Wave-frequency Flows Within a Near-bed Vegetation Canopy

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Abstract

We study water flows and wave dissipation within near-bed pneumatophore canopies at the wave-exposed fringe of a mangrove forest on Cù Lao Dung Island, in the Mekong Delta. To evaluate canopy drag, the three-dimensional geometry of pneumatophore stems growing upward from the buried lateral roots of Sonneratia caseolaris mangroves was reconstructed from photogrammetric surveys. In cases where hydrodynamic measurements were obtained, up to 84 stems per square meter were observed, with stem heights < 0.6 m, and basal diameters 0.01 – 0.02 m. The parameter $a = (\text{frontal area of pneumatophores blocking the flow})/(\text{canopy volume})$ ranged from zero to 1.8 m$^{-1}$. Within-canopy water velocity displayed a phase lead and slight attenuation relative to above-canopy flows. The phase lead was frequency-dependent, ranging from 0 – 30 degrees at the frequencies of energetic waves (> 0.1 Hz), and up to 90 degrees at lower frequencies. A model is developed for wave-induced flows within the vertically variable canopy. Scaling suggests that acceleration-induced forces and vertical mixing were negligible at wave frequencies. Consistent with theory, drag-induced vertical variability in velocity scaled with $\epsilon_f = T_w/(2\pi T_f)$, where $T_w =$ wave period, $T_f =$ $2/(C_D a |u|)$ is the frictional time scale, $C_D \approx 2$ is the drag coefficient, and $|u|$ is a typical flow speed. For given wave conditions ($|u|$ and $T_w$), theory predicts increasing dissipation with increasing vegetation density (i.e. increasing $a$), until a maximum is reached for order-one $\epsilon_f$. For larger $\epsilon_f$, within-canopy flow is so inhibited by drag that further increases in $a$ reduce within-canopy dissipation. For observed cases, $\epsilon_f \leq 0.38$ at energetic wave frequencies, so wave dissipation near the forest edge is expected.

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to increase with increasing pneumatophore canopy density, but under different wave conditions the most dense canopies may occasionally approach the dissipation maximum ($\epsilon_f \approx 1$). Predicted dissipation by the pneumatophore canopy was sufficient to attenuate most wave energy over distances slightly less (more) than 100 m into the marsh in 1 m (2 m) water depth.

Keywords:
wave dissipation, drag coefficient, mangrove, photogrammetry, estuaries, vegetation
Highlights

- Water velocity measured under waves in a Mekong Delta near-bed vegetation canopy.
- Within-canopy velocity consistent with a simple theory incorporating canopy drag.
- Theory predicts dissipation proportional to canopy density for observed cases.
- At higher canopy densities, predicted wave dissipation would decline.
- Predicted wave height halved tens (hundreds) of m into swamp in 1 m (2 m) depth.

1. Introduction

Aquatic vegetation can shelter coastlines from energetic waves currents (Jadhav et al., 2013; Riffe et al., 2011; Temmerman et al., 2013), and sometimes creates low-energy regions of sediment deposition (Bouma et al., 2007; Furukawa et al., 1997). Diverse coastal wetland ecosystems (Greenberg et al., 2007) are often productive regions of rapid carbon burial (Siikamäki et al., 2012), and many wetlands are threatened by pollution, land reclamation, and conversion to aquaculture. Mangrove swamps in particular have been impacted by widespread deforestation (Giri et al., 2011; Thu and Populus, 2007). In Vietnam’s Mekong Delta, reduced sediment supply is expected to compound the effects of rising sea levels on coastal evolution (Anthony et al., 2015; Nicholls et al., 1999). Improved understanding of interacting hydrodynamics and morphodynamics may prove relevant to management of mangrove ecosystems, coastal flooding and erosion.

We present observations from a mangrove-covered coast in the Southern Mekong Delta. This coast is exposed to the open ocean, and therefore to energetic waves. Previous researchers have measured attenuation of waves propagating into mangrove forests, finding substantial dissipation after tens or hundreds of meters of propagation (Bao, 2011; Horstman et al., 2014; Mazda et al., 2006). A variety of models have been developed to predict wave dissipation within vegetation. Dalrymple et al. (1984) modeled the drag forces exerted by vegetation, and the associated wave dissipation, using frictionless linear wave theory to estimate the wave-induced water flows.
(hereafter ‘wave orbital velocities’) past plant stems. This approach provides a good approximation in sufficiently sparse canopies, but frictionless wave theory is not applicable at very high canopy densities. For near-bed vegetation, Lowe et al. (2005) (hereafter LKM05) derived a general and practical model for dissipation and vertically averaged wave orbital velocities applicable to very dense canopies. Extensions were developed for two-layer canopies (Weitzman et al., 2015) and random waves, and tested or calibrated against observed wave attenuation (Jadhav et al., 2013; Lowe et al., 2007). Zeller et al. (2015) found that major model simplifications can often be justified: in many natural canopies, acceleration-dependence of drag (resulting from Froude-Krylov forces and added mass, e.g. Sumer and Fredsøe, 1997) and vertical turbulent mixing can both be neglected at wave frequencies. Building on the approach of Zeller et al. (2015), we examine the properties of a simple model for wave orbital velocities, and test the model against observations within natural near-bed canopies of mangrove pneumatophores.

Previous laboratory experiments have quantified enhanced dissipation (Pujol et al., 2013a) and reduction of wave orbital velocities (Lowe et al., 2005; Luhar et al., 2010; Pujol et al., 2013b; Weitzman et al., 2015) by drag within artificial canopies. In natural canopies, many stems, each having its own unique size and shape, often create substantial vertical variability. Such vertical variability is here quantified using recently developed photogrammetric techniques, which measure the three-dimensional geometry of all stems within sampled regions of canopy (Liénard et al., 2016). Combining data from photogrammetric surveys with wave orbital velocities measured within natural pneumatophore canopies, we test a simple vertically resolved model derived from the equations of Zeller et al. (2015). Modeling is then applied to analyze the ability of Mekong Delta pneumatophore canopies to shelter onshore regions from incoming waves.

The model is outlined in Section 2, in turn considering within-canopy velocity (Section 2.1) and dissipation (Section 2.2). The field site, instrumentation, and data analysis are outlined in Section 3. Results presented in Section 4, and summarized in Section 5.

2. Theory

2.1. Vertical Variability of Velocity Induced by Canopy Drag

The vegetation canopy is described by statistics \( n \) = number of stems per square meter, \( d \) = mean stem diameter, \( a \) = cross-sectional area of vegetation
blocking the flow per cubic meter of canopy (units m$^{-1}$), and $\phi = \text{proportion of volume occupied by solid stems}$. For roughly cylindrical vertical stems, $a \approx nd$ and $\phi \approx (\pi/4)nd^2$. Here $n$ is a function of elevation $z$ above the bed, because the number of stems usually decreases from a maximum at the bed to zero immediately above the elevation $h_v$ of the highest stem ($a$, $d$ and $\phi$ also vary with elevation).

As noted by Zeller et al. (2015), for many natural canopies $\phi \ll 1$, and at wave frequencies vertical mixing of momentum and acceleration-dependence of canopy drag forces are negligible (assumptions required for these simplifications are noted below and in Appendix A). For weakly nonlinear waves (|$u|$ $\ll c$, where $c$ = wave phase speed), the horizontal momentum equation (e.g, eq.1.25 of Zeller et al., 2015, with wave-induced momentum flux neglected) then reduces to

$$\frac{\partial u}{\partial t} + F_D + \frac{1}{\rho} \frac{\partial p}{\partial x} = 0,$$

where $t =$ time, $u =$ water velocity in horizontal direction $x$, $\rho =$ water density, $p =$ pressure, the canopy drag force per kilogram of water is

$$F_D = \frac{C_D}{2} |u|u,$$

$C_D$ is the drag coefficient for flow past a stem (here $C_D \approx 2$, Section 4), and $|u|$ is the magnitude of $u$. Therefore, acceleration [first term of (1)] results from vegetation drag (second term), and lateral pressure gradients (third term). Equation (1) is obtained by averaging over a horizontal region encompassing many stems (Zeller et al., 2015), but extending substantially less than one wavelength in any direction. For a velocity $u_b$ above the canopy, (1) reduces to

$$\frac{\partial u_b}{\partial t} + \frac{1}{\rho} \frac{\partial p}{\partial x} = 0.$$  

Let

$$T_f = \frac{2}{C_Da|u|}.$$

Substituting $F_D = u/T_f$ [which follows from (2) and (4)] into (1) shows that $T_f$ is a frictional timescale; if pressure forcing were negligible then drag would cause an initial velocity to decay over time of order $T_f$. Depth-uniform horizontal pressure gradients are eliminated by subtracting (3) from (1), leading
\[ \frac{\partial (u - u_b)}{\partial t} + \frac{u}{T_f} = 0. \]  
\[ \text{(5)} \]

From (5),
\[ \frac{\text{Typical magnitude of } u - u_b}{\text{Typical magnitude of } u} = O(\epsilon_f), \]
\[ \text{(6)} \]
where the ordering parameter
\[ \epsilon_f = \frac{T_w}{2\pi T_f} = \frac{C_D a |u| T_w}{4\pi}, \]
\[ \text{(7)} \]

\( T_w \) = typical wave period, and \( O(\epsilon_f) \) denotes a term of order \( \epsilon_f \). Therefore, if the frictional timescale \( T_f \) is much longer than the wave period \( T_w \), drag is insufficient to generate a large difference between within- and above-canopy velocities.

To clarify the assumptions underlying (1), let
\[ \epsilon_\nu = \frac{C_f |u| T_w}{2\pi h_w}. \]
\[ \text{(8)} \]

Perturbation analysis (Appendix A) resembling the scaling of Zeller et al. (2015) establishes (1) from more general equations [e.g. from equation (4) of LKM05] given \( u \ll c, \phi \ll 1 \) and \( \epsilon_\nu \ll \max\{\epsilon_f^{-1}, 1\} \). Under these conditions vertical mixing [neglected in (1) and (5)] is negligible through most of the canopy, although mixing remains significant in a thin neighborhood of the bed called the Wave Bottom Boundary Layer (WBBL, Mei, 1989).

For simplicity, we solve (5) for a time series discretely sampled at times \( t_j = j \Delta t \), with \( j \) between \(-N\) and \( N\). Velocities are represented by the Fourier series
\[ u = \sum_{j=-N}^{N} \langle u \rangle_T e^{i2\pi t/T_j}, \]
\[ \text{(9)} \]
\[ u_b = \sum_{j=-N}^{N} \langle u_b \rangle_T e^{i2\pi t/T_j}, \]
\[ \text{(10)} \]
where, for any variable \( \beta \), \( \langle \beta \rangle_T \) denotes the complex amplitude at period \( T_j = (2N + 1) \Delta t / j \). To derive a spectral model, we set \( |u| \) to the constant value \( (8/\pi)^{1/2} u_{rms} \) in (4), where \( u_{rms} \) is the root-mean-squared value of \( u \). Here
the factor \((8/\pi)^{1/2}\) was chosen to give the correct mean value for dissipation, given a Gaussian velocity distribution. Similar approximations have been used for WBBL models (Tolman, 1994), for spectral models of forces on cylinders (Borgman, 1967), for simulating forces on saltmarsh vegetation (Mullarney and Henderson, 2010) and for simulating wave dissipation in coral and vegetation canopies (Jadhav et al., 2013; Lowe et al., 2007). From (5), (9), and (10), within- and above-canopy velocities are related by

\[
\langle u \rangle_{T_j} = \hat{\Gamma}_j \langle u_b \rangle_{T_j},
\]

where the transfer function

\[
\hat{\Gamma}_j = \frac{1}{1 - i\epsilon_j},
\]

and the value of \(\epsilon_j\) for the \(j^{th}\) Fourier component is [c.f. (7)]

\[
\epsilon_j = \frac{T_j}{2\pi T_f} = \frac{C_D a |u| T_j}{4\pi}.
\]

For small \(\epsilon_j\), (12) predicts an \(O(\epsilon_j)\) phase lead (within-canopy velocity ahead of overlying velocity), and a smaller \(O(\epsilon^2)\) reduction in velocity amplitude. From (13), \(\epsilon_j\) is proportional to wave period \(T_j\), so both phase lead and amplitude attenuation increase with increasing period.

The solution (12) attributes all amplitude attenuation to canopy drag. However, even in the absence of drag, wave velocities decline with increasing depth unless the wavelength greatly exceeds the depth (i.e. unless waves are in shallow water, Mei, 1989). Frictionless linear wave theory predicts a ratio \(\zeta = \cosh(\kappa z_l)/\cosh(\kappa z_u)\) between velocities at lower and upper elevations \(z_l\) and \(z_u\), where \(k = 2\pi/\text{wavelength}\) is calculated from the dispersion relation \((2\pi/T_j)^2 = gk \tanh(kh)\) and \(g = 9.80\text{m/s}^2\) (Lowe et al., 2007). For the canopies considered here, such frictionless within-canopy depth attenuation will prove insignificant \((\zeta \approx 1)\) for wave frequencies below about 0.3 Hz. At higher frequencies, \(\zeta\) will drop substantially below 1, but for these cases \(\hat{\Gamma}_j \approx 1\). Therefore, a transfer function accounting for both frictional and frictionless vertical attenuation is

\[
\Gamma_j = \frac{\zeta}{1 - i\epsilon_j}.
\]

This prediction will be compared with observations.
2.2. Simulated Dissipation

Letting \( x \) be the wave propagation direction and neglecting bottom friction and directional spread, the depth-integrated wave energy balance at period \( T_j \) is

\[
\frac{\partial E_{T_j}c_{g,j}}{\partial x} = - \int_0^h \mathcal{E}_{T_j} \, dz \tag{15}
\]

where \( E_{T_j} \) is the spectral density of depth-integrated wave energy at period \( T_j \), \( c_{g,j} \) is the associated group velocity, and \( \mathcal{E}_{T_j} \) is the spectral density of dissipation by vegetation drag (a function of elevation). Let \( \Phi_{T_j}(X,Y) \) be the cross spectrum at period \( T_j \) between any two time series \( X \) and \( Y \) (so \( \Phi_{T_j}(X,X) \) is a power spectrum). The dissipation of wave energy by vegetation drag is the mean of \( u \mathcal{D}_s \) from (2) and (11) the spectral density of dissipation is [c.f. equation 27 of Lowe et al. (2007) and equation 7 of Jadhav et al. (2013)]

\[
\mathcal{E}_{T_j} = \frac{C_d}{2} a |u||\Gamma|^2 \Phi_{T_j}(u_b,u_b), \tag{16}
\]

where the velocity \( u_b \) immediately above the pneumatophores will be approximated by the velocity predicted at the bed by frictionless linear wave theory (as is appropriate for near-bed canopies). Expressions (13)–(14) for \( \Gamma \) depend on within-canopy \( |u| \), which itself depends on \( \Gamma \). To obtain an explicit predictive model for wave attenuation, an approximation for \( \Gamma \) in terms of above-canopy velocity \( |u_b| \) is now derived. Since \( \Gamma_j \) is a smooth function of \( T_j \), assuming a narrow-banded spectrum with peak period \( T_w \) the transfer function is approximated by a single value \( \Gamma_0 \) applicable to the full spectrum (this approach resembles Lowe et al., 2007). Substituting \( |\Gamma_0| = |u|/|u_b| \) into (13) and (14) then yields

\[
\Gamma_0 = \frac{1}{1 - i \epsilon_0 |\Gamma_0|}, \tag{17}
\]

where the dimensionless damping parameter

\[
\epsilon_0 = \frac{C_D a |u_b| T_w}{4 \pi} \tag{18}
\]

differs from \( \epsilon_f \) in that the above-canopy velocity \( u_b \) appears in (18) whereas the within-canopy velocity \( u \) appears in (7). For \( \epsilon_0 = 0 \) this yields \( \Gamma_0 = 1 \), and for \( \epsilon_0 > 0 \) some algebra yields

\[
|\Gamma_0|^2 = \frac{(1 + 4 \epsilon_0^2)^{1/2} - 1}{2 \epsilon_0^2}. \tag{19}
\]
The depth-average of $|\Gamma_0|$ was denoted $\alpha_w$ by LKM05. The model considered here does not correspond to any of the simple limits (canopy-independent, inertia-dominated, or unidirectional) considered by LKM05. Consistent with the general analysis of LKM05, the orbital displacement (denoted $A_{rms}^\infty$ by LKM05 and proportional to $|u|T_w$ here) is the sole hydrodynamic variable controlling $|\Gamma_0|$. However, $|\Gamma_0|$ here depends on only $\epsilon_0$, which differs by a factor of order $\phi^{1/2}C_D/(2\pi^{3/2})$ from the parameter $A_{rms}^\infty/S$ considered by LKM05, where the spacing between stems $S \approx n^{-1/2}$.

From (16), (18) and (19)

$$\mathcal{E}_{T_j} = \chi \frac{2\pi \Phi_{T_a}(u_b, u_b)}{T_w},$$

where the dimensionless dissipation

$$\chi = \frac{[(1 + 4\epsilon_0)\phi^{1/2} - 1]^3/2}{2^{3/2}\epsilon_0^2}. \quad (21)$$

Now $4\pi \chi$ is the ratio between the dissipation in one wave period and the above-canopy energy kinetic energy (both expressed per kilogram of water). For sufficiently sparse canopies ($\epsilon_0 \ll 1$), $\chi \approx \epsilon_0$, and since $\epsilon_0$ is proportional to $a$ (18), increasing vegetation density is associated with increasing dissipation (Figure 1). Expression (20) with $\chi = \epsilon_0$ could alternatively have been derived by simply neglecting all within-canopy attenuation of velocity (as was done, for example, by Dalrymple et al., 1984), so this limit is called the ‘unattenuated dissipation model’. As $\epsilon_0$ increases above about 0.4 (vertical dotted line, Figure 1), dissipation departs from the unattenuated prediction, and $\chi$ reaches a maximum of 1/2 when $\epsilon_0 = 2^{1/2}$. Further increase in $\epsilon_0$ reduces $\chi$ [$\chi \approx \epsilon_0^{-1/2}$ for $\epsilon_0 \gg 1$]. Such complex behaviour is possible owing to reduced wave orbital velocities, and resulting reduced dissipation, within very dense canopies. Lowe et al. (2007) previously derived a more general model for this reduction in wave dissipation resulting from reduced within-canopy wave orbital velocities. The analysis here is novel in deriving an analytic expression, which highlights the importance of $\epsilon_0$ for the limit of low $\phi$ and negligible vertical mixing.

Solution of the model (15), (18), (20), and (21) for arbitrary incident waves propagating into a forest with variable geometry and bathymetry requires numerical integration. However, for $\epsilon_0 \ll 1$ and constant depth the unattenuated model yields a simple analytic solution. From (18), (20), and
\( \chi \approx \epsilon_0 \), the depth-integrated dissipation

\[
\int_0^h \mathcal{E}_w \, dz = \frac{C_D |u_0|}{2} \Phi_{T_j}(u_b, u_b), \tag{22}
\]

where

\[
\lambda = \int_0^h a \, dz. \tag{23}
\]

Substituting (22) and the linear wave theory result

\( E_{T_j} = (h/2)[1 + \sinh(2kh)/(2kh)]\Phi_{T_j}(u_b, u_b) \) into (15) and solving yields

\[
E_{T_j} = \frac{E_0}{(1 + x/x_0)^2}, \tag{24}
\]

where the dissipation length scale

\[
x_0 = \frac{2hc_{g,j}[1 + \sinh(2kh)/(2kh)]}{C_D |u_0|} \tag{25}
\]

is the distance waves propagate into the marsh before their amplitude is halved by dissipation, \( E_0 \) is the energy at \( x = 0 \), and \( |u_0| \) is \( |u_b| \) at \( x = 0 \). This closely resembles a solution found by Dalrymple et al. (1984), here simplified for a near-bed canopy.

3. Field Site, Instrumentation, and Data Analysis

We present measurements of water velocity and vegetation geometry obtained along the forested, ocean-exposed coast of Cù Lao Dung Island in the Southern Mekong Delta. Within 0.8 m of the bed, dense canopies of pneumatophore stems grow upward from the buried lateral roots of the dominant Sonneratia caseolaris mangroves. Two control deployments C1 and C2 were conducted on the unvegetated tidal flats, offshore of the forest edge. The remaining deployments P1 – P5 sampled several locations within the forest, and several elevations within the pneumatophore canopy. Deployments C1 – C2 and P2 – P4 were conducted within 15 m of forest-edge location N 9°29.5090’, E 106°14.6377’, near the Southwest corner of Cù Lao Dung Island. Deployments P1 and P5 were conducted at the forest-edge location N 9°33.9296’, E 106°17.5562’, near the Northeast corner of the Island. The pneumatophore canopies near the forest edge, sampled during P1 – P5, were usually more
Figure 1: Dimensionless dissipation $\chi$ [thin black curve, see equations (20)–(21)] versus dimensionless damping parameter $\epsilon_0$, showing maximum dissipation for order-one $\epsilon_0$. Solid grey curve: approximation $\chi = \epsilon_0$, valid for $\epsilon_0 \ll 1$. Dashed grey curve: approximation $\chi = \epsilon_0^{-1/2}$, valid for $\epsilon_0 \gg 1$. Dotted black vertical line: $\epsilon_0 = 0.4$.

Figure 2: Image (a) and three-dimensional reconstruction (b) of low-density pneumatophore canopy P2. The grey quadrat, marking one square meter, was removed before the incoming tide flooded the sampling region. The Vector (yellow square) and Vectrino (orange triangle) current meters measured above- and within-canopy velocity. The aquadopp current meter (red diamond) was not used here.
dense (larger $n$ and $d$) than canopies farther into the forest interior. Additional deployment details are given by Mullarney et al. (2016); Norris et al. (2016) [this issue].

During each deployment, a pair of vertically displaced current meters simultaneously measured velocities at ‘lower’ ($0.019 \leq z_l \leq 0.38$ m) and ‘upper’ ($0.3 \leq z_u \leq 0.93$ m) elevations (Columns 1–4, Table 3). During P1–P5, the lower and upper instruments respectively measured velocity within and above the pneumatophore canopy. Deployment names indicate the relative canopy density at the lower instrument elevation, with the lowest density for P1 and the highest density for P5. Instruments deployed in the low density case P2 are shown in Figure 2a. ‘Deployments’ P1 and P5 actually refer to a single case, where a vertical stack of three instruments (one above the canopy and two within the canopy) were deployed for 20 hours, and two different time series were used to obtain maximum sampling duration (one 4.8-hour time series was centered on the first high tide observed during this deployment, and the second 4.5-hour time series was centered on the second high tide; spectra from these two time series we averaged to yield the results presented, although similar results are obtained if these time series are analyzed separately). The upper and middle instruments were compared in ‘Deployment P1’, whereas the upper and lower instruments were compared in ‘Deployment P5’. For remaining cases (C1–C2, P2–P4), measurement commenced soon after the upper instrument was submerged by the rising tide, and instruments were removed within a few hours, limiting sampling durations to $\leq 157$ minutes. Except for a few brief stoppages required for technical reasons, these time series were continuous. Deployments P3 and P4 were conducted at the same horizontal location on successive days, whereas each of C1, C2, and P2 was conducted at a different location. The lower instrument was always a Nortek Vectrino profiler, whereas the upper instrument was either a Vectrino profiler (deployments C1, P1, P5) or a Nortek Vector (C2, P2, P3, P4). Vectrino profilers measured velocity at 50 Hz every millimeter along a 35 mm profile. For deployments C1, P1, and P3, data from either end of each Vectrino profile were discarded, and velocities from the central 15 mm were averaged to yield a single 50 Hz time series for each Vectrino. For deployments C2, P2, P4, and P5, where the mid profile for the near-bed Vectrino was sometimes within 2 cm of the bed, velocities from to highest 5 mm of the profile were averaged, to minimize the influence of the WBBL (in these cases Vectrinos provided acoustic tracking of the bed, from which the mean sampling elevation $z_i$ was determined). Vectors measured point
velocity at 32 Hz, which was linearly interpolated to 50 Hz for comparison with Vectrino data. All Vectrinos were cabled to a single computer, whereas Vectors logged data internally. To synchronize Vectors with Vectrinos, Vector clock times were adjusted (usually by < 1 s) to ensure minimal phase shift between vertically displaced instruments at high (> 0.5 Hz) frequencies (these clock adjustments had minimal effect on phases at longer periods). Small (< 5 degree) errors in rotation about the vertical axis were removed by optimizing the correlation between upper and lower velocity time series. Results are presented for horizontal velocity $u$ in the mean wave direction (roughly onshore, and defined as the principal axis of measured 0.1 – 0.8 Hz velocity, Kuik et al., 1988). Low (< 70%) correlation data and spikes were replaced by linear interpolation.

Photographic surveys of pneumatophore canopies were conducted at low tide, within one day of corresponding current meter deployments. Photographs obtained from many (55 – 387) angles around 1 m² quadrats centered on the current meters were used to reconstruct a three-dimensional point cloud using the open-source photogrammetric software VisualSFM (Figure 2b). The diameter and location of every pneumatophore within each quadrat was estimated every 5 mm along vertical profiles extending from about $z = 0.03$ m to the top of the canopy, using the point cloud and the techniques developed by Liénard et al. (2016). For each of deployments P1 – P5, vertical profiles of vegetation statistics $n$, $d$, $a$ and $\phi$ were calculated from these data.

To test (14), empirical transfer functions between the upper velocity $u_u$ and the lower velocity $u_l$ were estimated. The variance of upper and lower velocities is quantified as a function of wave period by the power spectra $\Phi_{T_j}(u_u, u_u)$ and $\Phi_{T_j}(u_l, u_l)$. The cross-spectral matrix at period $T_j$ between the upper and lower velocities is

$$M(T_j) = \begin{bmatrix} \Phi_{T_j}(u_u, u_u), & \Phi_{T_j}(u_u, u_l) \\ \Phi_{T_j}(u_l, u_u), & \Phi_{T_j}(u_l, u_l) \end{bmatrix}. \quad (26)$$

The (complex) dominant eigenvector of $M(T_j)$, denoted $(\tilde{u}_u, \tilde{u}_l)$, is the dominant EOF for period-$T_j$ fluctuations in the two velocities (Henderson et al., 2001). An empirical transfer function was estimated from this dominant EOF as

$$\hat{\Gamma}_{T_j} = \frac{\tilde{u}_l}{\tilde{u}_u}. \quad (27)$$
The magnitude and argument of this complex transfer function are respectively called the ‘Gain’ and the ‘Phase’ [the phase simply equals the phase of the cross spectrum \( \Phi_T(u_u, u_l) \)]. Since the estimated transfer function is meaningful only when upper and lower layers are coherent, we also present the squared coherence \( |\Phi_T(u_u, u_l)|^2 / (\Phi_T(u_u, u_u)\Phi_T(u_l, u_l)) \). For each deployment, spectra were estimated using 50%-overlapping, hanning-windowed time series segments (Welch, 1967), yielding > 50 degrees of freedom.

To compare theoretical and empirical transfer functions [(14) and (27)], we write \( \varepsilon_j = \alpha |u|T_j/(4\pi) \), and for each deployment choose the single \( \alpha \) value that minimizes the squared error \( |\Gamma_T - \Gamma_T|^2 \) summed over all coherent frequencies (coherence \( > 0.5 \)). Here \( |u| \) is calculated as \( (8/\pi)^{1/2} u_{\text{rms}} \), where \( u_{\text{rms}} \) = root-mean-squared lower velocity. From (4) and (13) \( \alpha = C_D a \), so a plot of \( \alpha \) values obtained from multiple deployments against corresponding \( a \) values should show a positive correlation, with zero offset and slope equal to \( C_D \). Laboratory experiments with oscillating cylinders indicate that \( C_D \) depends on the stem Reynolds number \( R_e = 2u_{\text{rms}}d/\nu_m \) and the Keulegan-Carpenter number \( K_C = 2u_{\text{rms}}T_w/d \), where \( \nu_m \) = kinematic viscosity of water and \( T_w \) is a peak period. For \( R_e = O(10^3) \) and \( K_C = 50–300 \), experiments suggest \( C_D = 1–3 \) (e.g. Sumer and Fredsøe, 1997).

To examine whether drag-induced reduction of within-canopy wave orbital velocities substantially reduced dissipation, \( \varepsilon_0 \) was calculated from (18) and observations. The percentage overestimation in dissipation that would result from using the unattenuated model is \( (\varepsilon_0/\chi - 1) \times 100\% \).

<table>
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<th>( z_l, z_u )  (m)</th>
<th>( a )  (m(^{-1}))</th>
<th>( T_p ) (s)</th>
<th>( u_u )  (ms(^{-1}))</th>
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<td>0.14</td>
<td>2.2</td>
<td>0.28</td>
<td>0.16</td>
<td>0.015</td>
</tr>
<tr>
<td>P2</td>
<td>3/11/2015</td>
<td>37</td>
<td>0.028, 0.60</td>
<td>0.57</td>
<td>6.7</td>
<td>0.22</td>
<td>0.42</td>
<td>0.13</td>
</tr>
<tr>
<td>P3</td>
<td>3/8/2015</td>
<td>157</td>
<td>0.19, 0.93</td>
<td>0.58</td>
<td>5.9</td>
<td>0.20</td>
<td>1.0</td>
<td>0.17</td>
</tr>
<tr>
<td>P4</td>
<td>3/7/2015</td>
<td>129</td>
<td>0.021, 0.4</td>
<td>1.45</td>
<td>2.7</td>
<td>0.17</td>
<td>2.8</td>
<td>0.18</td>
</tr>
<tr>
<td>P5</td>
<td>3/13/2015</td>
<td>553</td>
<td>0.021, 0.81</td>
<td>1.78</td>
<td>2.2</td>
<td>0.28</td>
<td>4.2</td>
<td>0.38</td>
</tr>
</tbody>
</table>

Table 1: Summary of deployments. \( z_l, z_u \) = elevations of lower and upper velocity measurements; vegetation area \( a \) evaluated at elevation \( z_l \); \( T_p \) = peak period of upper velocity spectrum; \( u_u \) = root-mean-square upper velocity magnitude; \( \alpha \) = fitting parameter (theoretical value \( C_D a \)); \( \varepsilon_0 \) = dimensionless damping parameter.
4. Results

During the seven deployments, $a$ ranged from 0 to 1.78 m$^{-1}$, peak wave periods ranged from 2.0 to 6.7 s, and rms velocity magnitudes ranged from 0.17 to 0.53 ms$^{-1}$ (Table 3). During P1 – P5, stem Reynolds numbers ranged from $3.3 \times 10^3$ to $5.4 \times 10^3$, and Keulegan-Carpenter numbers ranged from 91 to 290.

Pneumatophore canopies were about 0.6 m high, with 45 – 84 stems m$^{-2}$, frontal area $a \leq 1.8$ m$^{-1}$, mean stem diameters $d \leq 0.028$ m, and solid volume fraction $\phi \leq 0.030$ (Figure 3). The lowest density canopy was measured at the location of deployment P2 (light grey curves, Figure 3, and pictured in Figure 2). Although profiles of stem number and diameter differed between the location of deployments P1 and P5 and the location of deployments P3 and P4, the two locations showed similar profiles of $a$ (solid black and dashed dark grey curves, Figure 3). In all cases, $\phi$ was sufficiently low that wake interference between stems (Tanino and Nepf, 2008) and acceleration-dependent forces were negligible.

During deployment C1, short period, moderate energy waves were observed (Table 3). As in all deployments, the velocity power spectrum was dominated by 0.1 – 1 Hz (hereafter ‘incident frequency’) waves (Figure 4a). Upper and lower velocities were nearly equal at most incident frequencies, with similar power spectra (Figure 4a), strong coherence (Figure 4b), gain
Figure 4: Comparison of upper (elevation $z = 0.30$ m) and lower ($z = 0.10$ m) velocities measured over unvegetated tidal flats, Deployment C1, 27 September 2014. (a) power spectra of upper (grey) and lower (black) velocities. (b–d): Squared coherence (b), gain (c), $<1$ indicates lower velocity smaller magnitude), and phase (c, positive indicates lower velocity leading) between upper and lower velocities. In (c)–(d), black (grey) dots indicate frequencies with squared coherence greater (less) than 0.5, and grey curves indicate fitted theoretical transfer function (14). In (c), black dashed curve, in this case indistinguishable form grey curve, indicates theoretical depth attenuation predicted by non-dissipative linear wave theory.
usually near 1 (Figure 4c) and phase near zero (Figure 4d). However, at the highest frequencies (>0.5 Hz) the gain dropped, consistent with the depth attenuation predicted by frictionless linear wave theory (black dots and black dashed curve, Figure 4c). At low frequencies (<0.1 Hz), energy levels were low (Figure 4a) and upper and lower velocities were only weakly coherent (Figure 4b). Since surface wave velocities are essentially depth-uniform at these frequencies, these vertically incoherent low frequency motions were likely not surface waves, but may have been turbulent eddies. Fitting the model (14) to the observed transfer function (27) yields a fitted transfer function near 1 (grey curves, Figure 4c,d) and a fitting parameter $\alpha$ near zero (eighth column, table 3), as expected in the absence of vegetation.

More energetic, longer period waves were observed during C2 (Table 3). As in deployment C1, upper and lower velocities were nearly equal at 0.1–
0.5 Hz, with some depth attenuation resembling the linear theory prediction at higher frequencies (Figure 5a–d). At lower frequencies (<0.1 Hz) a small phase shift was observed, with the lower velocity leading by about 5°. Gain also dropped below 1 at about 0.03 Hz. A gain <1 and a phase lead up to 25° are expected within the WBBL, and the lower current meter was near the bed (z_l = 0.019). Although the incident-frequency WBBL was likely too thin to be measured by the lower current meter, boundary layer thickness increases with wave period (Mei, 1989), so the gain and phase observed at frequencies <0.01 Hz may indicate the outer edge of the infragravity wave bottom boundary layer. Fitting the theoretical transfer function yields \Gamma near 1, and a fitting parameter \alpha that is small, although not as small as in case C1 (Table 3). Regardless of the small phase shift at low frequencies, both C1 and C2 indicate that, in the absence of vegetation, depth-dependence at energetic incident frequencies was minimal, except for frictionless depth attenuation at the highest frequencies.

In contrast to deployments C1 – C2 on the flats, clearer depth-dependence was observed within pneumatophores. Deployment P2, in a relatively low density canopy (Table 3), showed a phase shift of about 10° and 0.1 Hz (Figure 6d). Consistent with theory, phase decreased with increasing frequency. The model (14) provides a reasonable fit to observations (compare black dots and grey curves, Figure 6c,d), with a significantly non-zero fitting parameter \alpha (Table 3). Nevertheless, departures of the transfer function from the frictionless value \zeta were small and the damping parameter \epsilon_0 = 0.13 was substantially less than 1. Results were similar for deployment P3 (Figure 7), with clearer frictionless depth attenuation at high frequencies resulting from a relatively large separation of z_u and z_l. The greatest attenuation and phase shift were observed in the high density canopy of P5 (Figure 8, note the extended vertical axis range of Figure 8d). At low frequencies (<0.03 Hz), gain approached zero and phase approached 90° (Figure 8c,d), consistent with the \epsilon_j \rightarrow \infty limit of (14), although associated coherence was low (Figure 8b). The estimated \epsilon_0 = 0.38 was the largest for any deployment. Therefore, observations spanned the region to the left of the vertical black dotted line in Figure 1, where the unattenuated model provides a good approximation (solid grey and black curves match). For \epsilon_0 = 0.38, the unattenuated model is expected to overestimate dissipation by 20%.

As expected given the theoretical relationship \alpha = C_D a, values of \alpha obtained by fitting transfer functions for the seven deployments were correlated with corresponding a values (Figure 9). The best-fit slope suggests
Figure 6: As figure 4, but for deployment P2, vegetation density $a = 0.57$, 11 March 2015, upper and lower elevations $z = 0.60$ and 0.028 m.
$C_D = 2.1$, comparable to values for oscillating cylinders at these Reynolds and Keulegan-Carpenter numbers. As expected given dominance by canopy drag (rather than the vertical mixing responsible for WBBL vertical structure), there is no tendency for velocities measured within 3 cm of the bed (black symbols, Figure 9) to yield different $\alpha$ values than velocities measured at higher elevations (grey symbols).

For the observed small values of $\epsilon_0$, the unattenuated model (25) provides an estimate of the distance $x_0$ waves can propagate over uniform pneumatophore canopies before their amplitude is halved. Trapezoidal integration of measured $a$ profiles yields $\lambda$ (see 23) of 0.095 (location of P2), 0.29 (location of P3, P4) and 0.28 (location of P1, P5). Here, we focus on dissipation by pneumatophores, rather than tree trunks, because $a$ values are much higher within pneumatophore canopies. Given $C_D = 2$, depth $h = 2$ m, and observed wave conditions, associated wave decay distances were 510 m (P2), 180 m (P3), 200 m (P4), and 180 m (P1, P5). Although these decay distances are large, predictions are sensitive to the assumed water depth; for a depth of 1 m, decay distances are reduced to 180 m (P2), 65 m (P3), 74 m (P4), and 48 m (P1, P5).

5. Summary

On Cù Lao Dung Island in the Mekong Delta, wave orbital velocities measured within pneumatophore canopies were roughly consistent with a simple model (Zeller et al., 2015) that neglects vertical mixing and acceleration-dependent drag. Frequency dependence matched an analytic solution predicted using a linearized drag law. When solving for the dissipation of all wave energy (rather than frequency dependence) the model considered here is nonlinear, but sufficiently simple that an analytic solution relates within-canopy flow to above-canopy flow.

In low density canopies, modeled dissipation is proportional to canopy density, measured by $a$. Conversely, in sufficiently high density canopies, increasing density can reduce near-bed flows enough to reduce dissipation (dissipation proportional to $a^{-1/2}$). The transition between the low- and high-density regimes is controlled by a dimensionless parameter $\epsilon_0$ (18), which is a function of both canopy geometry and hydrodynamic conditions; dissipation is maximum at the transition between low- and high-density regimes, which occurs near $\epsilon_0 = 1.4$ (Figure 1). The pneumatophore canopies and hydrodynamic conditions we measured on Cù Lao Dung Island were in the low
Figure 7: As figure 4, but for deployment P3, vegetation density $a = 0.58$, 8 March 2015, upper and lower elevations $z = 0.93$ and 0.19 m.
Figure 8: As figure 4, but with extended y axis range, and for deployment P5, vegetation density $a = 1.78$, 13 March 2015, upper and lower elevations $z = 0.81$ and 0.021 m.
Figure 9: Fitting parameter $\alpha$ for hydrodynamic model versus measured vegetation density parameter $a$. Dashed line indicates theoretical relationship $\alpha = C_D a$ with best-fit $C_D = 2.1$. Deployment C1 grey circle, C2 black circle [partially obscured, near (0,0)], P1 grey diamond, P2 black square, P3 grey triangle, P4 black triangle, P5 black diamond. Black symbols: lower velocity measured at elevation $z_l < 0.03$ m above bed (grey symbols: $z_l > 0.03$ m).
density regime ($\epsilon_0 \leq 0.38$). This finding is consistent with the increase in within-canopy turbulent dissipation with increasing $a$ found by Norris et al. (2016) [this issue]. Nevertheless, $\epsilon_0$ was not much less than 1, so occasional departure from the low density regime remains likely. For one observed case (P5), a doubling of wave period and wave height, plausible in storm conditions, would yield $\epsilon_0 = 1.5$, at the boundary between low- and high-density regimes. In this case, using frictionless linear theory to estimate within-canopy velocity would yield wave dissipation estimates triple the true value. Elevated dissipation within pneumatophores may enhance sediment mobilization (Norris et al., 2016, this issue), whereas sheltering of the bed at high canopy densities could create conditions for rapid deposition. Rapid deposition can bury pneumatophores, reducing mangrove vigor or even causing death (Ellison, 1999; Moffett et al., 2015; Nardin et al., 2016). Therefore, the tendency for observed pneumatophore canopies to approach, but seldom exceed, the density of maximum dissipation may be adaptive.

By enhancing dissipation at the forest edge, pneumatophores reduce the height of waves propagating into the forest interior. Theory for low canopy density ($\epsilon_0 \ll 1$), resembling the model of Dalrymple et al. (1984), predicts strong depth-dependence to this sheltering effect, qualitatively consistent with previous observations (Mazda et al., 2006). For observed waves and pneumatophore canopies, simulations predict that waves dissipated substantially as they propagated tens of meters in 1 m water depth, or hundreds of meters in 2 m depth (propagation distances would be shorter under higher wave energies). However, the observations reported here were collected near the forest fringe, where canopies were relatively dense (Norris et al., 2016, this issue). Therefore, lower canopy densities might lead to reduced dissipation in the forest interior.

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Appendix A. Perturbation expansion

In the limit of small $\phi$ (denoted $\lambda_p$ by LKM05), subtracting our equation (3) from equation (4) of LKM05 yields

$$\frac{\partial u_d}{\partial t} + \frac{u}{T_f} - \nu \frac{\partial^2 u_d}{\partial z^2} = 0,$$  \hspace{1cm} (A.1)

where the defect velocity $u_d = u - u_b$ and the eddy viscosity $\nu = C_f |u_d| h_v$. Dimensionless variables, denoted by $\ast$, are defined by

$$t = \frac{T_w t_s}{2\pi},$$  \hspace{1cm} (A.2)

$$z = h_v z_s.$$  \hspace{1cm} (A.3)

Consider separately the cases $\epsilon_f \leq 1$ and $\epsilon_f > 1$.

Appendix A.1. Case $\epsilon_f \leq 1$

For moderate or weak attenuation of within-canopy velocity, define dimensionless variables by

$$u = u_0 u_\ast,$$  \hspace{1cm} (A.4)

$$u_d = \epsilon_f u_0 u_{d\ast},$$  \hspace{1cm} (A.5)

$$\nu = C_f \epsilon_f u_0 h_v \nu_\ast.$$  \hspace{1cm} (A.6)

Now (A.1) becomes

$$\frac{\partial u_{d\ast}}{\partial t_\ast} + u_\ast - \epsilon_f \epsilon \nu \frac{\partial^2 u_{d\ast}}{\partial z^2_\ast} = 0,$$  \hspace{1cm} (A.7)

so mixing is negligible if $\epsilon_f \epsilon \nu \ll 1$.

Appendix A.2. Case $\epsilon_f > 1$

For the case of large attenuation, we rescale

$$u_\infty = u_0 u_{\infty},$$  \hspace{1cm} (A.8)

$$u = \frac{u_0 u_\ast}{\epsilon_f},$$  \hspace{1cm} (A.9)

$$u_d \approx -u_\infty = -u_0 u_{\infty},$$  \hspace{1cm} (A.10)

$$\nu = C_f u_0 h_v \nu_\ast.$$  \hspace{1cm} (A.11)
Now (A.1) becomes

\[-\frac{\partial u_\infty}{\partial t_*} + u_* - \epsilon_\nu \nu_* \frac{\partial^2 u_d}{\partial z_*^2} = 0,\]

so mixing is negligible if $\epsilon_\nu \ll 1$.

References


