# Wrestling With a (Mathematical) Stance of Inquiry

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I take this opportunity to write about a topic I have been considering and studying—teachers' mathematical content knowledge—through the lens of a stance of inquiry. My purpose for doing so is twofold. First, I see connections between how we in our research group view mathematical content knowledge and stances of inquiry, and I would like to see whether explicating this is useful. Second, writing provides an opportunity to clarify one's thinking (and show me how much more I need to keep thinking), and as Duke Ellington once said, "I don't need time. What I need is a deadline." So thanks for the deadline and the reason for putting my thoughts down.

## What is a stance (of inquiry)?

I argue that a stance is related to other teacher characteristics, so I begin by briefly describing two of the most studied constructs in teacher education: knowledge and beliefs. Beliefs are "psychologically held understandings, premises, or propositions about the world that are felt to be true" (Richardson, 1996, p. 103*)*. Beliefs might be thought of as lenses through which one views the world (Philipp, 2007). People do not necessarily choose their beliefs, nor are they always aware of them, but they exist, affecting their views of the world. Knowledge is similar to beliefs, although educators often refer to knowledge as more local than beliefs and tend to view knowledge as *justified* (or, depending upon one's ontological stance, as *true*) belief. Both knowledge and belief tend to be thought of as teacher characteristics held somewhat independently of practice, and efforts to measure knowledge and belief tend to be grounded in decontextualized tasks, problems, or scales. The similarities between knowledge and beliefs are more important than the differences, and one might refer to them both as conceptions (Thompson, 1992), but however they are characterized, they are intimately related to one's stance as a teacher and learner of mathematics.

So what is a stance? The first two definitions of stance from dictionary.com (http://dictionary.reference.com/) are

- *the position or bearing of the body while standing: legs spread in a wide stance; the threatening stance of the bull.*
- *a mental or emotional position adopted with respect to something: They assumed an increasingly hostile stance in their foreign policy.*

Related to the first definition, which highlights the physical aspects of a stance, is an example from sports. Wrestlers begin a match from the standing position, and a wrestler's stance enables him (with apologies to all the female wrestlers) to make an aggressive move against his opponent or react to a move by his opponent. As such, a stance is a way to position oneself so as to act either proactively or reactively, and one selects a stance on the basis of his knowledge of technique, his strength, and his quickness. Further, although a wrestler generally approaches opponents from the same stance, he might slightly alter his stance or his moves from that stance on the basis of knowledge about his opponent.

Using the wrestling analogy, I highlight that a stance is "*adopted with respect to something*" (see second dictionary definition). However, some aspects of the physical analogy are, at best, questionable. A wrestler can learn and practice a stance somewhat independently of his knowledge of wrestling, but one's stance toward inquiry is intricately related to, and cannot be separated from, one's knowledge, beliefs, values, and so on. In other words, one does not simply learn a new inquiry stance, nor does one pick one out and try it on as one might do with a new physical stance.

**Mathematical Stance of Inquiry**

Much of what has been written about inquiry stances is about *pedagogical stances of inquiry*. For example, Franke et al. (2001) contended that the development of an inquiry stance provides teachers with opportunities for generative growth—a stance toward their own learning that enables them to take ownership of their knowledge; pose questions of interest to their local circumstances related to teaching and learning; generate answers to their own questions; and use their classrooms, students, and colleagues as sites for learning. Cochran-Smith and Lytle (1999) acknowledged that teaching from an inquiry stance is difficult, making every decision messy and potentially troubling and, further, "associated more with uncertainty than certainty, more with posing problems and dilemmas than with solving them, and also with the recognition that inquiry both stems from and generates questions" (1999, p. 294).

I consider another kind of inquiry stance: A *mathematical stance of inquiry*. Because no single, unifying view of mathematical knowledge exists, neither does a single mathematical stance of inquiry. For example, one who holds an instrumental understanding of mathematics (rules *without* reason, Skemp, 1978) approaches mathematics differently from the way one who holds a relational understanding (rules *with* reasons, Skemp, 1978) approaches mathematics. I adopt a definition of *mathematical proficiency* that my colleagues and I have found useful in our work. The

National Research Council (NRC, 2001), the operating arm of the National Academy of Sciences, published a consensus document in which they presented five interrelated strands that, together, comprise mathematical proficiency (see Figure 1). The first four are *conceptual understanding, procedural fluency, strategic competence* (the ability to formulate, represent, and solve mathematical problems), and *adaptive reasoning* (the capacity to think logically and to informally and formally justify one's reasoning). Closely related to, if not synonymous with, a positive mathematical stance of inquiry, is the fifth strand, *productive disposition,* "the tendency to see sense in mathematics, to perceive it as both useful and worthwhile, to believe that steady effort in learning mathematics pays off, and to see oneself as an effective learner and doer of mathematics" (NRC, 2001, p. 131). One holding a productive disposition would have confidence in his or her ability to learn mathematics and be free of debilitative anxiety toward mathematics. In our own work, we are expanding the construct of productive disposition to include mathematical integrity (DeBellis & Goldin, 1999). Mathematical integrity "describes an individual's affective psychological posture in relation to when mathematics is 'right,' when a problem is solved satisfactorily, when the learner's understanding is sufficient, or when mathematical achievement is deserving of respect or commendation" (DeBellis & Goldin, 1999, p. 253). A component of mathematical integrity involves knowing when one understands and when one does not, an aspect captured in this quote by Nobel laureate author Anatole France: "An education isn't how much you have committed to memory, or even how much you know. It's being able to differentiate between what you do know and what you don't."

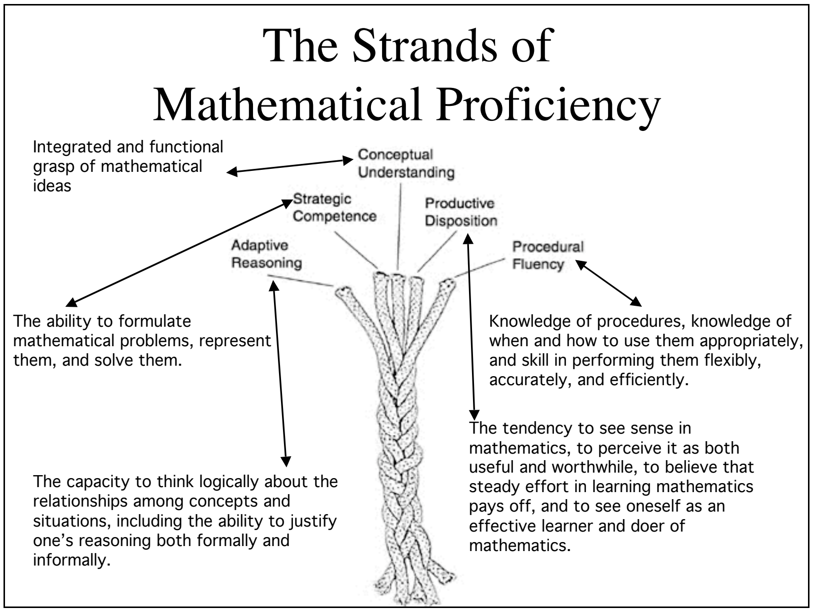


*Figure 1.* The strands of mathematical proficiency (*Adding It Up*, 2001).

As important as productive disposition appears to be, mathematics assessments, at all level, fail to include this. Not only do state-assessment writers, mathematicians, and K–12 classroom teachers ignore productive disposition but also few researchers include this component in their measures of mathematical content knowledge. Our research team understands this failure: Productive disposition is difficult to measure! When our research team set out to measure mathematical content knowledge of elementary school teachers, we developed items designed to address the strands of mathematical proficiency. (Figure 2 contains two items we developed.[1]) Although we think that our

[1] For those interested, I share two results from our study related to these two content items. First, teachers who had engaged in sustained professional development around a focus on children's mathematical thinking for at least 2 years performed better on these two tasks than teachers who were just as experienced and who had signed up for but had yet to begin the professional development. Second, whereas experienced teachers without professional development performed significantly better than prospective elementary school teachers on one of these tasks, they did not perform better on the other

items consistently addressed four of the five strands, we were initially unable to assess productive disposition with paper-and-pencil tasks.[2]

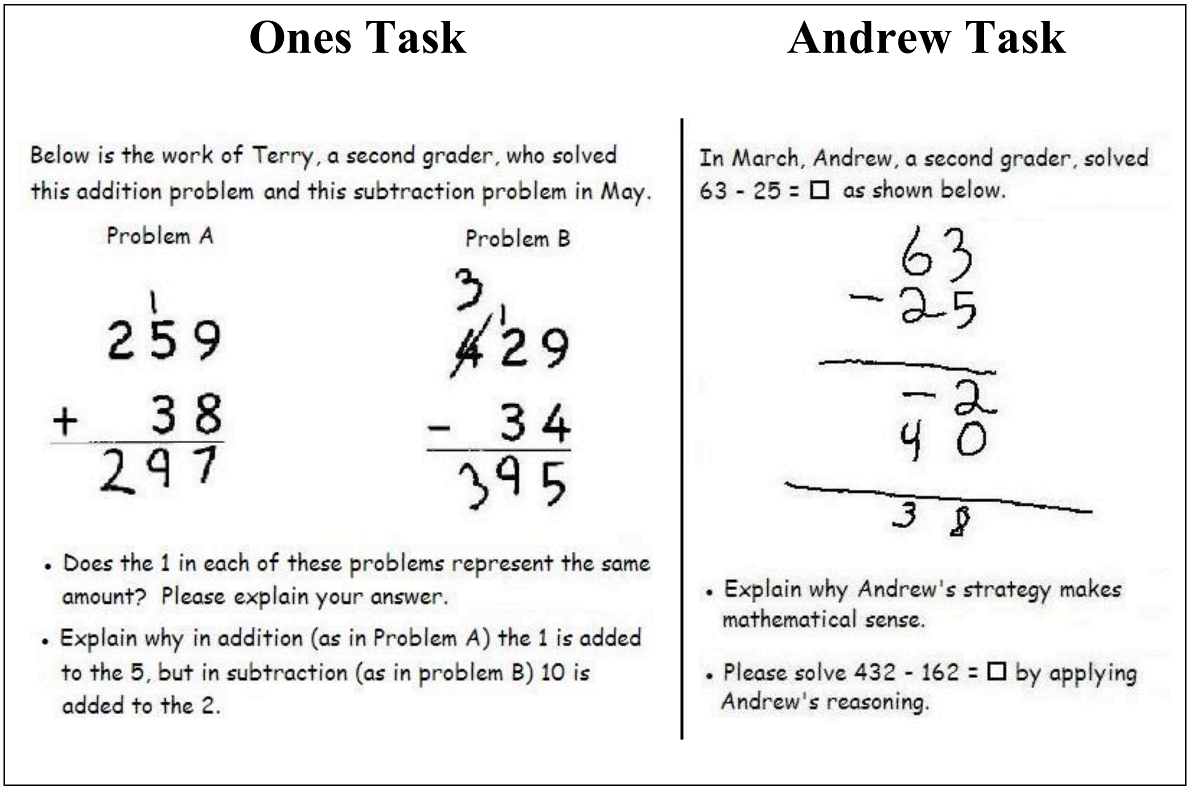


*Figure 2.* The Ones Task and the Andrew Task.

Mathematical productive disposition is at the heart of a mathematical stance of inquiry. An inquiry stance toward mathematics is associated with a sense that mathematics is useful in one's life; that one is capable, through effort, of learning mathematics; and that one can make an honest appraisal of his or her own understanding (or lack of understanding) of mathematics. Mathematical productive disposition does not develop independently of the other four components of mathematical proficiency.

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task. I leave it for you to conjecture which of these two tasks was privileged by teaching experience, which was not, and why.

[2] Recently we modified the One's task to address productive disposition, and we believe that we may have been successful. Furthermore, we are currently studying productive disposition in the context of teacher focus-group discussions.

Students seldom come to believe in their abilities to learn mathematics because they are told that they can do it; they believe that they can learn mathematics because they experience learning mathematics. Students become problem solvers by successfully solving problems, and they develop conceptual orientations by engaging with mathematics conceptually.

The inclusion of productive disposition as a component of mathematical proficiency is important as a statement that proficiency in mathematics has affective aspects. Furthermore, the relationship between affective factors and cognitive factors, though not entirely understood academically, is something with which all learners grapple. Mandler (1989) presented a theory that affective factors arise out of emotional responses to interrupted plans. According to Mandler, one approaches a task with a schema for how the task will be completed, and if the anticipated sequence of actions cannot be completed, the result is a physiological response. For Mandler, these affective factors were connected to one's knowledge and beliefs, because interpretations of interruptions vary from individual to individual. McLeod (1992) offered the example of a sixth-grade student solving a story problem. If the student believed that all mathematics problems should be solvable in a couple minutes but the student was unable to solve the problem in that period of time, the student might experience an arousal that he or she would interpret as negative. If these experiences were repeated frequently, the student might develop a negative attitude toward story problems, and, in many cases, such negative attitudes toward one aspect of mathematics generalize to negative attitudes toward mathematics in general or, perhaps even worse, toward the student's view of himself or herself as a mathematics learner. If, however, students believe that story problems can challenge even good problem solvers and require a longer period of time to solve, then arousal at an inability to quickly solve the problem might not be interpreted as negative but might prompt the student to delve further into the problem. The student's interpretation of the experience, not the experience itself, determines the outcome, a sentiment captured in the saying that life is 10% what happens to us and 90% how we react to it. But this saying oversimplifies, even trivializes, the complexities involved in that which leads to one's interpretations. One's interpretation is not simply selected off a shelf the way one, say, selects clothes from a rack. One's interpretation grows out of, and is based upon, years of engagement with the subject. Students' interpretations of their mathematical experiences are integrally related to their past experiences with mathematics. Furthermore, I contend that productive disposition is generally not supported in the teaching of mathematics, and, in fact, that the culture of mathematics teaching often works contrary to developing a positive mathematical disposition. For example, consider developing mathematical integrity. When teaching high school mathematics, I told my students, in an attempt to reward effort, that if they did not know what to do with a mathematics problem on a test, they should make an attempt and I would give them partial credit. I now wonder whether I was also rewarding students for writing meaningless mathematics without asking them to reflect upon the extent to which their work made sense. Might I have rewarded effort in another way while helping them become more reflective about what they did and did not know? What is the relationship between developing a mathematical stance of inquiry and developing other components of mathematical proficiency?

If a mathematical stance of inquiry is inseparable from one's experiences with other components of mathematical proficiency, what are the implications for developing a pedagogical stance of inquiry? What experiences are associated with teachers' being poised to view themselves as taking ownership of their knowledge (Franke et al., 2001) or with coming to view classroom decision making as often more about "posing problems and dilemmas than with solving them" (Cochran-Smith & Lytle, 1999, p. 294)? What knowledge, beliefs, and practices must codevelop with one's pedagogical stance of inquiry? To what extent do we provide experiences for teachers to develop these stances, and in what ways does the culture of school work against the development of pedagogical stances of inquiry?

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