Internal wave dissipation and mean flows in a sloping, stratified lakebed boundary layer

Stephen M. Henderson
John Harrison
Bridget Deemer
Observations in Lacamas Lake, WA

“Hypereutrophic”
Motivation: Nutrient Pollution ($\text{NO}_3^-$)
Estimating pollution removal (denitrification)

Removal of NO$_3^-$ pollution

⇒ creation of N$_2$ in bed

⇒ Elevation near-bed N$_2$

Turbulent diffusivity (Henderson)

Chemical flux = $-D \frac{\partial C}{\partial z}$

Vertical gradient in concentration (Harrison and Deemer)
Fluctuations in turbulent mixing

Layer-integrated measure of diffusivity

Big variations in near-bed mixing resulting from periodic stratification resulting from internal waves
Outline

1. Instrumentation
2. Lakewide internal waves
3. Boundary layer, deep lake
4. Boundary layer, thermocline
5. Summary/discussion of periodic stratification
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Tripods deployed on Lakebed

- Acoustic Doppler Profilers (1 & 2 MHz pulse-coherent Nortek Aquadopps)
- Acoustic Doppler Velocimeter (Nortek Vector)
- PME fast temperature
- Chemical sampling
- 1.5 m
- + Temperature loggers (RBR & Hobo)
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Full-depth velocity profiles

Acoustic Doppler Profiler (ADP)
Diurnal waves, upward phase propagation

Along-lake velocity (ms⁻¹)

Elevation above bed (m)

Time (day of 2011)
Diurnal waves, upward phase propagation

--- Theoretical internal wave propagation.

\[ c_z = 2\pi \sigma^2 \frac{\lambda_x}{\sqrt{N}}, \quad \sqrt{N} = \left[ -(g/\bar{\rho}) \frac{\partial \bar{\rho}}{\partial z} \right]^{1/2}, \quad \sigma = \text{frequency}, \quad \rho = \text{density} \]
Diurnal waves, upward phase propagation.

--- Theoretical internal wave propagation.

Fitted horizontal wavelength ($\lambda_x = 3000$ m) about twice lake length.
Diurnal waves, upward phase propagation.

- - - - - Theoretical internal wave propagation.
- - - - - Theoretical energy propagation.
Temperature Profiles

temperature transects measured along-and across-lake by repeatedly lowering and raising CTD from underway boat.
Along-lake: long wavelength
Across-lake: geostrophy

Geostrophy (thermal wind) observed for across-lake forces above bottom boundary layer

White circle: velocity into page
Black circle: out of page
Radius proportional to speed

\[ \frac{\partial u}{\partial z} = (\rho f)^{-1} g \frac{\partial \rho}{\partial y}, \]

Velocity gradient:
- Observed
- Inferred from thermal wind
Like seiches, the observed waves had wavelength exceeding lake length.

Unlike standard seiches, the observed waves propagated vertically.
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Unlike standard seiches, the observed waves propagated vertically.

Why?
Nodal structure?

- Velocity spectra peak at elevation 1m, with second peak ~4.5m. Antinodes?

spectra measured by upward- and downward-looking ADCPs
Blanked out elevations (1.5-1.8m) span location of ADCP
Nodal structure

- Fitting vertically-propagating waves to observations 1.7 – 4.6 m above bed \((A_{up}=R* A_{down})\), wavelength = 1284 m), reproduces observed power spectra.

Note reflection coefficient R is frequency-dependent fitting parameter - this asks what reflection coefficient best fits data

Diagram:

- Observed
- Fitted

Power (m^2 s^{-1})

Elevation (m)

Frequency (day^{-1})
Upward phase propagation

- Upward phase propagation consistent with downward energy propagation.
Partial reflection

Non-dissipative seiches would have $R=1$

Blue dots: fitted (ignore black line for now)
Energy balance

$F_I =$ Incident wave energy flux

$F_R =$ Reflected wave energy flux

$D =$ Turbulent dissipation

From hourly mean velocity, fitting waves to upward ADP measurements

$F_I - F_R = D$

From turbulent velocity statistics, downward ADP
Predicting Reflection

\[ R = \frac{\text{Reflected amplitude}}{\text{Incident amplitude}} \]

\[ R_{\text{pred}}(\sigma) = \frac{1 - \alpha}{1 + \alpha} \]

\[ \alpha = K \frac{u_{\text{rms}}}{c} \]

\[ K = 2 \left( \frac{8}{\pi} \right)^{1/2} C_D \]

- Reflection was weak because vertical wave speed small \((c=10^{-4}\text{ms}^{-1})\), because Lacamas lake is small.

- Published data indicates vertical propagation in some other small lakes.

I think this sort of vertical propagation might be widespread in small lakes Leading-order departure from standard seiche idea.