of orange slices in combined microwave/air drying. *Innov Food Sci Emerg Technol* 4:203–209
86. Salagnac P, Glouannec P, Lecharpentier D (2004) Numerical modeling of heat and mass transfer in porous medium during combined hot air, infrared and microwaves drying. *Int J Heat Mass Transf* 47:4479–4489
87. Saltiel C, Datta AK (1999) Heat and mass transfer in microwave processing. *Adv Heat Transfer* 33:1–94
88. Sanga ECM, Mujumdar AS, Raghavan GSY (2002) Simulation of convection-microwave drying for a shrinking material. *Chem Eng Process* 41:487–499
89. Schiffmann RF (1995) Microwave and dielectric drying. In: Mujumdar AS (ed) *Handbook of industrial drying*, 2nd edn. Marcel Dekker, New York
90. Schiffmann RF (2006) In: Willert-Porada M (ed) *Advances in microwave and radio frequency processing*. Springer, New York
91. Schubert H, Regie M (2006) Novel and traditional microwave applications in the food industry. In: Willert-Porada M (ed) *Advances in microwave and radio frequency processing*. Springer, New York
92. Sharma GP, Prasad S (2004) Effective moisture diffusivity of garlic cloves undergoing microwave-convective drying. *J Food Eng* 65:609–617
93. Sherwood TK (1929) The drying of solids-I. *Ind Eng Chem* 21:12–16
94. Shivhare U, Raghavan V, Bosisio R, Giroux M (1993) Microwave drying of soybean at 2.45 GHz. *J Microw Power Electromagn Energy* 28:11–17
95. Sipahioglu O, Barringer SA (2003) Dielectric properties of vegetables and fruits as a function of temperature, ash, and moisture content. *J Food Sci* 68:234–239
96. Skansi D, Bajza Z, Arapovic A (1995) Experimental evaluation of the microwave drying of leather. *J Soc Leather Technol Chem* 79:171–177
97. Souraki BA, Mowla D (2008) Experimental and theoretical investigation of drying behavior of garlic in an inert medium fluidized bed assisted by microwave. *J Food Eng* 88:438–449
98. Souraki BA, Andrés A, Mowla D (2009) Mathematical modeling of microwave-assisted inert medium fluidized bed drying of cylindrical carrot samples. *Chem Eng Process* 48:296–305
99. Soysal Y (2004) Microwave drying characteristics of parsley. *Biosyst Eng* 89:167–173
100. Stuchly SS, Stuchly MA (1980) *Microwave drying: potential and limitations*. In: Mujumdar AS (ed) *Advances in drying*. Hemisphere Publishing Corporation, New York
101. Sun E, Datta A, Loba S (1995) Composition-based prediction of dielectric properties of foods. *J Microwave Power Electromagn Energy* 30:205–212
102. Sutar PP, Prasad S (2007) Modeling microwave vacuum drying kinetics and moisture diffusivity of carrot slices. *Dry Technol* 25:1695–1702
103. Tang J, Feng H, Lau M (2002) Microwave heating in food processing. In: Young X, Tang J (eds) *Advances in bioprocessing engineering*. World Scientific, Singapore
104. Therdthai N, Zhou W (2009) Characterization of microwave vacuum drying and hot air drying of mint leaves (*Mentha cordifolia* Opiz ex Fresen). *J Food Eng* 91:482–489
105. Tong CH, Lund DB (1990) Effective moisture diffusivity in porous materials as a function of temperature and moisture content. *Biotechnol Prog* 6:67–75
106. Tong CH, Lentz RR, Rossen JL (1994) Dielectric properties of pea puree at 915 MHz and 2450 MHz as a function of temperature. *J Food Sci* 59:121–122
107. Torringa HM, Dijk EJV, Bartels PV (1996) Microwave puffing of vegetables: Modeling and measurements. In: *Proceedings of 31st microwave power symposium*, pp 16–19
108. Tulasidas TN, Raghavan GSV, Norris ER (1993) Microwave and convective drying of grapes. *Trans ASAE* 36:1816–1865
109. Tulasidas TN, Raghavan GSV, Mujumdar AS (1995) Microwave drying of grapes in a single mode cavity at 2450 MHz—II: quality and energy aspects. *Dry Technol* 13:1973–1992
110. Turner IW, Jolly PG (1991) Combined microwave and convective drying of a porous material. *Dry Technol* 9:1209–1269
111. Turner IW, Rudolph V (1992) Convective and microwave enhanced drying of glass beads. In: Mujumdar AS (ed) *Drying'92*. Elsevier, Amsterdam
112. Turner IW, Puiggali JR, Jomaa W (1998) A numerical investigation of combined microwave and convective drying of a hygroscopic porous material: a study based on pine wood. *Chem Eng Res Des* 76:193–209
113. Turner IW, Perré P (2004) Vacuum drying of wood with radiative heating: II. Comparison between theory and experiment. *AIChE J* 50:108–118
114. Wang J, Zhang JP, Wang JP, Xu NZ (1999) Modeling simultaneous heat and mass transfer for microwave drying on apple. *Dry Technol* 17:1927–1934
115. Wei CK, Davis HT, Davis EA, Gordon J (1985) Heat and mass transfer in water-laden sandstone: microwave heating. *AIChE J* 31:842–848
116. Welt BA, Tong CH, Rossen JL (1992) An apparatus for providing constant and homogeneous temperatures in low viscosity liquids during microwave heating. *Microw World* 13:9–13
117. Whitaker S (1977) Simultaneous heat, mass, momentum transfer in porous media: a theory of drying. *Adv Heat Transf* 13: 119–203
118. Wu HW, Tao Z, Chen GH, Deng HW (2004) Conjugate heat and mass transfer process within porous media with dielectric cores in microwave freeze drying. *Chem Eng Sci* 59:2921–2928
119. Yan WQ, Zhang M, Huang LL, Tang JM, Mujumdar AS, Sun JC (2010) Study of the optimization of puffing characteristics of potato cubes by spouted bed drying enhanced with microwave. *J Sci Food Agric* 90:1300–1307
120. Yang APP, Yang TCS, Taub IA (1998) Effectiveness of metallic shielding in improving the uniformity of microwave heating. In: *Proceedings of 33rd microwave power symposium* pp 90–92
121. Zhang M, Tang J, Mujumdar AS, Wang S (2006) Trends in microwave-related drying of fruits and vegetables. *Trends Food Sci Technol* 17:524–534
122. Zhang QO, Jackson TH, Ugan A (2000) Numerical modeling of microwave induced natural convection. *Int J Heat Mass Transf* 43:2141–2154