Depth Dependence of Nearshore Currents and Eddies

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Key Points.

- Twelve ADPs resolved the vertical, across- and along-shore structure of surfzone currents and eddies.
- Transition to depth-uniform currents with increasing breaking predicted by model for breaker-induced turbulent mixing.
- Eddies (<0.01 Hz velocity fluctuations) showed greater depth-dependence than alongshore currents.

Abstract. The three-dimensional (across-shore, alongshore, and vertical) structure of hourly-mean currents and <0.01 Hz eddies was measured on a natural beach using 12 Acoustic Doppler Profilers. Both eddies and alongshore currents became relatively depth-uniform inside the surfzone. Eddies consistently showed greater depth dependence than alongshore currents. A two-layer model, derived by perturbation expansion of the wave-averaged shallow water equations, yielded separate equations for depth-averaged and depth-dependent velocity components. For alongshore currents, depth dependence is generated by opposite forcing on lower and upper layers, respectively by bottom friction (quantified by timescale $\lambda_b^{-1}$) and waves or winds. This generation is balanced by mixing between upper and lower layers (timescale $\lambda_m^{-1}$), yielding a ratio between depth-dependent and depth-averaged alongshore currents equal to $\lambda_b/\lambda_m$. Established models for bottom friction and breaker-induced interfacial mixing predicted a surfzone reduction in $\lambda_b/\lambda_m$ consistent with the observed reduction in alongshore current depth dependence. Scatter around trends was considerable. Alongshore variability was significant for depth-averaged across-shore currents and for depth-dependence of
currents and eddies. Predominantly seaward across-shore Eulerian currents were usually bottom intensified (or surface intensified) far inside (or outside) the surfzone. This depth-dependence was consistent with wave forcing far inside the surfzone, whereas winds were likely significant offshore. In the surfzone, the mixing timescale \( \lambda_m^{-1} \) was shorter than the eddy period, so a quasi-steady balance was predicted between forcing and mixing of eddy depth dependence. Observed eddy depth dependence exceeded predictions for eddies generated by shear production (i.e. shear instabilities), possibly indicating generation of surfzone eddies by random breaking waves.
1. Introduction

Near the shore, strong water flows generated by breaking waves transport sediments and shape coastlines [Nielsen, 1992]. Fluctuating nearshore flows also transport and mix pollutants and organisms [Grant et al., 2005; Spydell et al., 2007]. Many researchers have studied the dynamics of nearshore currents and eddies [e.g. Dean and Dalrymple, 2002], here respectively defined as hourly-mean velocities, and slow (< 0.01 Hz) departures from hourly means. Here, we separate velocity profiles into depth-averaged and depth-dependent components, and focus on depth dependence. Depth dependence is important partly because sediment transport depends most strongly on the near-bed velocity, rather than the depth-averaged velocity [e.g. Conley and Beach, 2003]. Furthermore, depth-dependence may influence both horizontal and vertical mixing [Svendsen and Putrevu, 1994; Kumar and Feddersen, 2016b]. Depth-dependence is in turn influenced by vertical mixing, so observations of surfzone depth dependence provide a test of models for breaker-induced mixing. Field tests of models for depth dependence are less extensive than corresponding tests for the depth-averaged flow.

Since depth-dependent and depth-averaged flows interact, we first briefly summarize key results for depth averaged flows. We consider long, straight beaches [for alongshore non-uniform cases, see Wilson et al., 2013; Hansen et al., 2015]. Depth-averaged alongshore currents are governed primarily by a balance between convergence of the wave-induced momentum flux [the ‘radiation stress’, Longuet-Higgins and Stewart, 1964] and bottom friction [Longuet-Higgins, 1970a, b; Feddersen et al., 1998; Ruessink et al., 2001]. Bottom friction plays a smaller role in the mean depth-averaged across-shore momentum balance,
which is primarily between gradients in radiation stress and pressure, with pressure gra-
dients resulting from an across-shore sea surface slope [‘wave setup’, Longuet-Higgins and
Stewart, 1964; Bowen et al., 1968; Stive and Wind, 1982; Raubenheimer et al., 2001]. Mass
balance ensures cancellation between a seaward Eulerian mean flow (the ‘undertow’) and
a shoreward wave-induced mass flux [Svendsen, 1984].

Eddies, whose depth dependence has often been neglected, propagate alongshore at
about the speed of the mean current [Oltman-Shay et al., 1989; Noyes et al., 2004; Spy-
dell, 2016]. Eddy kinetic energy usually exceeds potential energy [Noyes et al., 2004, and
our Section 4.3 below], roughly consistent with a rigid-lid approximation. Early models
suggested that eddies form as instabilities of the alongshore current, which gain energy
by ‘shear production’, i.e. by an eddy-induced down-gradient momentum flux that mixes
the across-shore profile of the alongshore current [Bowen and Holman, 1989; Dodd and
Thornton, 1990; Allen et al., 1996]. Alternatively, eddies could be generated by fluc-
tuations in radiation stress (hereafter ‘wave forced’ eddies, Haller et al. [1998]). In the
absence of depth dependence, rigid-lid eddies respond to only the rotational, non-divergent
component of wave forcing, which at leading order results from dissipation of breaking
waves [Longuet-Higgins, 1973; Peregrine, 1999; Feddersen, 2014]. Field observations con-
firm generation of vertical vorticity by breakers [Clark et al., 2012]. Very low frequency
(< 0.004 Hz) eddies observed on an alongshore-non-uniform natural beach were explained
by wave forcing [Macmahan et al., 2004; Reniers et al., 2007]. In idealized numerical
experiments with alongshore-uniform beaches, wave forcing of vorticity fluctuations is
significant (for wave-averaged simulations of Long and Özkan Haller [2009]) or dominant
(for wave-resolving simulations of Feddersen [2014]) relative to shear production. How-
ever, eddies have also been observed in laboratory experiments with regular waves, which impose no forcing at eddy frequencies [Reniers et al., 1997].

The depth-averaged currents and eddies discussed above are modified by depth dependence. For currents, depth dependence is often analyzed using one-dimensional, vertical profile ('1dv') models. Simulated depth dependence of the undertow [Svendsen, 1984; Stive and Wind, 1986; Svendsen et al., 1987] is influenced by the depth dependence of eddy viscosity. Laboratory [Svendsen, 1987; Ting and Kirby, 1994; Scott et al., 2005; Yoon and Cox, 2010] and field [George et al., 1994; Bryan et al., 2012; Feddersen, 2007, 2012a, b] observations confirm theoretical predictions [Battjes, 1983] that breaking waves inject substantial turbulence. This near-surface injection of turbulence increases the ratio between mid-water-column and nearbed eddy viscosities [Cox and Kobayashi, 1997; Uchiyama et al., 2010], reducing the role of the bottom stress [except in a nearbed boundary layer, Stive and Wind, 1986; Svendsen et al., 1987] and yielding relatively strong seaward nearbed flows.

Models have also examined the depth-dependence of forcing by radiation stress gradients. This forcing can be divided into a surface stress or near-surface momentum injection, and a depth-uniform component [Stive and Wind, 1986; Rivero and Arcilla, 1995; Newberger and Allen, 2007a; Uchiyama et al., 2010]. For alongshore forcing on alongshore-uniform beaches, the depth-uniform component is zero, and breaker-induced forcing appears entirely as a surface stress [Deigaard, 1993] or near-surface momentum injection [Uchiyama et al., 2010]. The mean alongshore Reynolds stress (i.e. the vertical turbulent flux of alongshore momentum) $\tau_y = -\nu(\partial\langle v \rangle/\partial z)$ is then independent of depth, so mean shear of the alongshore current $\partial\langle v \rangle/\partial z$ is inversely proportional to eddy
viscosity $\nu$. Therefore, if breaking increases mixing in the mid water column, relative to mixing near the bed, then shear is concentrated near the bed and overlying velocity becomes relatively depth-uniform [for simulations of alongshore current profiles influenced by breaker-injected turbulence, see Feddersen and Trowbridge, 2005].

Field observations of depth-dependent currents are qualitatively consistent with the models outlined above [Haines and Sallenger, 1994; Garcez Faria et al., 1998, 2000]. For across-shore currents in the surfzone, neglecting bottom stress degrades model skill only slightly [Garcez Faria et al., 2000]. Depth dependence of alongshore currents, which is relatively weak above a thin nearbed layer, has been quantified by fitting logarithmic profiles to observations [Garcez Faria et al., 1998]. Depth-dependence, quantified in such analysis by fitted bottom roughness, is maximum in the trough onshore of a shore-parallel sandbar. This variability in depth dependence might result from variable breaker injection of turbulence, rather than variable physical bottom roughness [Feddersen et al., 2003; Feddersen and Trowbridge, 2005]. To test a 1dv model that incorporated breaker-injected turbulence, Reniers et al. [2004] first selected depth-averaged wave forcing to reproduce observed depth-averaged currents. The model then explained nearly half of the mean squared difference between depth-averaged across-shore currents and currents measured at specific depths. For the small depth dependence of alongshore currents skill was lower, but usually positive.

Fully three-dimensional (3d) circulation models have incorporated radiation stresses and injection of turbulence associated with wave breaking, as well as terms neglected in 1dv models such as Craik-Leibovich (CL) vortex forces [Craik, 1970; Craik and Leibovich, 1976; Leibovich, 1983] and advection of momentum by the Eulerian mean current. Such models
have positive skill in the few field cases against which they have been tested [Newberger and Allen, 2007b; Uchiyama et al., 2010]. For a single day with small wave angles (14° in 8 m depth) and a well developed sandbar, a 3d model predicted that CL forces and Eulerian advection play minor roles in the across-shore balance, but are substantial in the alongshore [Uchiyama et al., 2010]. However, these two terms almost cancel [because CL terms introduce momentum advection by the Stokes drift, which opposes Eulerian advection, Uchiyama et al., 2009, and our appendix A], leaving approximate alongshore depth-averaged balance between radiation stresses and bottom friction, as is commonly assumed [e.g. Feddersen et al., 1998].

Several models have simulated depth-dependence of eddies. Relative to depth-averaged simulations, eddy energy was reduced when depth dependence was accounted for using a quasi 3d model [this model assumes a quadratic velocity profile and a depth-uniform eddy viscosity, Zhao et al., 2003]. Fully 3d simulations yield similar results [Newberger and Allen, 2007b]. Both the quasi- and fully-3d models discussed above assumed regular waves, and therefore imposed zero eddy-frequency wave forcing. Failure of these models to produce energetic eddies led Newberger and Allen [2007b] to suggest that wave forcing may be an important source of eddy energy (consistent with the 2d modeling discussed above). Kumar and Feddersen [2016a, b] applied time-varying depth-uniform forcing to a 3d model and found that energetic eddies were generated from the surfzone onto the inner continental shelf. Published field observations of surfzone eddy depth dependence are lacking.

Most model-data comparisons discussed above used velocities measured by a vertical stack of current meters mounted on a sled. The sled was moved to several across-shore
locations to sample across-shore variability. This approach provides excellent resolution of depth dependence. However, a single sled does not yield weeks of continuous synoptic sampling, as can be provided by fixed arrays of instruments. Conversely, large fixed arrays [e.g. Elgar et al., 1997; Noyes et al., 2004] have not resolved depth variability. Here, 12 ADPs were deployed in across- and along-shore arrays to resolve vertical, across- and along-shore structure of currents and eddies. To provide simple predictions for comparison with observations, a two-layer model is derived (Section 2) from the shallow water equations for depth-dependent flow (Section 2.1). Velocities are forced by winds and waves (Section 2.2), and opposed by eddy viscosity and bottom friction (Section 2.3). An ordering expansion (Section 2.4), consistent with the magnitudes of terms measured in the surfzone, yields simple leading-order models for depth-averaged currents (Section 2.5), depth-averaged eddies (Section 2.6), depth-dependent currents (Section 2.7) and depth-dependent eddies (Section 2.8). The depth-averaged analysis recovers models developed by previous researchers and depth-dependent equations for currents are two layer equivalents of previous 1dv models, whereas the depth-dependent equations for eddies are novel. Observations are then introduced (Section 3) with discussion of the field site, instrumentation (Section 3.1), and data analysis (Section 3.2). Results (Section 4) are presented in turn for winds and waves (Section 4.1), depth-averaged currents (Section 4.2), depth-averaged eddies (Section 4.3), depth-dependent currents (Section 4.4) and depth-dependent eddies (Section 4.5). The predicted ordering was confirmed for depth-dependent currents and eddies, with 1dv terms usually dominant in the surfzone. Both eddies and currents showed significant depth dependence outside the surfzone but became relatively depth-uniform inside the surfzone, consistent with predictions for mixing by breaker-injected
turbulence. Eddy velocities showed greater depth-dependence than alongshore currents, a fact consistent with theory if forcing generates lateral convergence and divergence of depth-dependent velocities. Finally (Section 5) results are summarized, and we discuss implications for nearshore mixing, as well as potential applicability to other beaches.

2. Models for depth-dependent currents and eddies

2.1. General two-layer shallow water model

Let \( h \) = mean water depth, \( t \) = time, and \((x, y, z)\) = (across-shore, alongshore, vertical) coordinates (defined so \( z = 0 \) and \(-h\) at the surface and the bed). Assume an alongshore-uniform beach \((\partial h/\partial y = 0)\). Let \((u, v, w)\) = components \((x, y, z)\) of velocity. Coordinate \( z \) will be replaced by \( \sigma = z/h \) (unlike Mellor [2008, 2011], this \( \sigma \) coordinate is fully Eulerian), and \( w \) will be replaced by \( \omega = w - u\sigma \partial h/\partial x \). Let \( \eta \) = mean sea surface elevation, \((\tau_x, \tau_y)\) = upward turbulent fluxes of \( u \) and \( v \) (i.e Reynolds stresses), and \((X, Y)\) = forcing by waves in directions \((x, y)\) (i.e. Radiation stress gradients and CL forcing, Section 2.2).

Neglecting Coriolis, the wave-averaged barotropic, rigid-lid shallow water equations are [Blumberg and Mellor, 1987]

\[
\begin{align*}
\frac{\partial h u}{\partial t} + \frac{\partial h u^2}{\partial x} + \frac{\partial h u v}{\partial y} + \frac{\partial u \omega}{\partial \sigma} + \frac{\partial \tau_x}{\partial \sigma} + g h \frac{\partial \eta}{\partial x} &= h X, \\
\frac{\partial h v}{\partial t} + \frac{\partial h u v}{\partial x} + \frac{\partial h v^2}{\partial y} + \frac{\partial v \omega}{\partial \sigma} + \frac{\partial \tau_y}{\partial \sigma} + g h \frac{\partial \eta}{\partial y} &= h Y, \\
\frac{\partial h u}{\partial x} + \frac{\partial h v}{\partial y} + \frac{\partial \omega}{\partial \sigma} &= 0,
\end{align*}
\]

where \( \partial/\partial x \) indicates a derivative holding \( \sigma \) (rather than \( z \)) constant.

There is zero Lagrangian-mean flow across the sea surface, so

\[
\omega|_{\sigma=0} = \frac{\partial h \bar{u}_s}{\partial x} + \frac{\partial h \bar{v}_s}{\partial y}
\]
where \((\overline{u}, \overline{v})\) is the depth-averaged horizontal Stokes drift, and \(\omega|_{\sigma=0}\) denotes \(\omega\) evaluated at \(\sigma = 0\) [c.f. Hasselmann, 1971; Newberger and Allen, 2007a]. Zero flux through the seabed yields \(\omega|_{\sigma=-1} = 0\).

Two-layer analysis will provide a simple model for comparison with field observations.

For any variable \(\alpha\), let

\[
(\alpha)_u = \int_{-0.5}^{0} \alpha d\sigma, \tag{5}
\]

\[
(\alpha)_\ell = \int_{-1}^{-0.5} \alpha d\sigma. \tag{6}
\]

The depth-average is

\[
\overline{\alpha} = (\alpha)_u + (\alpha)_\ell, \tag{7}
\]

while depth-dependence is measured by

\[
\Delta \alpha = (\alpha)_u - (\alpha)_\ell. \tag{8}
\]

The across-shore velocities averaged over upper and lower layers are respectively \(2(u)_u = \overline{u} + \Delta u\) and \(2(u)_\ell = \overline{u} - \Delta u\). The ‘dimensionless depth dependence’ \(\Delta u/\overline{u}\) is zero for depth-uniform flow, is positive (negative) for surface-intensified (bottom-intensified) flow, has magnitude 1 if \((u)_u = 0\) or \((u)_\ell = 0\), and has magnitude > 1 if \((u)_u\) and \((u)_\ell\) flow in opposite directions.

To evaluate layer-averages of momentum flux terms [second and third terms of (1) – (2)], we approximate velocity with linear functions of depth:

\[
u = \overline{v} + 4\Delta v(\sigma + 0.5), \tag{9}
\]

\[
u = \overline{v} + 4\Delta v(\sigma + 0.5). \tag{10}
\]
From (5) – (10)

\[ \overline{uv} = \overline{u} \overline{v} + \frac{4}{3} \Delta u \Delta v, \]  
\[ \Delta (uv) = \overline{u} \Delta v + \overline{v} \Delta u. \]  

Replacing \( v \) with \( u \) in (11) – (12) yields expressions for \( \overline{u^2} \) and \( \Delta (u^2) \) [similarly, replacing \( u \) with \( v \) yields \( \overline{v^2} \) and \( \Delta (v^2) \)].

Depth-averaging (1) – (3) and using (11) yields

\[
\frac{\partial h \overline{u}}{\partial t} + \frac{\partial h [\overline{u^2} + (4/3) \Delta u^2]}{\partial x} + \frac{\partial h [\overline{u} \overline{v} + (4/3) \Delta u \Delta v]}{\partial y} + (u \omega + \tau_x)|_{\sigma=0} - \tau_x|_{\sigma=-1} + hg \frac{\partial \eta}{\partial x} = h \overline{X},
\]
\[
\frac{\partial h \overline{v}}{\partial t} + \frac{\partial h [\overline{u} \overline{v} + (4/3) \Delta u \Delta v]}{\partial x} + \frac{\partial h [\overline{u^2} + (4/3) \Delta u^2]}{\partial y} + (v \omega + \tau_y)|_{\sigma=0} - \tau_y|_{\sigma=-1} + hg \frac{\partial \eta}{\partial y} = h \overline{Y},
\]
\[
\frac{\partial h \Delta u}{\partial x} + \frac{\partial h \Delta v}{\partial y} + \omega|_{\sigma=0} = 0,
\]

Applying \( \Delta \) to (1) – (3) and using (12) yields

\[
\frac{\partial h \Delta u}{\partial t} + \frac{\partial (2h \overline{u} \Delta u)}{\partial x} + \frac{\partial (h \overline{u} \Delta v + \overline{v} \Delta u)}{\partial y} + (u \omega + \tau_x)|_{\sigma=0} + \tau_x|_{\sigma=-1} - 2(u \omega + \tau_x)|_{\sigma=-0.5} = h \Delta X,
\]
\[
\frac{\partial h \Delta v}{\partial t} + \frac{\partial (h \overline{u} \Delta v + \overline{v} \Delta u)}{\partial x} + \frac{\partial (2h \overline{v} \Delta v)}{\partial y} + (v \omega + \tau_y)|_{\sigma=0} + \tau_y|_{\sigma=-1} - 2(v \omega + \tau_y)|_{\sigma=-0.5} = h \Delta Y,
\]
\[
\frac{\partial h \Delta u}{\partial x} + \frac{\partial h \Delta v}{\partial y} + \omega|_{\sigma=0} - 2\omega|_{\sigma=-0.5} = 0,
\]

Pressure, being depth-uniform, is absent from (16) – (17).

These equations are wave-averaged. We will further Reynolds decompose into mean currents and eddies, i.e. for any \( \alpha \),

\[ \alpha = \langle \alpha \rangle + \alpha', \]

where angle brackets denote a 1-hour average. We assume statistical stationarity and alongshore-uniformity (\( \partial \langle \alpha \rangle / \partial t = \partial \langle \alpha \rangle / \partial y = 0 \)).
2.2. Forcing

The surface stress \((\tau_x, \tau_y)|_{\sigma=0}\) results from forcing by winds or breaking waves, so the total forcing of the upper layer is

\[
(\mathcal{X})_u = (X)_u - h^{-1}\tau_x|_{\sigma=0},
\]

\[
(\mathcal{Y})_u = (Y)_u - h^{-1}\tau_y|_{\sigma=0}.
\]

Corresponding depth-averaged forcing is \([\overline{\mathcal{X}}, \overline{\mathcal{Y}}] = [(X)_u + (X)_\ell, (Y)_u + (Y)_\ell]\), and depth-dependent forcing is \([\Delta \mathcal{X}, \Delta \mathcal{Y}] = [(X)_u - (X)_\ell, (Y)_u - (Y)_\ell]\).

We apply the breaker-induced surface stress or near-surface momentum injection [which has magnitude \(D/c\), where \(D = \text{wave dissipation rate} \) and \(c = \text{phase speed}\), Stive and Wind, 1986; Deigaard, 1993] entirely to the upper layer. This forcing acts in the wave phase propagation direction \(\theta\), so in the surfzone where winds can be neglected,

\[
\Delta \mathcal{Y}/\Delta \mathcal{X} = \tan(\theta),
\]

CL forces can be added, but prove small in our perturbation analysis (Appendix A). Total forcing also contains a depth-uniform forcing by the radiation stress gradient [Newberger and Allen, 2007a; Uchiyama et al., 2010] which can be neglected when calculating \((\Delta \mathcal{Y}, \Delta \mathcal{X})\) because it is removed by the \(\Delta\) operator. Furthermore, the alongshore component of the depth-uniform forcing is zero [Deigaard, 1993; Newberger and Allen, 2007a], so \(\langle (Y)_\ell \rangle = 0\) and from (7)–(8)

\[
\langle \overline{\mathcal{Y}} \rangle = \langle \Delta \mathcal{Y} \rangle.
\]
2.3. Parameterizations of turbulent mixing

The bed stress \((\tau_x, \tau_y)|_{\sigma=-1}\) and the mid-water-column stress \((\tau_x, \tau_y)|_{\sigma=-0.5}\) are parameterized by damping coefficients \(\lambda_b\) and \(\lambda_m\), defined by

\[
(\tau_x, \tau_y)|_{\sigma=-1} = -(h \lambda_b/2) [(u)_{\ell}, (v)_{\ell}], \tag{24}
\]

\[
(\tau_x, \tau_y)|_{\sigma=-0.5} = -(h \lambda_m/2) (\Delta u, \Delta v). \tag{25}
\]

Now \(\lambda_b^{-1}\) is a time scale for damping of the lower layer flow by bottom friction, and \(\lambda_m^{-1}\) is a time scale for dissipation of \((\Delta u, \Delta v)\) by friction between upper and lower layers.

We evaluate \(\lambda_b\) using the drag coefficient parameterization of Feddersen et al. [2000], i.e. \(\langle \tau_y \rangle|_{\sigma=-1} = -C_D(a_1 \sigma_T + a_2 |\langle (v)_{\ell} \rangle|) \langle (v)_{\ell} \rangle\), where \(C_D = 2 \times 10^{-3}\) [within the range of estimates, Feddersen et al., 1998; Feddersen and Trowbridge, 2005] is the bottom drag coefficient, \(\sigma_T = (\sigma_u^2 + \sigma_v^2)^{1/2}\) is the root-mean-square (rms) water speed associated with waves (\(\sigma_u\) and \(\sigma_v\) are standard deviations in across- and along-shore directions), and empirical coefficients \(a_1 = 0.66\) and \(a_2 = 0.87\) [we neglect differences between Feddersen’s near-bed velocity measurements and \((v)_{\ell}\)]. Now from (24),

\[
\lambda_b = (2C_D/h)(a_1 \sigma_T + a_2 |\langle (v)_{\ell} \rangle|). \tag{26}
\]

For \(\lambda_m\), an eddy viscosity model

\[
(\tau_x, \tau_y)|_{\sigma=-0.5} = -\nu \left( \frac{\partial u}{\partial z}, \frac{\partial v}{\partial z} \right) \bigg|_{\sigma=-0.5} \tag{27}
\]

with (9), (10), (25) and (27) yields

\[
\lambda_m = 8\nu/h^2. \tag{28}
\]

The modeled eddy viscosity resulting from wave breaker-injected turbulence is

\[
\nu_w = s D^{1/3} H s/\gamma, \tag{29}
\]
where $H_s =$ significant wave height, $\varsigma = 0.076e^{-(\gamma h/H_s)\alpha/6}$, the empirical coefficient $\alpha = 4.38$, and $\gamma = 0.48$ is the maximum value of $H_s/h$ far inside the surfzone (Section 4.1). Here $\varsigma$ is based on (28) and the eddy viscosity at depth $h/2$. To estimate this eddy viscosity, we modified the model of Feddersen [2012b] to use a turbulent length scale $0.16H_s/\gamma$ instead of $0.16h$, and we chose the coefficient $0.076$ [which in Feddersen’s notation equals $(0.15\alpha\delta^4)^{1/3}$] to reproduce Feddersen’s result in the limit $H_s = \gamma h$ (Fedderson’s model was developed for this limit; see also Svendsen [1987]; Kobayashi et al. [2005]; Yoon and Cox [2010]). The Thornton and Guza [1983] model yielded breaker dissipation $D$, with model parameters tuned to fit the energy balance $\partial q_x/\partial x = -D$, where the across-shore wave energy flux $q_x$ was estimated from ADP measurements (Section 3.2). To account for mixing in non-breaking cases, a background value $6\lambda_b$ (justified in Section 2.7) was added to the breaker-injected mixing $\lambda_w = 8\nu_w/h^2$ to yield the total damping parameter

$$\lambda_m = \lambda_w + 6\lambda_b.$$  

(30)

2.4. Scaling

Scaling assumptions are first listed, then justified. Readers not interested in scaling can skip to Section 2.5. Let $L =$ surfzone width, $H =$ typical depth, and let the ordering
parameter $\epsilon \ll 1$. Order-one dimensionless variables, denoted by $^*$, are defined by:

$$x = L x^*, \quad (31)$$

$$y = L y^*, \quad (32)$$

$$t = \epsilon^{-1} (L/c) t^*, \quad (33)$$

$$h = H h^*, \quad (34)$$

$$\langle \bar{v} \rangle = \epsilon c \langle \bar{v}^* \rangle, \quad (35)$$

$$\langle \Delta v \rangle = \epsilon^2 c \langle \Delta v^* \rangle, \quad (36)$$

$$\bar{v}' = \epsilon^2 c \bar{v}'^*, \quad (37)$$

$$\Delta v' = \epsilon^3 c \Delta v'^*, \quad (38)$$

$$\langle \bar{u} \rangle = \epsilon^2 c \langle \bar{u}^* \rangle, \quad (39)$$

$$\langle \Delta u \rangle = \epsilon^2 c \langle \Delta u^* \rangle, \quad (40)$$

$$\bar{u}' = \epsilon^2 c \bar{u}'^*, \quad (41)$$

$$\Delta u' = \epsilon^3 c \Delta u'^*, \quad (42)$$

$$\langle \omega \rangle = \epsilon^2 c (H/L) \langle \omega^* \rangle, \quad (43)$$
\[ \omega' = \epsilon^2 c \langle H/L \rangle \omega'_{ss}, \quad (44) \]
\[ \langle u_s' \rangle = \epsilon^2 c \langle u_{ss}' \rangle, \quad (45) \]
\[ \langle \Delta u_s \rangle = \epsilon^2 c \langle \Delta u_{ss} \rangle, \quad (46) \]
\[ u_s' = \epsilon^2 c u_{ss}', \quad (47) \]
\[ \Delta u_s' = \epsilon^4 c \Delta u_{ss}', \quad (48) \]
\[ \langle X \rangle = \epsilon^2 \left( \frac{c^2}{L} \right) \langle X' \rangle, \quad (49) \]
\[ \langle \Delta X \rangle = \epsilon^2 \left( \frac{c^2}{L} \right) \langle \Delta X' \rangle, \quad (50) \]
\[ X' = \epsilon^2 \left( \frac{c^2}{L} \right) X'_{ss}, \quad (51) \]
\[ \Delta X' = \epsilon^3 \left( \frac{c^2}{L} \right) \Delta X'_{ss}, \quad (52) \]
\[ \lambda_b = \epsilon (c/L) \lambda_{bs}, \quad (53) \]
\[ \lambda_m = (c/L) \lambda_{ms}. \quad (54) \]

\( \mathcal{Y} \) and \( v_s \) are scaled in the same manner as \( \mathcal{X} \) and \( u_s \).

This scaling is justified as follows. Phase speed \( c \) is greater than \( \langle \tau \rangle \), justifying (35) (e.g. \( c \approx \left[ (9.8 \text{ ms}^{-2}) (4 \text{ m}) \right]^{1/2} \approx 6 \text{ ms}^{-1} \) in 4 m depth). Observed alongshore eddy and mean velocities are weakly depth-dependent and eddy velocities are weaker than alongshore mean currents (Sections 4.2–4.5), leading to (36)–(38). For across-shore velocity, eddy and depth-dependent mean velocities are comparable to corresponding alongshore velocities, but mean across-shore flows (unlike mean alongshore flows) have substantial depth-dependence, leading to (39)–(42) (Sections 4.2–4.5). Eddies have along- and across-shore scales comparable to the surf zone width, and propagate alongshore with speed roughly \( \langle \tau \rangle \), leading to (31), (32) and (33) [Oltman-Shay et al., 1989; Noyes et al., 2004]. Scaling (45) follows from depth-integrated mass balance between the undertow (39) and the wave...
mass flux [Svendsen, 1984], and implies $O(\epsilon c)$ wave-frequency velocity [here $O(\alpha)$ denotes a term of order $\alpha$]. Scaling (46) implies Boussinesq (nearly shallow water) waves [Peregrine, 1967]. Radiation stress is $O(cu_s)$ [Longuet-Higgins, 1973], implying (49). Forcing by breakers has leading order depth-dependence [Stive and Wind, 1986; Deigaard, 1993; Uchiyama et al., 2010], implying (50). Assuming small eddy-frequency modulations in $(u_s, v_s)$ and $(\chi, \psi)$ leads to (51), (52), (47), and (48). Choices (43), (44), (53) and (54) prove necessary for consistency in Sections 2.5–2.8. The scaling $\lambda_b \ll \lambda_m$ will prove essential to the weak depth-dependence of eddies and alongshore currents.

2.5. Depth-averaged currents

Time-averaging (13)–(14) and applying (24), (25), (27), (28), (31)–(54) yields

$$g \frac{\partial \langle \eta \rangle}{\partial x} - \langle \chi \rangle = 0,$$

$$\frac{\lambda_b \langle \bar{v} \rangle}{2} - \langle \psi \rangle = 0,$$

where smaller $O(c^3)$ terms are neglected (for an illustration of the perturbation expansion used to derive such leading-order expressions, see the derivation of the leading-order depth-dependent mean alongshore balance in Appendix B). Here (55) is the standard across-shore balance between wave setup and radiation stress, while (56) is the standard alongshore balance between bed stress and radiation stress. We will use (56) to estimate the forcing $\langle \psi \rangle$.

Time-averaging (4) and (15), applying (31)–(54), then integrating across-shore and noting zero flux through the shore yields the standard balance between the wave mass flux and the undertow: $\langle \bar{u} \rangle + \langle \bar{u}_s \rangle = 0$ (to which a roller mass flux could be added).
2.6. Depth-averaged eddies

Retaining only time-dependent terms in (13), (14), (4) and (15), and applying (24), (25), (27), (28), (31)–(54) yields

\[
\frac{\partial \bar{u}'}{\partial t} + \langle \bar{v} \rangle \frac{\partial \bar{u}'}{\partial y} + \lambda_b \bar{u}' / 2 + g \frac{\partial \eta'}{\partial x} - \mathcal{X}' = 0, 
\]

(57)

\[
\frac{\partial \bar{v}'}{\partial t} + \bar{u} \frac{\partial \langle \bar{v} \rangle}{\partial x} + \langle \bar{v} \rangle \frac{\partial \bar{v}'}{\partial y} + \lambda_b \bar{v}' / 2 + g \frac{\partial \eta'}{\partial y} - \mathcal{Y}' = 0, 
\]

(58)

\[
\frac{\partial h \bar{u}'}{\partial x} + \frac{\partial h \bar{v}'}{\partial y} = 0, 
\]

(59)

where neglected higher-order terms are \(O(\epsilon^4)\) in (57)–(58), and \(O(\epsilon^3)\) in (59). These are standard linearized wave-forced, depth-averaged equations for rigid-lid eddies [e.g. Dodd et al., 1992, with wave forcing added]. The non-divergent leading-order \((h \bar{u}', h \bar{v}')\) respond only to the rotational, non-divergent part of \((\mathcal{X}', \mathcal{Y}')\). By definition, the depth-dependent velocities \((\Delta u, \Delta v)\) make no contribution to the depth-integrated water flux, so they do not appear in (59), are unaffected by the rigid-lid condition, and can converge or diverge.

2.7. Depth-dependent currents

Time-averaging (16)–(17) and applying (24), (25), (27), (28), (31)–(54) yields

\[
\lambda_m \langle \Delta u \rangle - \langle \Delta \mathcal{X} \rangle = R_{\langle \Delta u \rangle}, 
\]

(60)

\[-\lambda_b \langle \bar{v} \rangle / 2 + \lambda_m \langle \Delta v \rangle - \langle \Delta \mathcal{Y} \rangle = R_{\langle \Delta v \rangle}, 
\]

(61)

where the left of (60)–(61) are \(O(\epsilon^2)\), the \(O(\epsilon^3)\) residuals

\[
R_{\langle \Delta u \rangle} = \lambda_b (\langle \bar{u} \rangle - \langle \Delta u \rangle) / 2, 
\]

(62)

\[
R_{\langle \Delta v \rangle} = -\langle \Delta u \rangle \frac{\partial \langle \bar{v} \rangle}{\partial x} - \frac{\lambda_b \langle \Delta v \rangle}{2}, 
\]

(63)

and \(O(\epsilon^4)\) terms are neglected. For derivation of (61), (63) see Appendix B. Residuals \(R_{\langle \Delta u \rangle}\) and \(R_{\langle \Delta v \rangle}\) are neglected in modeling, but are noted so their magnitudes can be
evaluated to check model errors. Time-averaged mass conservation (18) confirms (43), but otherwise is neglected.

Stive and Wind [1986] developed a 1dv undertow model incorporating breaker-induced surface stress and momentum mixing parameterized using an eddy viscosity, while noting that bottom friction is a small player in the across-shore momentum balance. Since surfzone $\langle \Delta x \rangle$ results from a breaker induced surface stress (or near-surface momentum injection) and $\lambda_m \langle \Delta u \rangle$ represents eddy viscous mixing, (60) is a two-layer equivalent of the Stive and Wind [1986] model. Similarly, (61) incorporating bed stress, eddy viscosity, and alongshore surface stress (respectively first, second, and third terms on left), is a two-layer equivalent of many 1dv alongshore current models [e.g. Feddersen and Trowbridge, 2005].

Neglecting $R_{\langle \Delta v \rangle}$, (61) yields

$$\frac{\langle \Delta v \rangle}{\langle v \rangle} = \frac{\lambda_b}{2\lambda_m} \left(1 + \mu_{\langle v \rangle}\right), \tag{64}$$

where

$$\mu_{\langle v \rangle} = \frac{2\langle \Delta y \rangle}{\lambda_b \langle v \rangle} \tag{65}$$

is the ratio between contributions to depth dependence from wave forcing and bottom friction. From (23) and (56)

$$\mu_{\langle v \rangle} = 1, \tag{66}$$

so wave forcing and bottom friction contribute equally to vertical structure. Now (64) becomes

$$\frac{\langle \Delta v \rangle}{\langle v \rangle} = \frac{\lambda_b}{\lambda_m}. \tag{67}$$
Therefore, alongshore depth dependence is generated by equal and opposite forcing of the lower layer (bottom friction) and the upper layer (winds and waves). This generation at rate $\lambda_b\langle \bar{v} \rangle$ is balanced by mixing $\lambda_m\langle \Delta v \rangle$, yielding (67).

In the absence of breaking, the eddy viscosity model $\nu = \kappa u^* z (2h - z)/(2h)$ [Reniers et al., 2004] yields a velocity profile $v = (u^*/\kappa)\{\log(z/z_0) - \log((2h - z)/(2h - z_0))\}$, where $z_0 =$ bottom roughness length. Substituting this into (5) – (8) and assuming apparent roughness $z_0 = 10^{-2}$ m [e.g. Drake et al., 1992] yields $\langle \Delta v \rangle / \langle \bar{v} \rangle \approx 1/6$ in 4 m depth. From (30) and (67), $\langle \Delta v \rangle / \langle \bar{v} \rangle = \lambda_b/(\lambda_w + 6\lambda_b)$, which approaches 1/6 in the absence of breaking, justifying the background value $6\lambda_b$ chosen for $\lambda_m$ outside the surfzone.

Neglecting $R_{\langle \Delta u \rangle}$, (60) yields

$$\frac{\langle \Delta u \rangle}{\langle \bar{v} \rangle} = \frac{\lambda_b}{2\lambda_m} \mu_{\langle u \rangle},$$

where

$$\mu_{\langle u \rangle} = \frac{2\langle \Delta \mathbf{X} \rangle}{\lambda_b \langle \bar{v} \rangle} = O(\epsilon^{-1}) \gg 1.$$  

(69)

This would resemble (64) if, instead of equaling 1, $\mu_{\langle u \rangle}$ were $\gg 1$. The contrast between $\mu_{\langle u \rangle} \gg 1$ and $\mu_{\langle v \rangle} = 1$ can be traced to the fact that pressure $(g\eta)$ limits $\langle \bar{u} \rangle$ (55) but not $\langle \bar{v} \rangle$ (56). In contrast, pressure, being depth-uniform, does not limit $\langle \Delta u \rangle$ (60) [or $\langle \Delta v \rangle$, (61)]. Although $\mu_{\langle u \rangle} \gg 1$, across-shore bottom friction remains important in a bottom boundary layer [Svendsen et al., 1987], which is not resolved here.

To relate $\langle \Delta u \rangle$ to mixing parameterizations, (22), (60), (65), (66) and (67) are combined to yield

$$2\tan(\theta)\frac{\langle \Delta u \rangle}{\langle \bar{v} \rangle} = \frac{\lambda_b}{\lambda_m},$$

(70)

where wind forcing has been neglected, as is appropriate inside the surfzone.
2.8. Depth-dependent eddies

Retaining only time-dependent terms in (16) – (17) and applying (24), (25), (27), (28), (31) – (54) yields

\[-\lambda_b \overline{v}'/2 + \lambda_m \Delta u' - \Delta x' = R_{\Delta u'}, \tag{71}\]
\[-\lambda_b \overline{v}'/2 + \lambda_m \Delta v' - \Delta y' = R_{\Delta v'}, \tag{72}\]

where the left of (60) – (61) are \(O(\epsilon^3)\), the \(O(\epsilon^4)\) residuals

\[R_{\Delta u'} = -\frac{\partial \Delta u'}{\partial t} - \langle \overline{v} \rangle \frac{\partial \Delta u'}{\partial y} - \overline{u} \frac{\partial \langle \Delta u \rangle}{\partial x} - \langle \Delta u \rangle \frac{\partial \overline{u}'}{\partial y} - \lambda_b \Delta u', \tag{73}\]
\[R_{\Delta v'} = -\frac{\partial \Delta v'}{\partial t} - \langle \overline{v} \rangle \frac{\partial \Delta v'}{\partial y} - \Delta u \frac{\partial \langle \overline{v} \rangle}{\partial x} - \overline{u} \frac{\partial \langle \Delta v \rangle}{\partial x} - \langle \Delta v \rangle \frac{\partial \overline{v}'}{\partial y} - \lambda_b \Delta v', \tag{74}\]

and \(O(\epsilon^5)\) terms are neglected. Residuals are neglected in modeling, but are noted so their magnitudes can be evaluated to check model errors. Time-dependent mass conservation (18) confirms (44), but otherwise is neglected.

At leading order, (71) and (72) for fluctuating eddy velocities resemble the steady-state balance (61) for the alongshore current. Eulerian and advective accelerations (\(\partial / \partial t\) and \(\langle \overline{v} \rangle \partial / \partial y\)) are absent from (71) and (72) at leading-order, because the timescale for interfacial mixing \(\lambda_{m}^{-1}\) is assumed to be smaller than the typical eddy period (a few hundred seconds). This short timescale for interfacial mixing will prove consistent with the model (Section 2.3) for breaker-induced eddy viscosity. This scaling is also implied by assumptions that (i) bottom friction appears at leading order in (71)–(72), and (ii) interfacial friction is stronger than bottom friction, as required to produce weak vertical variability of the alongshore current (67).

Neglecting \(R_{\Delta u'}\) and \(R_{\Delta v'}\) in (71)–(72), moving forcing \((\Delta x', \Delta y')\) and bottom friction \((\lambda_b \overline{u}'/2, \lambda_b \overline{v}'/2)\) terms to the right hand side, squaring, averaging, taking the square root,
and rearranging yields

\[
\frac{\langle \Delta u'^2 \rangle^{1/2}}{\langle \bar{u}'^2 \rangle^{1/2}} = \frac{\lambda_b}{2\lambda_m} \left( 1 + 2r_{\Delta \chi, \pi} \mu_{u'} + \mu_{u'}^2 \right)^{1/2},
\]

(75)

\[
\frac{\langle \Delta v'^2 \rangle^{1/2}}{\langle \bar{v}'^2 \rangle^{1/2}} = \frac{\lambda_b}{2\lambda_m} \left( 1 + 2r_{\Delta \gamma, \pi} \mu_{v'} + \mu_{v'}^2 \right)^{1/2},
\]

(76)

where

\[
\mu_{u'} = \frac{2\langle \Delta \chi'^2 \rangle^{1/2}}{\lambda_b \langle \bar{u}'^2 \rangle^{1/2}},
\]

(77)

\[
\mu_{v'} = \frac{2\langle \Delta \gamma'^2 \rangle^{1/2}}{\lambda_b \langle \bar{v}'^2 \rangle^{1/2}},
\]

(78)

and, for any two variables \(\alpha_1, \alpha_2\), the correlation 
\(r_{\alpha_1, \alpha_2} = \frac{\langle \alpha_1' \alpha_2' \rangle}{\langle \alpha_1'^2 \rangle^{1/2} \langle \alpha_2'^2 \rangle^{1/2}}\). If 
\(r_{\Delta \chi, \pi} = r_{\Delta \gamma, \pi'} = 1\) then (75) and (76) resemble (64).

Consider three cases:

Case 1: eddies are generated by shear production, without significant wave forcing. Now \(\mu_{u'}, \mu_{v'} \approx 0\) and (75) – (76) reduce to 
\(\langle \Delta u'^2 \rangle^{1/2} / \langle \bar{u}'^2 \rangle^{1/2} = \langle \Delta v'^2 \rangle^{1/2} / \langle \bar{v}'^2 \rangle^{1/2} = \lambda_b / (2\lambda_m)\). Dimensionless depth dependence of eddies is half that of the alongshore current (because both wave forcing and bottom friction contribute to alongshore current depth dependence, but in this case only friction generates eddy depth dependence).

Case 2: wave forcing and bottom friction make reinforcing, equal contributions to vertical structure \((r_{\Delta \chi, \pi} = r_{\Delta \gamma, \pi'} = \mu_{u'} = \mu_{v'} = 1)\). Now (75) – (76) become
\(\langle \Delta u'^2 \rangle^{1/2} / \langle \bar{u}'^2 \rangle^{1/2} = \langle \Delta v'^2 \rangle^{1/2} / \langle \bar{v}'^2 \rangle^{1/2} = \lambda_b / \lambda_m\). Eddies have the same dimensionless depth dependence as the alongshore current. Decorrelation \((r_{\Delta \chi, \pi}, r_{\Delta \gamma', \pi'} < 1)\) would reduce vertical structure.

Case 3: Wave forcing dominates generation of vertical structure. Now \(\mu_{u'}, \mu_{v'} \gg 1\) and (75) – (76) resemble (68). Dimensionless depth-dependence of the eddies greatly exceeds that of the alongshore current.
Analysis of across- and along-shore currents (Section 2.7) suggests how Cases 2 and 3 might arise. For mean across-shore currents $\mu_\langle u \rangle \gg 1$ because pressure limited $\langle u \rangle$ (but not $\langle \Delta u \rangle$). Similarly, if eddies were forced by convergent, irrotational $(\vec{X}', \vec{Y}')$ and $(\Delta \vec{X}', \Delta \vec{Y}')$, then pressure would limit $(\pi', \nu')$ but not $(\Delta u', \Delta v')$, leading to $\mu_\nu' \gg 1$ and strong vertical structure (Case 3). Conversely, for purely rotational and non-divergent $(\vec{X}', \vec{Y}')$ and $(\Delta \vec{X}', \Delta \vec{Y}')$ pressure plays no role, as in the alongshore current case (Case 2). Numerical simulations suggest a mix of rotational and divergent forcing (Nirnimesh Kumar, pers. com.), which is expected to yield eddy dimensionless depth dependence slightly exceeding that of the alongshore current.

3. Field Observations and Data Analysis

3.1. Site Description and Instrumentation

The major deployment was conducted in 2011 on natural beach at Duck, North Carolina [the site of numerous previous experiments, e.g. Garcez Faria et al., 1998, 2000; Birkemeier et al., 2001; Feddersen, 2007]. A smaller trial deployment was conducted in 2010 on an Oregon beach. During the Oregon deployment, under intense wave breaking, agreement was found between ADP and nearby Acoustic Doppler Velocimeter (ADV) measurements (Appendix C). Further discussion outside Appendix C refers only to the Duck deployment.

Waves were measured in 8 m depth using an array of bottom-mounted pressure gauges maintained by staff of the US Army Corp of Engineers Field Research Facility. Using an amphibious vehicle [Lee and Birkemeier, 1993], bathymetry was surveyed on October 17 and November 16 (yeardays 290 and 319, Figure 1). Wave conditions were regularly imaged by cameras maintained by Oregon State University’s Coastal Imaging Laboratory. Winds were measured by an anemometer mounted on a pier about 400 m south of the...
ADP array. Several times per week, a CTD was lowered from the pier, in 8 m depth, to estimate density profiles.

Twelve Nortek Aquadopp ADPs measured pressure and vertical profiles of velocity at locations X1–X7 in an across-shore array, and Y1–Y6 in an alongshore array, with X4 and Y2 denoting a common element of both arrays (Figure 1a). A 1 MHz ADP at X7 recorded velocity in 0.66 m vertical rangebins, whereas all others were 2 MHz units recording in 0.33 m bins. Each ADP was mounted 0.3–0.6 m above bed on a vertical iron pipe. ADPs sampled continuously from October 18 to November 6, at 1 Hz for locations X1–X7, Y1–Y2, and at 0.5 Hz for Y3–Y6. For each ADP, orientations measured using an external compass were modified by a time-constant offset of $< 4^\circ$, chosen to ensure that mean wave angles were consistent with Snell’s law for refraction [e.g. Herbers et al., 1999]. Pipe elevations were surveyed, ADP elevations were measured relative to the pipes, and corrections of $< 5$ cm were applied to ensure that the estimated mean sea surface was flat on days with small waves and winds. Acoustic reflections from the sea surface and bed were used to estimate changes in seabed elevation during the deployment (Appendix D).

### 3.2. Data Analysis

The sea surface elevation above each ADP was estimated every second from measured atmospheric and water pressures, using linear wave theory to account for depth dependence of pressure, and velocities above the surface were discarded. An entire hour’s data from a velocity rangebin was discarded whenever either (i) the bin was above the instantaneous surface for $> 2.5\%$ of the hour, or (ii) the mean bin elevation was too near the mean sea surface, i.e. when $\left( z_j + 0.33 \text{ m} - z_A \right) > 0.9(\langle \eta \rangle - z_A) $, where $z_j$, $z_A$ and $\langle \eta \rangle$ are elevations of the bin, ADP, and mean sea surface. In remaining cases, any discarded
Data points were replaced by temporal linear interpolation to calculate hourly velocity statistics.

Data were filtered by frequency band. For a given ADP during a given hour, let \( u_j \) be the hour-long across-shore velocity time series measured at the \( j \)-th rangebin, with hourly mean \( \langle u_j \rangle \). A linear function of time was fitted to \( u_j \), and departures from this trend were filtered into 'eddies' (frequencies < 0.01 Hz, denoted \( u'_j \)), an 'infragravity' band (0.01 – 0.05 Hz, denoted \( u''_j \)), and 'waves' (0.05 – 0.25 Hz, denoted \( u'''_j \)). Frequencies > 0.25 Hz were discarded. Similar filtering was applied to alongshore velocity \( v_j \) and pressure \( p \). At Y3 – Y6, where ADP sampling frequency was relatively low, wave-frequency observations were discarded.

For each ADP, the wave-frequency sea level fluctuation \( \eta''' \) was calculated from pressure \( p''' \) using linear wave theory. The significant wave height \( H_s \) was calculated as four times the standard deviation of this wave-frequency sea level, denoted \( 4 \langle \eta'''^2 \rangle^{1/2} \). For each ADP, the depth-integrated across-shore wave energy flux \( q_x \) was calculated by averaging estimates obtained from the lowest five rangebins, i.e. \( q_x = (1/5) \sum_{j=1}^{5} q_{x,j} \). To estimate \( q_{x,j} \) the depth-integrated energy flux spectrum was calculated from the co-spectrum between \( \eta \) and \( u_j \), using linear theory to account for depth dependence, and integrated over all wave frequencies. A similar calculation yielded an alongshore flux \( q_y \). The mean wave angle \( \theta \) was calculated as the direction of vector \( (q_x, q_y) \), with positive \( \theta \) indicating waves approaching from north of shore-normal. Although 1 Hz ADP velocity time series are noisy, estimates of mean wave angle are not (e.g. Appendix C), because many degrees of freedom are collected when integrating co-spectra over all wave frequencies (co-spectra are unbiased, since noise in \( \eta \) and velocity are uncorrelated). Lateral separation between...
acoustic beams ($<1.4\,\text{m}$) was small compared with typical wavelengths (e.g. $35\,\text{m}$ for $6\,\text{s}$ waves in $4\,\text{m}$ depth) and was neglected.

If wave propagation shoreward from $8\,\text{m}$ depth matched non-dissipative linear theory for peak-frequency waves, the wave height in depth $h$ would be

$$H_{nd} = \left[ \frac{c_{g8} \cos(\theta_8)}{c_{gh} \cos(\theta_h)} \right]^{1/2} H_8, \quad (79)$$

where $c_{g8}$ and $c_{gh}$ are peak-frequency group velocities in $8\,\text{m}$ depth and depth $h$, calculated from linear theory, and $\theta_8$ and $\theta_h$ are corresponding wave angles. The angle in $\theta_8$ was measured by the pressure array, and $\theta_h$ was estimated from Snell’s law: $\sin(\theta_8)/c_8 = \sin(\theta_h)/c_h$, where $c_8$ and $c_h$ are peak-frequency phase speeds in depths $8\,\text{m}$ and $h$. The ratio $H_{nd}/h$ will be used to determine across-shore instrument locations relative to the edge of the surfzone (Section 4.1).

Bin averages will be used to evaluate consistent relationships between pairs of variables $\alpha$ and $\beta$ measured at a single ADP. Divide the range of $\beta$ values into equally spaced increments centered on $\beta_1, \beta_2, \beta_3, \ldots$, such that $\beta_{j+1} = \beta_j + \delta$. The bin-average $\|\alpha\|_{\beta_j}$ is then the average of $\alpha$ over all hours when $\beta_j/2 - \delta < \beta < \beta_j + \delta/2$. Data will often be binned by values of $\beta = H_{nd}/h$. To streamline notation, when binning by $H_{nd}/h$ we omit the subscript, so $\|\alpha\|$ denotes $\|\alpha\|_{H_{nd}/h}$.

Velocity variances are biased high by instrument noise. To assess and remove bias, the covariance between velocities measured in adjacent rangebins was estimated. If velocity varies little between adjacent bins then estimated covariances resemble variances, but are nearly unbiased because the noise in acoustic measurements is almost uncorrelated between bins.
The elevation \(\sigma = -0.5\) between upper and lower layers (Section 2.1) was evaluated as seabed elevation plus half the hourly mean depth \(h\). Raw upper [or lower] layer velocities, denoted \(2(u)_{ur}\) [or \(2(u)_{lr}\)] were estimated by averaging velocities above (below) \(\sigma = 0.5\). If observations spanned < 30% of either layer, data were discarded. This led to removal of most data from X1 and X2, so these ADPs are discarded from all bin-averaged analysis. Depth-averaged flows \((\bar{u}, \bar{v})\) were calculated as \([(u)_{ur} + (u)_{lr}, (v)_{ur} + (v)_{lr}]\).

Depth-dependence \((\Delta u, \Delta v)\) was calculated as \(B[(u)_{ur} - (u)_{lr}, (v)_{ur} - (v)_{lr}]\), where \(B = h/[2(z_u - z_l)]\), with \(z_u\) and \(z_l\) the mean elevation of velocity bins used to calculate \([(u)_{ur}, (v)_{ur}]\) and \([(u)_{lr}, (v)_{lr}]\) \((B > 1\) because velocity data were absent near the surface and the bed).

The hourly raw variance of \((u')_{ur}\) is

\[
\langle (u')^2_{ur} \rangle = \left(\left(\frac{1}{2N_U} \sum_{j \in U} u'_j\right) \left(\frac{1}{2N_U} \sum_{k \in U} u'_k\right)\right) = \frac{1}{4N^2_U} \sum_{j, k \in U} \langle u'_j u'_k \rangle , \tag{80}
\]

where \(U\) is the set of \(N_U\) functioning velocity bins in the upper layer. This estimate is biased high by instrument noise owing to cases \(j = k\). Discarding the biased elements and averaging the rest gives

\[
\langle (u')^2_{ur} \rangle \approx \frac{1}{4N_U(N_U - 1)} \sum_{j, k \in U, \ j \neq k} \langle u'_j u'_k \rangle . \tag{81}
\]

Similar approaches were used to obtain unbiased estimates of all quantities involving layer averages \((\langle \bar{u}^2 \rangle, \langle \bar{v}^2 \rangle, \langle \Delta u'^2 \rangle, \langle \Delta v'^2 \rangle, \langle \bar{u'} \Delta u' \rangle \text{ etc.})\)

To calculate the residual \(R_{\Delta v}\) neglected in (61), let \(\alpha_j\) be the value of any variable \(\alpha\) at \(X_j\), so the estimate of \(\alpha\) at midway between \(X_j\) and \(X_{j+1}\) is \((\alpha)_{j+1/2} = (\alpha_{j+1} + \alpha_j) / 2\), and the difference between \(\alpha\) at \(X_j\) and \(X_{j+1}\) is \(\partial_{j+1/2} \alpha = \alpha_{j+1} - \alpha_j\). Now the estimated
\[
\hat{R}_{(\Delta v)} = \langle (\Delta u) \rangle_{j+1/2} \frac{\partial j_{+1/2}}{\partial j_{+1/2}} - \frac{(\lambda_b \langle \Delta v \rangle)_{j+1/2}}{2}.
\] (82)

A similar expression was used to evaluate \(\hat{R}_{(\Delta u)}\). To estimate the rms magnitude of \(R_{\Delta u'}\), we assume terms are uncorrelated, approximate \(\partial \alpha'/\partial t \sim \alpha'/T\) where \(T = 200\) s is a typical eddy period, and neglect poorly resolved alongshore derivatives to obtain

\[
\hat{R}_{\Delta u'} = \left[ \frac{(\Delta u'^2)_{j+1/2}}{T^2} + \langle \bar{u}'^2 \rangle_{j+1/2} \left( \frac{\partial j_{+1/2}}{\partial j_{+1/2}} \langle \Delta u \rangle \right)^2 \right. \\
\left. + \langle (\Delta u) \rangle_{j+1/2} \left( \frac{\partial j_{+1/2}}{\partial j_{+1/2}} \bar{u}' \right)^2 \right]^{1/2} + (\lambda_b^2 \langle \Delta u'^2 \rangle)_{j+1/2}. \] (83)

A similar expression was used to estimate \(R_{\Delta u'}\).

4. Results

4.1. Waves, Winds, and Stratification

Time series summarizing conditions observed throughout the deployment are presented in Figure 2. Winds ranged from 1 to 21 ms\(^{-1}\) (Figure 2a). Stratification \(\Delta \rho = \text{density at } 6\) m depth - surface density ranged up to 0.4 kg m\(^{-3}\) (Figure 2b). For the smallest wave heights \(H_s < 0.4\) m (e.g. yearday 300, Figure 2c), breaking occurred only within a few meters of shore (Figure 3a). For moderate heights \(H_s \approx 1.5\) m (e.g. day 302), breaking extended part-way across the array X1 – X7 (Figure 3b). The largest wave heights reached \(H_s = 4\) m in 8 m depth, although heights at the ADP array were smaller owing to breaking extending offshore of the ADP array (Figures 2c and 3c).

Across-shore profiles are contrasted for selected hours on days 300, 301, and 309 (Figure 4). Tidal stages were similar for the selected cases (nearly indistinguishable grey lines, Figure 4a). The large waves observed after day 301 (Figure 2c) caused onshore erosion...
and offshore accretion (black lines Figure 4a). Outside the surfzone, observed wave height $H_s$ matched the prediction $H_{nd}$ (79) of non-dissipative theory [e.g. matching black and grey lines in Figure 4b, day 300 (circles), and day 301 at X7 (cross at $x = 407$ m)]. In the outer surfzone, $H_s/H_{nd}$ dropped slightly below 1, owing to intermittent breaking [e.g. day 301, X4−X6 at $250 < x < 330$ m, Figure 4b]. In the ‘saturated surfzone’, $H_s/H_{nd}$ dropped substantially below 1, owing to widespread breaking (e.g. day 309, and day 301, X1−X3 at $x \leq 220$ m). Binning data by $H_{nd}/h$ and averaging (Section 3.2) establishes the consistency of these trends. Outside the surfzone ($H_{nd}/h < 0.35$), $\|H_s\|/\|H_{nd}\| \approx 1$ (Figure 5). In the saturated surfzone ($H_{nd}/h > 0.6$), $\|H_s\|/\|H_{nd}\| \approx \gamma h/H_{nd}$, consistent with the saturated breaking condition $H_s = \gamma h$ with fitted $\gamma = 0.48$ (Figure 5; separation of lines for $H_{nd}/h > 0.6$ indicates weak dependence of $\gamma$ on location). The outer surfzone ($0.35 < H_{nd}/h < 0.6$) is a region of unsaturated breaking.

At Y1−Y2, mean wave angles ranged from oblique propagation from the north ($\theta = 30^\circ$, day 301) to oblique propagation from the south ($\theta = -30^\circ$, day 293, Figure 2d). Missing data results from burial of both Y1 and Y2, owing to bed accretion. Mean angles decreased slightly as waves propagated across the array and into shallow water (Figure 4c), consistent with theory [e.g. Herbers et al., 1999].

### 4.2. Depth-averaged Currents

Consistent with forcing by radiation stresses, the depth-averaged alongshore current $\langle v \rangle$ usually flowed to the south (north) when waves approached from the north (south, Figure 2d,e, Figure 4c,d), and was strongest inside the surfzone [e.g. day 302 (crosses), Figure 4d]. On day 301, $|\langle v \rangle| \approx 1.5$ ms$^{-1}$ under oblique, moderate energy waves (Figure 2c,d,e, Figure 4c,d). Although waves were larger on day 309, angles were almost
shore-normal, and alongshore currents were comparable to day 301. The depth-averaged across-shore current was usually smaller than the alongshore current and flowed seaward ($\langle \bar{u} \rangle > 0$, Figure 2e).

To quantify alongshore variability, let $\langle \bar{u} \rangle_y$ be the average along $Y_1 - Y_6$ of $\langle \bar{u} \rangle$, and let $\langle \bar{u} \rangle_y' = \langle \bar{u} \rangle - \langle \bar{u} \rangle_y$. Across-shore currents showed substantial alongshore variability (for all times, rms $\langle \bar{u} \rangle_y'$ and $\langle \bar{u} \rangle_y$ respectively 0.055 ms$^{-1}$ and 0.16 ms$^{-1}$), whereas alongshore currents were nearly alongshore-uniform (rms $\langle \bar{v} \rangle_y'$ and $\langle \bar{v} \rangle_y$ respectively 0.037 ms$^{-1}$ and 0.43 ms$^{-1}$).

The Richardson number $Ri = gh\Delta \rho/|\rho(\langle \bar{u} \rangle^2 + \langle \bar{v} \rangle^2)|$ suggests suppression of mixing by stratification outside the surfzone, but minimal stratification inside the surfzone. We estimated $\Delta \rho = \rho|_{z=-h} - \rho|_{z=-0.5 \text{ m}}$ at X1–X7 by assuming that density profiles at the ADPs equaled profiles in 8 m depth, with linear interpolation of the coarse $\rho$ time series. This likely overestimates $Ri$, since $\Delta \rho$ likely decreases onshore of 8 m [Hally-Rosendahl et al., 2014; Kumar and Feddersen, 2016b]. Bin-averaged $Ri$ reached 0.1 outside the surfzone, but was below 0.05 inside the both the saturated and outer surfzones for X1–X6 (at X7, $Ri$ reached 0.07 in the outer surfzone, but was below 0.04 for larger $H_{nd}/h$, not shown).

### 4.3. Depth-averaged Eddies

Standard deviations of across- and along-shore eddy velocities ($\langle \bar{u}^2 \rangle^{1/2}$ and $\langle \bar{v}^2 \rangle^{1/2}$) ranged from 0.01 ms$^{-1}$ to 0.25 ms$^{-1}$, with the largest values observed during storms (Figure 2g) and inside the surfzone (Figure 4e). Depth-averaged eddy velocities were fairly alongshore-uniform [for entire deployment, rms $\langle \bar{u}^2 \rangle^{1/2}$, $\langle \bar{v}^2 \rangle^{1/2}$, and $\langle \bar{u}^2 \rangle^{1/2}$ and $\langle \bar{v}^2 \rangle^{1/2}$].
respectively $0.0056 \text{ ms}^{-1}$, $0.0051 \text{ ms}^{-1}$, $0.075 \text{ ms}^{-1}$ and $0.057 \text{ ms}^{-1}$]. The ratio between eddy kinetic and potential energies is $R = h(\langle \overline{u'^2} \rangle + \langle \overline{v'^2} \rangle)/(g\langle \overline{\eta'^2} \rangle)$. Bin-averaged $R$ ranged from 1 – 8 outside the surfzone to about 20 in the saturated surfzone, indicating dominance of non-gravity motions [not shown, c.f. Lippmann et al., 1999; Noyes et al., 2004].

4.4. Depth-Dependent Currents

Consistent with previous observations and theory [Putrevu and Svendsen, 1993; Reniers et al., 2004], across-shore currents were usually surface-intensified outside the surfzone (e.g. Figure 6a–g; at X4, $\langle \Delta u \rangle > 0$ in 72% of cases when $H_{nd}/h < 0.35$) and bottom intensified inside the surfzone (e.g. Figures 7a–d and 8a–g; at X4, $\langle \Delta u \rangle < 0$ in 98% of cases when $H_{nd}/h > 0.6$), although bottom intensification was sometimes weak in high energy conditions (e.g. Figure 8a–g). Of hours at X4 outside the surfzone with $\langle \Delta u \rangle < 0$, winds blew onshore in 86% of cases (for $\langle \Delta u \rangle > 0$, winds were onshore for 52% of cases), suggesting significant wind forcing. Alongshore currents showed substantial depth-dependence outside the surfzone (Figure 6h–n), but became relatively depth-uniform inside (Figures 7,8h–n). Alongshore currents were usually surface-intensified ($\langle \Delta v \rangle$ and $\langle \overline{v} \rangle$ the same sign, Figure 2e,f), although an unexplained case of substantial bottom intensification was observed on day 309.

Depth-dependent currents $\langle \Delta u \rangle$ and $\langle \Delta v \rangle$ had similar magnitudes, and both displayed significant alongshore variability (grey curves, Figures 6–8d,k; rms $\langle \Delta u \rangle^y$, $\langle \Delta v \rangle^y$, $\langle \overline{\Delta u} \rangle^y$ and $\langle \overline{\Delta v} \rangle^y$ respectively $0.0092 \text{ ms}^{-1}$, $0.0080 \text{ ms}^{-1}$, $0.022 \text{ ms}^{-1}$, and $0.023 \text{ ms}^{-1}$).

Bin-averaged dimensionless depth-dependence of the alongshore current $\langle \Delta v \rangle/\langle \overline{v} \rangle$ decreased from about 0.17 outside the surfzone to nearly zero in the saturated surfzone.
[filled black symbols, Figure 9; to avoid cancellation of the denominator when \( \langle v \rangle \) reverses, we plot \( \| \text{sign}(\langle v \rangle) \langle \Delta v \rangle \| / \| \text{abs}(\langle v \rangle) \| \), where \( \text{abs}(\langle v \rangle) \) denotes absolute value, and \( \text{sign}(\langle v \rangle) = 1 \) or \(-1\) when \( \langle v \rangle > 0 \) or \(< 0 \). Although \( \langle \Delta u \rangle \) and \( \langle \Delta v \rangle \) had comparable magnitudes (Figure 2f), dimensionless depth dependence was greater for the across-shore current (i.e. \( \langle \Delta u \rangle / \langle v \rangle > \langle \Delta v \rangle / \langle v \rangle \), compare filled pink and black symbols, Figure 9) because the depth-averaged current was predominantly alongshore (\( \langle v \rangle > \langle u \rangle \)). The alternative normalization for \( \langle \Delta u \rangle \) suggested by (67) and (70) was roughly consistent with observations in the saturated surfzone [i.e. \( \|2\text{sign}(\langle v \rangle) \tan(\theta) \langle \Delta u \rangle \| / \| \text{abs}(\langle v \rangle) \| \) and \( \langle \Delta v \rangle / \langle v \rangle \) of similar magnitude, compare dashed red lines with filled black symbols for \( H_{nd}/h > 0.6 \), Figure 9].

In the outer surfzone, depth-dependence of the undertow departed from (70) as \( \langle \Delta u \rangle \) reversed sign. Both \( \langle \Delta u \rangle \) and \( \langle \Delta v \rangle \) were scattered about bin averages (not shown), with large scatter for the across-shore current outside the surfzone, possibly owing to reversals in winds.

The model (67), simulating mixing of the alongshore current profile by breaker-injected turbulence, predicts a depth dependence \( \langle \Delta v_{\text{predicted}} \rangle = (\lambda_b/\lambda_m) \langle v \rangle \). At Y1–Y6, this prediction resembled observations, although scatter was considerable, with unexplained bottom-intensification observed on days 308–309 (compare thin black and dotted red lines, Figure 2f). When the dimensionless depth dependence \( \langle \Delta v \rangle / \langle v \rangle \) was bin-averaged by \( \lambda_b/\lambda_m \) values, observations were consistent with (67), although scatter was considerable (filled symbols, Figure 10; we calculated \( \| \text{sign}(\langle v \rangle) \langle \Delta v \rangle \| \lambda_b / \lambda_m / \| \text{abs}(\langle v \rangle) \| \lambda_b / \lambda_m \) to avoid cancellation in the denominator). It is not significant that the best-fit slope is so near 1 [alternative plausible background values for \( \lambda_m \), other than \( 6\lambda_b \) (Section 2.7), would yield slightly different slopes], but the correlation, with order-one slope, is encouraging.
Outside the surfzone, simulated interfacial mixing by breakers was negligible (grey curve, Figure 2i). Inside the surfzone, simulated breaker-induced interfacial mixing was intense ($\lambda_w \gg \lambda_b$), with the mixing timescale $\lambda_w^{-1}$ often reduced to just over 10 s, less than a typical eddy period.

Consistent with the assumed ordering, the neglected residual $R_{(\Delta u)}$ in the depth-dependent mean across-shore momentum equation was small compared with retained terms (whether averaged outside the surfzone, in the outer surfzone, or in the saturated surfzone, $\text{abs}(\hat{R}_{(\Delta u)}) < 10\%$ of the mean $\text{abs}(\lambda_{M} (\Delta u))$, not shown). The neglected residual in the alongshore balance was also usually small [the mean $\text{abs}(\hat{R}_{(\Delta v)}) < 10\%$ of the mean $\text{abs}(\lambda_{M} (\Delta v))$ at all locations outside surfzone, in outer surfzone, and at X5–X7 in saturated surfzone]. However, $\text{abs}(\hat{R}_{(\Delta v)})$ did reach a maximum of $20\%$ of the mean $\text{abs}(\lambda_{M} (\Delta v))$ at X3–X4 in the saturated surfzone. The term $\langle \Delta u \rangle (\partial \langle v \rangle / \partial x)$ [Svendsen and Putrevu, 1994] dominated $R_{(\Delta v)}$. Including this term did not significantly change the results shown in Figure 10 (not shown).

4.5. Depth-Dependent Eddies

Depth-dependence in standard deviation of eddy velocity, clear under low-energy conditions (Figure 11), was reduced under moderate (Figure 12) and high (Figure 13) energies. Consistent with manufacturer’s estimates, the difference between unbiased covariance-based and biased variance-based estimates of standard deviation suggest noise variance in single-bin eddy-frequency ADP measurements of $(0.01 \text{ m/s})^2$ and $(0.014 \text{ m/s})^2$ for 2 MHz and 1 MHz ADPs respectively (circles and crosses, Figure 11). This noise, clear in the low energy case, was not significant for intermediate of high energies (Figures 12, 13). All eddy statistics other than the circles in Figures 11–13 use unbiased estimators.
Standard deviation of depth-dependent eddy velocity ranged from $<0.005\text{ ms}^{-1}$ to about $0.025\text{ ms}^{-1}$ (Figure 2h). Layer averages used for $(\langle \Delta u'^2 \rangle, \langle \Delta v'^2 \rangle)$ estimates have more degrees of freedom than the single bins plotted in Figures 11–13. Nevertheless, the smaller values were barely above the noise floor, and unbiased estimators produced a few (5%) negative $(\langle \Delta u'^2 \rangle, \langle \Delta v'^2 \rangle)$ estimates (not plotted). Bin averages collect many more degrees of freedom, were relatively stable, and never produced negative $(\langle \Delta u'^2 \rangle, \langle \Delta v'^2 \rangle)$.

Depth-dependent velocities displayed substantial alongshore-non-uniformity [for entire deployment, square root of rms $(\langle \Delta u'^2 \rangle)^{1/2}, (\langle \Delta v'^2 \rangle)^{1/2}, (\Delta u'^2)^{1/2} / (\langle u'^2 \rangle)^{1/2}$ and $\overline{(\Delta v'^2)^{1/2}}$ respectively 0.0069 ms$^{-1}$, 0.0056 ms$^{-1}$, 0.014 ms$^{-1}$ and 0.011 ms$^{-1}$].

Dimensionless depth dependence for eddy velocities ranged from 0.5 outside the surfzone to $<0.1$ inside the saturated surfzone, with the same trend observed for both $(\Delta u'^2)^{1/2} / (\langle u'^2 \rangle)^{1/2}$ and $(\Delta v'^2)^{1/2} / (\langle v'^2 \rangle)^{1/2}$ at all across-shore locations (unfilled symbols, Figure 9). A weak local maximum in depth dependence near the outer edge of the surfzone might result from enhanced generation of depth-dependence at the onset of breaking.

However, depth dependence then grows stronger for the smallest $H_{nd}/h$, possibly owing to density stratification. Eddy depth-dependence exceeded that of the alongshore current (compare blue, green, and black symbols, Figure 9), and also exceeded simulated $\lambda_b / \lambda_m$ (Figure 10), consistent with theory for eddies generated by a mixture of rotational and irrotational forcing (Case 3, Section 2.8), and inconsistent with theories for generation purely by shear production or rotational forcing (Cases 1 and 2). The high eddy depth dependence cannot be explained by instrument noise because variance estimators are unbiased.
Coherence between upper-layer velocities \(2[(u')_u, (v')_u]\) and lower-layer velocities \(2[(u')_\ell, (v')_\ell]\) approached 1 during storms, and dropped to about 0.75 under low-energy conditions (not shown). The coherence between \((\pi', \pi')\) and \((\Delta u', \Delta v')\) was below 0.5 (Figure 14a). Corresponding phases were usually less than 90 degrees outside the surfzone, indicating surface intensification, and exceeded 90 degrees inside the saturated surfzone, indicating bottom intensification (Figure 14b). Phases transitioned through 90 degrees in the outer surfzone, indicating that near-bed velocities lagged near-surface velocities. Neither bottom intensification, nor the observed lag of near-bed velocity, would be expected if vertical structure were primarily generated by bottom friction.

Consistent with assumed ordering, the neglected residuals \(R_{\Delta u'}\) and \(R_{\Delta v'}\) were usually smaller than retained terms in the surfzone [for both saturated and outer surfzones, both \(\|\hat{R}_{\Delta u'}^2\|^{1/2}/\|\lambda_m^2 \langle \Delta u'^2 \rangle \|^{1/2}\) and \(\|\hat{R}_{\Delta v'}^2\|^{1/2}/\|\lambda_m^2 \langle \Delta v'^2 \rangle \|^{1/2}\) were < 0.19 at all across-shore locations, not shown]. In contrast, outside the surfzone residuals became large \((\|\hat{R}_{\Delta u'}^2\|^{1/2}/\|\lambda_m^2 \langle \Delta u'^2 \rangle \|^{1/2}\) and \(\|\hat{R}_{\Delta v'}^2\|^{1/2}/\|\lambda_m^2 \langle \Delta v'^2 \rangle \|^{1/2}\) ranged from 0.38 to 0.69). The neglected terms were dominated by \((\partial \Delta u'/\partial t, \partial \Delta v'/\partial t)\). The importance of these terms could be anticipated from the fact that the mixing timescale \(\lambda_m^{-1}\) is comparable to the eddy period outside the surfzone (solid black curve, Figure 2i). Of the other residual terms outside the surfzone, \(\langle \Delta u \rangle \partial \bar{u}' / \partial x\) and \(\langle \Delta u \rangle \partial \bar{v}' / \partial x\) were moderately small (< 0.31 of retained terms), and other terms were smaller (< 0.15 of retained terms).

5. Summary and Discussion

A two-layer analysis highlights the role of depth-dependent mixing in controlling the depth-dependence of nearshore currents and eddies. Like previous models [e.g. Reniers et al., 2004; Feddersen and Trowbridge, 2005; Uchiyama et al., 2010], this analysis incor-
porates breaker-injected turbulence. Breakers increase mixing at mid depths, relative to mixing near the bed, homogenizing surfzone velocity profiles. Observed depth dependence of currents was consistent with this prediction. For alongshore currents, theory predicts that opposite forces on lower and upper layers (bottom friction, and wind or wave forcing) generate depth dependence, and are balanced by interfacial turbulent mixing. For across-shore currents, depth-uniform horizontal pressure gradients oppose the depth-averaged flow, but not the depth-dependent flow, increasing the dimensionless depth dependence (i.e the ratio between depth-dependent and depth-average velocities), consistent with previous models [e.g. Svendsen et al., 1987].

For eddies, depth dependence also declined with increasing breaking. Observed eddy depth-dependence consistently exceeded that of the alongshore current. This observation was inconsistent with theory for eddies generated by shear production but might be explained if surfzone eddies were wave-forced, given forcing that included a significant divergent component. This is because pressure opposes the depth-averaged, but not the depth-dependent, eddy response to divergent forcing. Observed phases between depth-averaged and depth-dependent eddy velocities differed from expectations for depth dependence generated by bottom friction, but might be explained by forcing. This paper emphasizes the transition to depth-uniformity with increasing wave breaking, but outside the surfzone stratification likely played an important role.

The diffusivity for surfzone-scale lateral mixing is often $O(1 \, \text{m}^2\text{s}^{-1})$ [Spydell et al., 2007; Clark et al., 2010]. This mixing is likely dominated by essentially depth-uniform eddies, although shear dispersion [Svendsen and Putrevu, 1994; Pearson et al., 2009] associated with depth dependent currents and eddies might also play a role. Using velocities mea-
sured at a single depth, Clark et al. [2010] estimated that the shear dispersion contribution was small in a natural surfzone. From bin averaging, our observations suggest along- and across-shore diffusivities for shear dispersion by currents, equal to $(\langle \Delta u \rangle^2/\lambda_m, \langle \Delta v \rangle^2/\lambda_m)$, were both $O(10^{-1} \text{m}^2\text{s}^{-1})$ outside the surfzone, and $O(10^{-2} \text{m}^2\text{s}^{-1})$ inside. Such small diffusivities suggest that shear dispersion played a small role in the surfzone of this beach, although we cannot rule out shear dispersion as a significant factor outside the surfzone.

For both mean currents and eddies, surfzone observations were consistent with predicted dominance of depth-dependent momentum balances by steady-state forcing and vertical mixing (i.e. the terms included in steady 1dv models), with minimal role for lateral advection or time dependence. This dominance by forcing and steady-state mixing results from a short interfacial mixing timescale (tens of seconds). Such rapid mixing is consistent with previous estimates of surfzone eddy viscosity [e.g. applying (28) to eddy viscosities of Uchiyama et al., 2010; Clark et al., 2010]. Rapid mixing is also suggested by observations showing $<5$ s lag between passage of breaking waves and a resulting increase in vertical vorticity in the mid water column, albeit in just 1.2–2 m depth [Clark et al., 2012]. Outside the surfzone, the interfacial mixing timescale becomes comparable to an eddy period, and time dependence becomes important to eddy-frequency momentum balances. Previous 3d models for flow on beaches with shore-parallel sandbars have found a significant role for across-shore advection [Newberger and Allen, 2007a; Uchiyama et al., 2010]. In contrast, observations presented here, collected over relatively uniform depths in the absence of a sandbar, suggest only a small role for for lateral advection. This contrast might be explained if the sharp lateral gradients associated with sandbars increase the importance of lateral advection.
The trends reported above are based on bin-averaging, and individual 1-hour averages showed significant scatter about these trends. A variety of neglected processes, possibly including alongshore variability, might be responsible for this scatter. Depth-averaged across-shore currents and depth-dependent currents and eddies often showed clear alongshore variability, whereas depth-averaged alongshore currents and eddies were more alongshore-uniform.

Appendix A: Craik-Leibovich Forcing

The mean CL term \( \langle \vec{u}_S \times \vec{\zeta} \rangle = \langle \vec{u}_S \rangle \times \langle \vec{\zeta} \rangle + \langle \vec{u}_S' \times \vec{\zeta}' \rangle \) is here shown to be negligible under the assumed ordering (here \( \vec{\zeta} \) is the vorticity; similar analysis can establish that eddy-frequency fluctuations in CL forcing are also negligible). Since \( (u', v', w') \leq (\langle u \rangle, \langle v \rangle, \langle w \rangle) \) and \( (u'_S, v'_S, w'_S) \ll (\langle u_S \rangle, \langle v_S \rangle, \langle w_S \rangle) \), we will evaluate only \( \langle \vec{u}_S \rangle \times \langle \vec{\zeta} \rangle \). Using alongshore uniformity, the mean vorticity is

\[
\langle \vec{\zeta} \rangle = -\frac{\partial \langle v \rangle}{\partial z} \vec{x} + \left( \frac{\partial \langle u \rangle}{\partial z} - \frac{\partial \langle w \rangle}{\partial x} \right) \vec{y} + \frac{\partial \langle v \rangle}{\partial x} \vec{z},
\]

(A1)

where \( \vec{x}, \vec{y}, \vec{z} \) are unit vectors. The alongshore component of \( \langle \vec{u}_S \rangle \times \langle \vec{\zeta} \rangle \) is

\[
(\langle \vec{u}_S \rangle \times \langle \vec{\zeta} \rangle)_y = -\langle u_S \rangle \langle \zeta_z \rangle + \langle w_S \rangle \langle \zeta_x \rangle,
\]

(A2)

where \( \zeta_x \) and \( \zeta_z \) are horizontal and vertical components of \( \vec{\zeta} \). From (A1) and (A2)

\[
(\langle \vec{u}_S \rangle \times \langle \vec{\zeta} \rangle)_y = -\langle u_S \rangle \frac{\partial \langle v \rangle}{\partial x} - \langle w_S \rangle \frac{\partial \langle v \rangle}{\partial z},
\]

(A3)

which represents advection of alongshore momentum by the Stokes drift [Uchiyama et al., 2009, found a depth-averaged form of this result]. Noting that \( \partial \langle v \rangle / \partial z \approx 4 \langle \Delta v \rangle / h \) [from (10)] and \( w_S = O(u_S H/L) \) (from non-divergence of the mean Stokes drift), using (31)–
(48) yields
\[
\langle \langle \vec{u}_S \times \vec{\zeta} \rangle \rangle_y = -\frac{\epsilon^3 \epsilon^2}{L} [\langle \bar{u} \rangle + O(\epsilon)] \left[ \frac{\partial \langle \bar{v} \rangle}{\partial x} + 4 \epsilon (\sigma + 0.5) \frac{\partial \langle \Delta v_x \rangle}{\partial x} \right] - \frac{\epsilon^4 \epsilon^2}{L^4} 4 \langle w_S \rangle \langle \Delta v_x \rangle,
\]
(A4)

where the \( O(\epsilon) \) term in the first brackets represents depth-dependence of the Stokes drift.

The depth-uniform \( O(\epsilon^3) \) component of (A4) is smaller than resolved terms in (56) [if (56) were carried to the next order, this advection by the Stokes drift would roughly cancel Eulerian advection]. The depth-dependent \( O(\epsilon^4) \) components of (A4) are smaller than the \( O(\epsilon^3) \) residual \( R_\langle \Delta v \rangle \) in (61).

Appendix B: Perturbation expansion for depth-dependent mean alongshore momentum equation

Time-averaging (17) yields
\[
\frac{\partial}{\partial x} [h(\langle \bar{u} \rangle \langle \Delta v \rangle + \langle \bar{u}' \rangle \langle \Delta v' \rangle + \langle \bar{v} \rangle \langle \Delta u \rangle + \langle \bar{v}' \rangle \langle \Delta u' \rangle)]
+ \langle \langle v \rangle \langle \omega \rangle + \langle v' \rangle \langle \omega' \rangle + \langle \tau_y \rangle \rangle|_{\sigma = 0} + \langle \tau_y \rangle \rangle|_{\sigma = -1}
- 2 \langle \langle \bar{v} \rangle \langle \omega \rangle + \langle \bar{v}' \rangle \langle \omega' \rangle + \langle \tau_y \rangle \rangle|_{\sigma = -0.5} = h \langle \Delta Y \rangle,
\]
(B1)

where temporal and alongshore derivatives have been eliminated using stationarity and alongshore uniformity. Using (21), (24) – (25) and (31) – (54),
\[
\frac{\partial}{\partial x} [h_\epsilon (\epsilon^4 \langle \bar{u} \rangle \langle \Delta v \rangle + \epsilon^5 \langle \bar{u}' \rangle \langle \Delta v' \rangle + \epsilon^3 \langle \bar{v} \rangle \langle \Delta u \rangle + \epsilon^5 \langle \bar{v}' \rangle \langle \Delta u' \rangle)]
+ \epsilon^3 \langle \langle \bar{v} \rangle \rangle + \epsilon^5 \langle \langle \bar{v}' \rangle \langle \Delta u' \rangle \rangle|_{\sigma = 0} + \epsilon^5 \langle \langle \bar{v}' \rangle + 2 \epsilon \Delta v \rangle \langle \omega \rangle |_{\sigma = 0} - \epsilon^2 \frac{h \lambda_{bs} \lambda_{ms} \epsilon}{2} \langle \bar{v} \rangle - \epsilon \langle \Delta v \rangle
- 2 \epsilon^3 \langle \langle \bar{v} \rangle \rangle + \epsilon^5 \langle \langle \bar{v}' \rangle \langle \omega \rangle \rangle|_{\sigma = -0.5} + 2 \epsilon^2 \frac{h \lambda_{ms} \epsilon}{2} \langle \Delta v \rangle = \epsilon^2 h_\epsilon \langle \Delta Y \rangle,
\]
(B2)

where we have used \( v|_{\sigma = 0} = \bar{v} + 2 \Delta v \) (10). Truncating at \( O(\epsilon^3) \) yields
\[
- \frac{\lambda_{bs}}{2} \langle \bar{v} \rangle + \lambda_{ms} \langle \Delta v \rangle - \langle \Delta Y \rangle = \epsilon R_\langle \Delta v \rangle,
\]
(B3)
where the dimensionless residual
\[
R(\Delta v_*) = -\left( \frac{1}{h_*} \right) \frac{\partial h_* \langle \bar{v}_* \rangle \langle \Delta u_* \rangle}{\partial x_*} - \frac{\langle \bar{v}_* \rangle(\langle \omega_* \rangle|_{\sigma=0} - 2\langle \omega_* \rangle|_{\sigma=-0.5}) - \frac{\lambda_{h_*}}{2}\langle \Delta v_* \rangle. \tag{B4}
\]
Rearranging (B4)
\[
R(\Delta v_*) = -\langle \Delta u_* \rangle \frac{\partial \langle \bar{v}_* \rangle}{\partial x_*} - \frac{\lambda_{h_*}}{2}\langle \Delta v_* \rangle - \frac{\partial h_* \langle \Delta u_* \rangle}{\partial x_*} + \langle \omega_* \rangle|_{\sigma=0} - 2\langle \omega_* \rangle|_{\sigma=-0.5} \tag{B5}
\]
which, using the time average of (18), simplifies to
\[
R(\Delta v_*) = -\langle \Delta u_* \rangle \frac{\partial \langle \bar{v}_* \rangle}{\partial x_*} - \frac{\lambda_{h_*}}{2}\langle \Delta v_* \rangle. \tag{B6}
\]
Returning to dimensional variables, (B3) and (B6) become (61) and (63).

Appendix C: Test of ADP velocity measurements under intense wave breaking

On Agate Beach, Oregon, during 16–17 January 2010, two 2 MHz Nortek Aquadopp ADPs were deployed within several meters of a Nortek Vector Acoustic Doppler Velocimeter (ADV) (Figure 15a). Instruments were deployed and recovered during successive low low tides, and intense wave breaking was observed over the instruments throughout the deployment. The mean water depth reached just over 2 m (Figure 15b), and offshore significant wave height increased from 3.5 m (when wave image of Figure 15a was recorded) to 6.2 m (National Data Buoy Center, Station 46050). One ADP quickly ceased recording owing to a bad battery. The remaining ADP yielded mean currents, mean wave angles, and eddy velocity statistics in agreement with adjacent ADV measurements (Figure 15c–g).

Appendix D: Estimation of bed elevation from ADP backscatter intensity

Owing to reflection from the water surface, ADP profiles of backscatter intensity often showed a peak at a range \( \eta - z_A \) above the instrument (labeled 1 in Figure 16a), where
$z_A < 0$ is the ADP elevation. A second peak at range $2(\eta - z_A) + (z_A - z_B)$ (labeled 2 in Figure 16) resulted from multiple reflection (from the water surface, from the bed, then again from the water surface, and back to the ADP, here $z_B =$ bed elevation). For every second and every rangebin, we calculated a coordinate

$$\xi = \text{range} - 2(\eta - z_A)$$

(D1)

using 1 Hz $\eta$ estimated from pressure and linear theory. After removing the mean decay with range, backscatter was interpolated onto a regular $\xi$ grid. The multiple reflection then appeared as a peak at constant $\xi$ in two-minute running mean backscatter (labeled 2 in Figure 16b). Each day, an ADP elevation above the bed $z_A - z_B$ was calculated as the $\xi$ value associated with this peak. Outside the surfzone, or in the outer surfzone, this procedure often yielded clear bed elevation estimates, but far inside the surfzone estimates were often not obtained because clear reflection peaks could not be identified (the peak at range $\eta - z_A$ caused by single reflection from the surface was also unclear far inside the surfzone, possibly owing to the presence of bubbles). Values of $z_A - z_B$ obtained on days with clear multiple reflections, which ranged between 0 and 1 m at X3–X7 (with 99% of values $< 0.6$ m), were interpolated using splines to yield continuous time series. Estimated $z_A - z_B$ were consistent with diver measurements made during instrument deployment, and in cases where ADPs were buried by bed accretion, estimated $z_A - z_B$ smoothly approach zero prior to burial.

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Figure 1. Seabed elevations (m, NAVD88) measured on yeardays 290 (a) and 319 (b).

Red dots mark locations of ADPs X1–X7 and Y1–Y6.
Figure 2. Time series observed during deployment. (a) Wind velocity (grey: seaward, black: alongshore). (b) Density difference from surface to 6 m depth. (c) Wave height at Y1–Y2 (grey) and in 8 m depth (black). (d) Wave angle at Y1–Y2. (e)–(h): velocities, across-shore grey, alongshore black, averaged over functioning instruments from alongshore array Y1–Y6. (e) Depth-averaged current, (f) depth-dependent current, with red dotted line indicating prediction (67), (g) Standard deviation of depth-averaged eddy velocity, (h) Standard deviation of depth-dependent eddy velocity, (i) Estimated mixing parameters $\lambda_b$ (dashed black), $\lambda_w$ (grey) and $\lambda_m = \lambda_w + 6\lambda_b$ (solid black) at Y1–Y2.
Figure 3. Rectified photographs of surfzone during low (a, yearday 300), moderate (b, yearday 302) and high (c, yearday 309) energy conditions. Red circles show locations X1–X7 and Y2–Y6. Imaged region extends 600 m alongshore by 480 m across-shore.
Figure 4. Across-shore profiles measured 7–8 am, yearday 300 (circles, dashed lines), 5–6 am, yearday 301 (crosses, solid lines) and 12–1 am, and yearday 309 (pluses, dotted lines). (a): Elevation $z$ of seabed (black) and mean sea surface (grey). (b): Significant wave height $H_s$ (black) and shoaled non-dissipative wave height $H_{nd}$ (grey). (c): wave angle $\theta$. (d): depth-averaged alongshore current $\langle \mathbf{v} \rangle$. (e): standard deviation of depth-averaged eddy frequency across-shore (grey) and alongshore (black) velocity.
Figure 5. Ratio between observed and nondissipative wave heights \([H_s/H_{nd}, (79)]\) versus dissipation parameter \(H_{nd}/h\). Data binned by \(H_{nd}/h\), each symbol a bin average of all data at one ADP. Grey curve indicates saturated breaking condition \(H_s = \gamma h\). Vertical dotted lines indicate boundaries between outside surfzone \((H_{nd}/h < 0.35)\), outer surfzone, and saturated surfzone \((H_{nd}/h > 0.6)\).
Figure 6. Profiles of across-shore (a–g) and alongshore (h–n) mean current, 7–8 am, yearday 300 (significant wave height 0.30 m in 8 m depth). Black circles: currents from ADPs X1–X7, (Figure 1), from X1 on left (panels a,h) to X7 on right (panels g,n). Light grey curves, panels d,k: currents from alongshore array (Y1, Y3–Y6). Thick grey horizontal lines indicate seabed and mean sea surface.
Figure 7. As Figure 6, but for 5–6 am, yearday 301 (significant wave height 1.7 m in 8 m depth).
Figure 8. As Figure 6, but for 12–1 am, yearday 309 (significant wave height 4.0 m in 8 m depth).
Bin-averaged dimensionless depth-dependence of mean currents and eddies versus dissipation parameter $H_{nd}/h$. Filled black and large pink symbols respectively $\langle \Delta v \rangle / \langle \bar{v} \rangle$ and $\langle \Delta u \rangle / \langle \bar{u} \rangle$. Dashed red lines: $2 \tan(\theta) \langle \Delta u \rangle / \langle \bar{v} \rangle$ (70). Unfilled blue and green symbols $\langle \Delta u'^2 \rangle^{1/2} / \langle \bar{v}^2 \rangle^{1/2}$ and $\langle \Delta u'^2 \rangle^{1/2} / \langle \bar{u}^2 \rangle^{1/2}$ (blue and green curves coincide and are difficult to distinguish, indicating very similar scaling for eddy across- and along-shore velocities). Each symbol represents a bin average, with only bins containing $\geq 8$ data points plotted. Vertical dotted lines indicate boundaries between outside surfzone ($H_{nd}/h < 0.35$), outer surfzone, and saturated surfzone ($H_{nd}/h > 0.6$).
Figure 10. Bin-averaged dimensionless observed depth dependence $\langle \Delta v \rangle / \langle \overline{v} \rangle$ (filled black symbols), $\langle \Delta v'^2 \rangle^{1/2} / \langle \overline{v'^2} \rangle^{1/2}$ (unfilled black) and $\langle \Delta u'^2 \rangle^{1/2} / \langle \overline{u'^2} \rangle^{1/2}$ (unfilled grey), versus ratio $\lambda_b / \lambda_m$ between simulated near-bed and interfacial mixing parameters. For mean current $\langle \Delta v \rangle / \langle \overline{v} \rangle$, dashed red line indicates agreement with (67).
Figure 11. Profiles of across-shore (a–g) and alongshore (h–n) root-mean-squared eddy velocity, 7–8 am, yearday 300 (significant wave height 0.30 m in 8 m depth). Black circles: calculated from raw variance, across-shore array X1–X7, with onshore instrument X1 on left (a,h), and offshore instrument X7 on right (g,n). Black crosses: calculated from covariance between adjacent rangebins. Grey curves, panels d,k: alongshore array Y1, Y3–Y6. Thick grey horizontal lines indicate seabed and mean sea surface.
Figure 12. As Figure 11, but for 5–6 am, yearday 301 (significant wave height 1.7 m in 8 m depth).
Figure 13. As Figure 11, but for 12–1 am, yearday 309 (significant wave height 4.0 m in 8 m depth).
Figure 14. Bin-averaged coherence and phase between depth-averaged and depth-dependent eddy velocities. Black: between $\Delta u'$ and $\overline{v}'$. Grey: between $\Delta u'$ and $\overline{u}'$. Zero phase indicates surface intensification, $180^\circ$ phase indicates bottom intensification, positive quadrature indicates lag of near-bed velocity.
Figure 15. ADP test deployment on Agate beach, Oregon. (a): Deployment location photographed early in deployment, inset shows ADV and ADPs displaced alongshore, photographed at low tide. (b) Hourly mean water depth at deployment location. (c)–(g): Comparison of ADV (solid) and ADP (dashed, from rangebin overlapping ADV sampling elevation) measurements of (c) hourly mean current (black seaward, grey alongshore), (d) mean wave angle, (e) variance of across-shore eddy velocity, (f) variance of alongshore eddy velocity, (g) covariance between across- and along-shore eddy velocity.
Figure 16. Backscatter intensity measured at X4, yearday 300. (a): backscatter versus range, showing surface reflection (labeled 1), and first multiple reflection from surface and bed (labeled 2). Grey dashed line: range to surface, estimated from 2-minute running mean pressure. (b): backscatter, with mean range-dependent attenuation removed, as a function of coordinate $\xi$ (D1), showing surface reflection (labeled 1), and first multiple reflection from surface and bed (labeled 2). Black line at $\xi = 0.30$ m: estimated bed elevation. Grey dashed line: $\xi$ coordinate of the first reflection from surface, inferred from pressure.